

NONLINEAR RESPONSE ANALYSES OF IRREGULAR GROUNDS BY USE OF THE PSEUDO-SPECTRAL METHOD

Makoto KAMIYAMA ¹, Keiichi FUKUCHI ²

ABSTRACT

This paper deals with nonlinear response analyses of irregular ground using the pseudo-spectral method. We formulated an iterative numerical method for analyzing nonlinear earthquake response of irregular grounds. It is based on the idea of the so-called equivalent linear method as well as on the efficiency of calculation peculiar to the pseudo-spectral technique. The method was applied to some model grounds with typically irregular structures to investigate the interaction between the material nonlinearity and configuration nonlinearity. It is concluded that the material nonlinearity diminishes the effects due to the irregularity of ground structures.

Keywords: Nonlinear, response analysis, irregular ground, pseudo-spectral method

INTRODUCTION

As a result of recent observations of strong ground motions, there is no doubt that highly soft soils demonstrate the material nonlinearity during strong motions. For example, the motion records obtained by a vertical array observation system at Port Island in Kobe City during the 1995 Kobe Earthquake showed remarkably nonlinear behaviors in the soft surface layers.

In the analyses of strong ground motions, however, the stress-strain relations of soils have been widely assumed to be linear. Especially seismologists generally dealt with strong ground motions due to the linear constitutive law of soil materials. As opposed to them, geotechnical engineers have assumed soft soils to behave nonlinearly during strong motions, based on their experimental data in laboratories. In the geotechnical engineering field, such nonlinear behaviors of soft soils during strong motions were a few decades ago accepted as a kind of common sense and various methods of response analyses have been developed to deal with nonlinear motions of grounds. Among these methods, the so-called equivalent-linear method, represented by the computer code SHAKE (Schnabel et al, 1972), have been popular with geotechnical engineers because of its simplicity. Despite being simplified, SHAKE sometimes produces analysis results relevant relatively to observed records. However, such a simplified method naturally includes some defects due mainly to its simplicity: responses in shorter period range are underestimated while peak acceleration is overestimated. This motivates us to develop more sophisticated method so as to match observed records while maintaining the simplicity of the equivalent linear method (Kamiyama and Matsukawa, 2002). The defects of SHAKE result from not only the simplicity employed in the stress-strain relation but also its another assumption that ground motions of horizontal layers are caused by vertically incident wave from the bed rock. Contrary to geotechnical engineers, seismologists have targeted strong motions of irregularly structured grounds not to be horizontally layered even though assuming their stress-strain relations are linear. As for the configuration of grounds, seismologists might have adopted a right way because grounds actually have irregular structures not to be horizontal.

¹ Professor, Department of Environmental Information Engineering, Tohoku Institute of Technology, Japan, Email: mkamiyam@tohotech.ac.jp

² Former Student, Department of Civil Engineering, Tohoku Institute of Technology, Japan.

The above discussions show that response analyses of strong ground motions should include not only the nonlinearity of soil materials but also the irregularity of ground structures. As described above, we already have developed a nonlinear analysis method for horizontally layered grounds so as to sophisticate the equivalent-linear method (Kamiyama and Matsukawa, 2002). The technique is basically due to the Pseudo-Spectral Method (PSM). PSM easily enables to expand response analyses from one-dimensional to two- or three-dimensional problems, which adapt with irregularly structured grounds. This paper presents a new analysis method of nonlinear response for irregular grounds using PSM. There are some trials that nonlinear analyses of irregular grounds were performed by the equivalent-linear method (Furumoto, Sugito and Yashima). These past trials have used constant material parameters that are not time-dependent. As opposed to such a treatment, this paper uses non-stationary parameters of materials.

PSEUDO-SPECTRAL METHOD AND ITS APPLICATION TO NONLINEAR PROBLEMS

PSM is a numerical method applied to analyses of ground motions using both techniques of the Fourier Transform (FT) and the Finite Difference Method (FDM). It solves the equations controlling ground motions with aid of FT in the space domain. At the same time, FDM is used in the time domain to provide solutions for the equations from time to time (Kamiyama et al, 2001). We present here an outline of the method for the anti-plane problem of ground motions; the SH wave problem in which irregularly structured grounds are dealt with in the two-dimensional space.

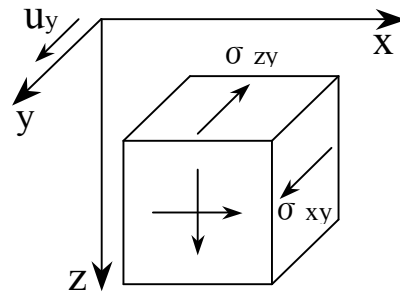


Figure 1. Motion parameters for the SH wave problem

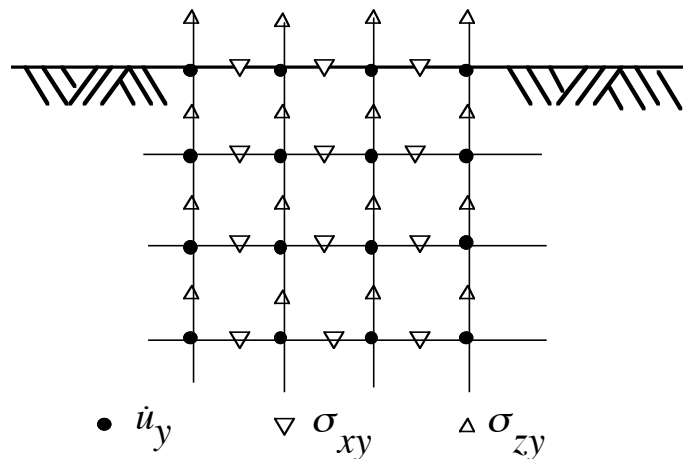


Figure 2. A schematic figure of staggered grid points

The SH wave problem consists of one displacement and two stresses controlling its motion equations. Figure 1 shows a schematic figure for a soil element subjected to SH wave motions. The motion equations can be expressed in terms of main parameters such as a particle velocity and stresses:

$$\frac{\partial}{\partial t} \mathbf{u} = \begin{bmatrix} 0 & 1/\rho & 0 \\ \mu & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{\partial}{\partial x} \mathbf{u} + \begin{bmatrix} 0 & 0 & 1/\rho \\ 0 & 0 & 0 \\ \mu & 0 & 0 \end{bmatrix} \frac{\partial}{\partial z} \mathbf{u} + \mathbf{f}_y \quad (1)$$

where $\mathbf{u} = [\dot{u}_y, \sigma_{xy}, \sigma_{zy}]$, \dot{u}_y is the particle velocity, σ_{xy} and σ_{zy} are the shear stresses in Figure 1, x and z are the spatial coordinates, respectively, in the horizontal and vertical directions. t is time, ρ is the density, μ is the rigidity of material and \mathbf{f}_y is the external force per the unit mass.

Equation 1 is a first order system of differential equations that can be solved numerically by use of PSM. The space and time in Equation 1 are first discretized. We made here the spatial discretization based on the staggered grid system, which alternates grid points halfway for the particle velocity and shear stresses to obtain numerically stable solutions. Figure 2 shows an example of the grid scheme made by the staggered system. As shown in Figure 2, grid points are different each for \dot{u}_y , σ_{xy} and σ_{zy} in the SH wave problem. Using a grid system such as in Figure 2, the spatial derivatives in Equation 1 are calculated by the FT technique in space while the temporal derivatives are approximated by the central difference. Such usual operations in the PSM method can provide a time stepping solution to Equation 1: the time histories of accelerations, velocities, displacements, strains and stresses are obtained at each grid point.

The PSM method outlined above takes no effects resulting from damping of materials into account. We can induce damping effects into the PSM method according to Graves (Graves, 1996). Graves showed that the effects of the damping factor of soils are induced into the time domain by applying an attenuation coefficient at each stepping time. The attenuation coefficient $A(x, z)$ is determined by the damping ratio of soil materials as follows:

$$A(x, z) = \exp(-2\pi h(x, z) f_o \Delta t) \quad (2)$$

where $h(x, z)$ is the damping ratio of soil materials at x and z , f_o is the central frequency of the motions and Δt is the stepping time.

Note here that the time stepping calculations by Equations 1 and 2 never require stationary values in time for the material parameters μ and h . This means that the PSM method provides a procedure for nonlinear response analyses. That is, the material parameters μ and h , which determine the relations between strains and stresses, can have time-dependent values in Equations 1 and 2. Therefore, provided that the time histories of absolute values of strains are given in addition to the strain-dependencies of μ and h , we can induce non-stationary values of μ and h so as to vary in time and depend on strains, leading to a numerical way of nonlinear response analyses. This is similar to the basic idea of the equivalent linear response analysis, expanded by the inducement of non-stationary variations to the rigidity and damping ratio. Accordingly, the iteration technique is effective to obtain final solutions of nonlinear responses, similarly to the equivalent linear method. The method outlined in the above is performed according to the flow chart shown in Figure 3. Each phase in the flow is as follows:

First, rigidities μ and damping ratios h are given at each grid point in reference to geo-physically surveyed data at the site in hand. At the same time, strain-dependent relations of μ and h are also set up at each grid point based on some experimental data relevant to the site. Then, a response analysis is conducted to obtain time histories of accelerations and strains at each grid point using the Pseudo-Spectral Method. The strain histories are further converted to their complex envelopes that give information for the time variations of absolute strains. We next estimate the time histories of μ and h

at each grid point in conjunction of the time variations of absolute strains with the strain-dependent relations of μ and h prescribed above. The parameters μ and h in this phase are renewed from the previous ones. They are no longer constant in time but are time-varying reflecting the time histories of absolute strains. These renewed parameters of μ and h are used in the next response and give new histories of accelerations and strains at each grid point. In the phase, the response results are judged on whether or not they converge with respect to the previous responses. If the judgment phase is not satisfied, we iterate each phase of the above until the responses converge within a set-up condition.

In the process of inducing strain-dependencies of rigidity and damping ratio for phase IV and phase V in Figure 3, we have two candidates of strains like e_{xy} and e_{yz} , as shown in Figure 1. This means that there are two ways of inducing the strain dependency: one is to induce the two strains independently and the other is to induce only the larger strain among the two components of strains. This choice depends on how to deal with the anisotropy of material dynamics. We here adopt the latter on the assumption that the strain dependencies of rigidity and damping ratio are isotropic. Therefore, the larger strain among the two strains determines the values of rigidity and damping ratio at each grid point and at each stepping time.

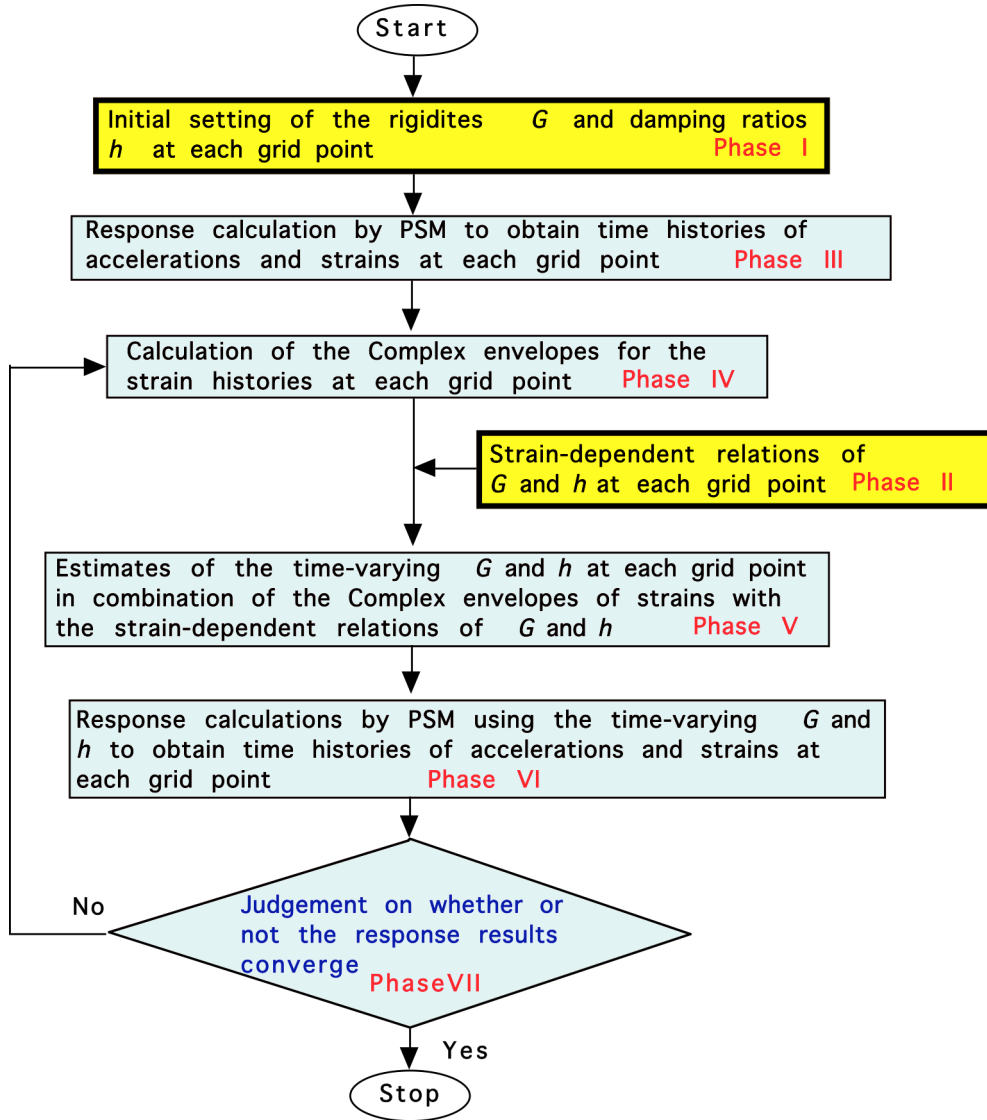


Figure 3. Flow chart of nonlinear response analysis by PSM

APPLICATION TO MODEL GROUNDS

In order to investigate its availability and efficiency, the present method of nonlinear response analysis was applied to various kinds of model grounds. We here give two representative cases of model grounds in consideration of paper space. The model grounds are illustrated in Figure 4. The No.1 ground is composed of one soft surface layer, which has a form of symmetric basin, over the bed half-space. The other model: the No.2 ground has an asymmetric basin over the bed half space. In these models, only the soft surface layer has a nonlinear constitutive law of material dynamics. The strain-dependent relations of rigidity and damping ratio assumed for the surface layer are shown in Figure 5. In addition to the strain dependencies of material parameters, S wave velocities and Q values for the surface and bed layers are also presented in Figure 4. Note here that the No.1 ground and No.2 ground have, respectively, different S wave velocities for the surface and bed layers; the contrast of S wave velocities between the surface and bed layers is more remarkable for the No.1 ground compared with the No.2 ground. We repeatedly simulated motion responses for both grounds with various kinds of incident waves. Among such simulations, we here present motion responses resulted from the vertically incident wave of Ricker wavelet with peak accelerations of 10, 50 and 100 cm/sec^2 and central period of 0.3 sec. The spatial interval of grid points and the stepping time were, respectively, set to be 2.5 m and 0.005 sec to satisfy the stability condition of the PSM analysis.

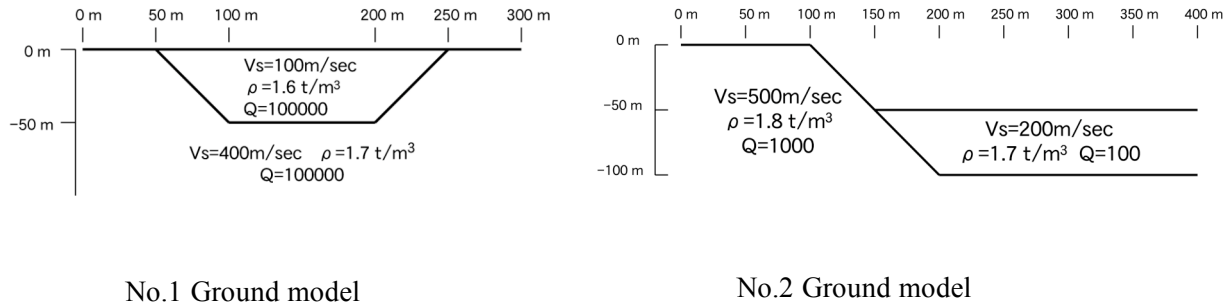


Figure 4. Analyzed model grounds

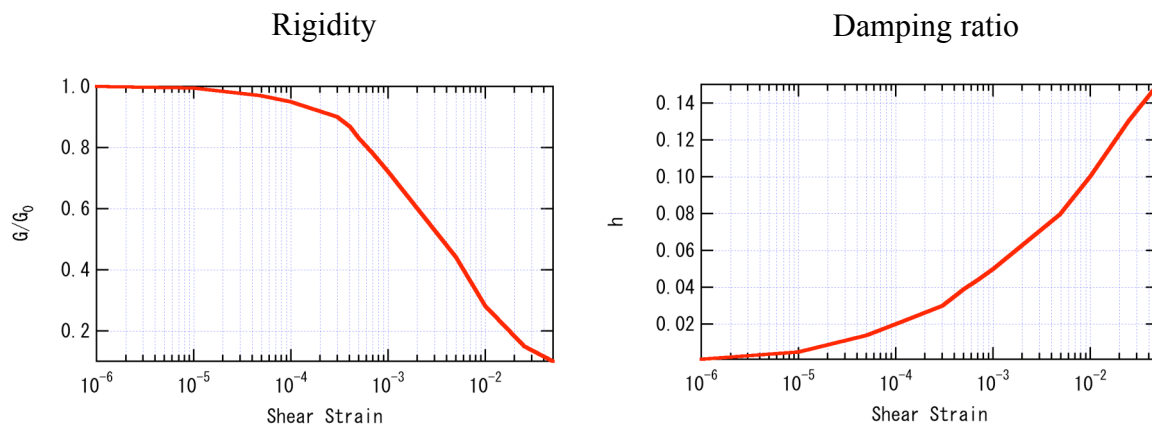


Figure 5. Strain dependencies of rigidity and damping ratio for surface layer

No.1 ground

The response analyses for the No.1 ground are shown in Figure 6. The left one in Figure 6 shows the linear response results obtained by assuming that the surface and bed layers are both linear for their

dynamic behaviors. On the other hand, the right one in Figure 6 is the nonlinear response in which the surface layer behaves according to the strain-dependencies shown in Figure 5. The time histories of accelerations obtained at representative points on the surface against the peak acceleration of 100 cm/sec^2 of incident wave are plotted for both of the linear and nonlinear cases in Figure 6. Both analyses were made to compare linear and nonlinear responses.

In the linear case of Figure 6, the irregularity of the surface layer plays an important role in generating some secondary waves that travel horizontally as a form of surface waves. The horizontally traveling waves overlap with the direct waves vertically propagating from the bed layer and reflect repeatedly at both edges of surface layer. They show little attenuation because of the less damping of the surface layer and give remarkably large amplitudes at points and at times where the direct waves overlap in almost the same phase. As opposed to such linear responses, the nonlinear results show less horizontally traveling waves because of their damping effects due to the strain dependency of damping ratio. To exemplify the difference between the linear and nonlinear responses, Figure 7 compares the motion responses obtained at the mid point of the symmetric basin for both cases. It is found in Figure 7 that the effects of material nonlinearity are more remarkable for the secondary waves, which appear in the later phases of the total motions, rather than the direct waves of vertical propagation apparent in the initial phases. This means that the geometrical irregular effects of ground structure; that is, the effects of the geometrical nonlinearity are weakened by the material nonlinearity. Figure 8 is a comparison of spectra between the linear and nonlinear responses. Figure 8 shows that ground motions generally attenuate in spectral amplitudes and lengthen in predominant periods as a result of the material nonlinearity.

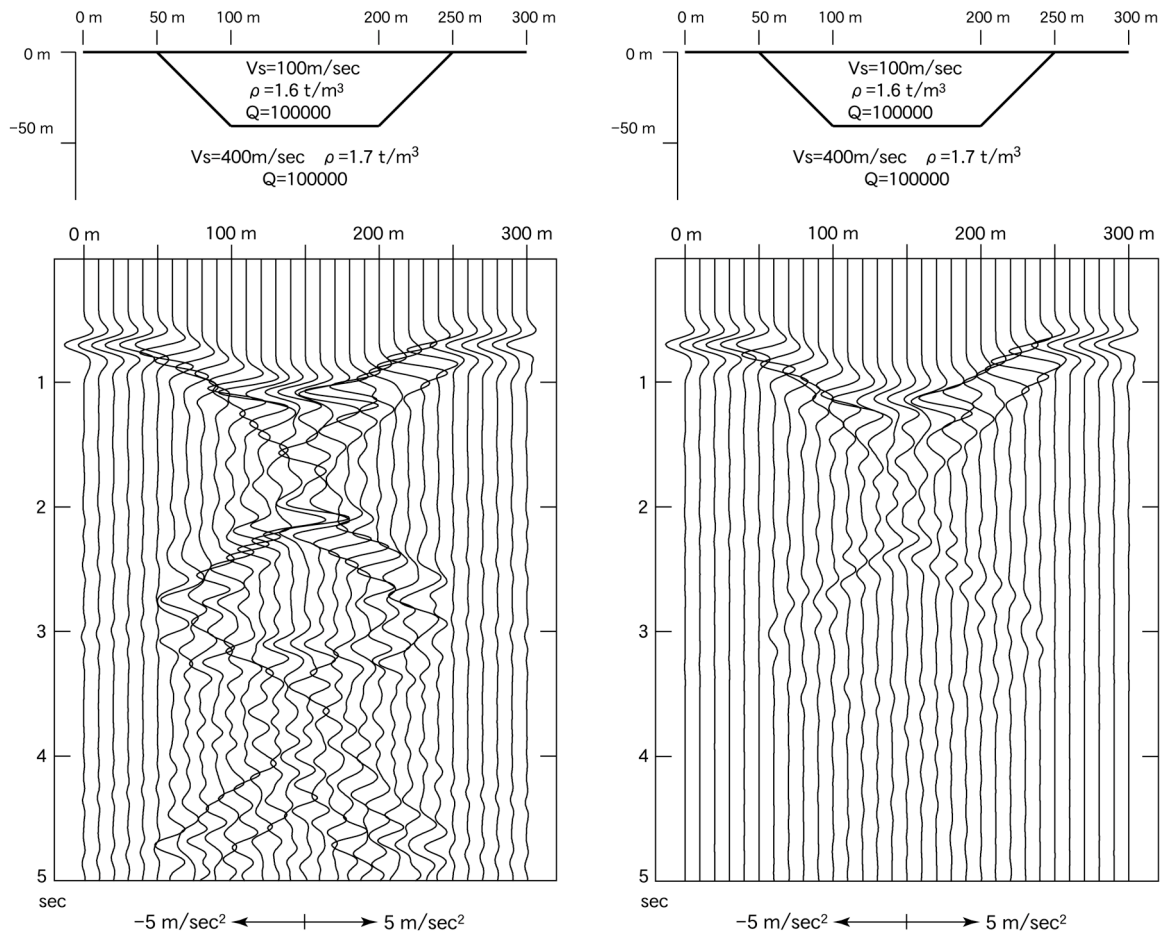


Figure 6. Response results for No.1 ground (left: linear response, right: nonlinear response)

The nonlinear effects of soil materials might be dependent on the amplitudes of incident motions. Figure 9 illustrates the nonlinear responses obtained by the three cases of peak amplitudes such as 10, 50 and 100 cm/sec^2 for the incident waves. It is shown in Figure 9 that the horizontal traveling waves described above attenuate in proportion to the amplitudes of incident waves. Figure 10 is a typical comparison of these nonlinear responses obtained at the mid-point of the basin. Figure 11 also illustrates comparatively the dynamic orbits of the stress-strain relations for strain e_{xy} and stress σ_{xy} estimated at the mid-point of the basin with the depth of 0 m. It is clear in Figures 10 and 11 that the greater the amplitudes of incident waves are, the smaller the rigidities μ become and the larger the damping ratios h are: resulting in that the horizontally traveling waves propagate with less speeds and less amplitudes in response to the amplitudes of incident waves.

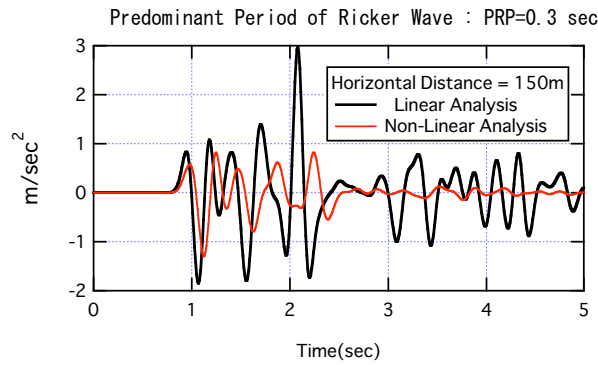


Figure 7. A comparison of responses between linear and nonlinear analyses

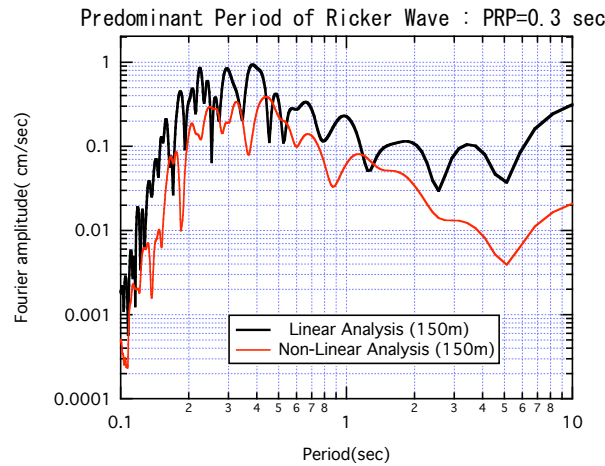


Figure 8. A comparison of response spectra between linear and nonlinear analyses

No.2 ground

The No.2 ground is a asymmetrical basin with less impedance ratio between the surface and bed layers compared with the No.1 ground model. The linear and nonlinear responses are compared in Figure 12. These responses resulted from the incident wave having the peak acceleration of 100 cm/sec^2 and central period of 0.3 sec as well as the strain-dependencies shown in Figure 5. In comparison with Figure 6, this case indicates little differences of the nonlinear responses against the linear responses in which the surface layer was assumed to behave linearly. Especially, the secondary waves due to the irregularity of basin structure exist with less attenuation for the nonlinear responses even though the

same amplitude of incident wave and same strain-dependencies of surface soils as in Figure 6. This means that responses due to the nonlinearity of soil materials for irregularly structured grounds are greatly affected by the irregularity itself as well as by the impedance between the surface and bed layers.

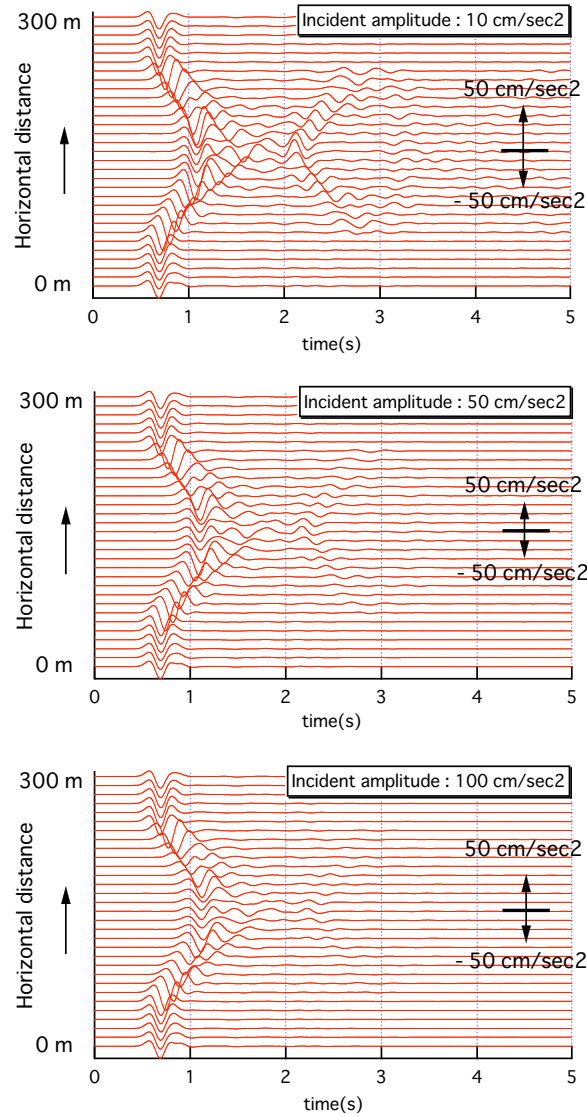


Figure 9. Nonlinear responses resulting from different amplitudes of incident waves

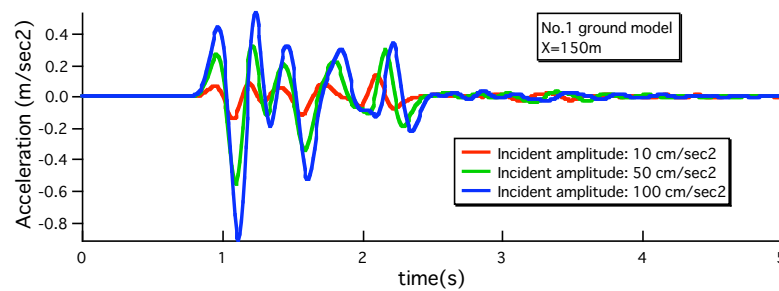


Figure 10. A comparison of nonlinear responses due to different amplitudes of incident waves

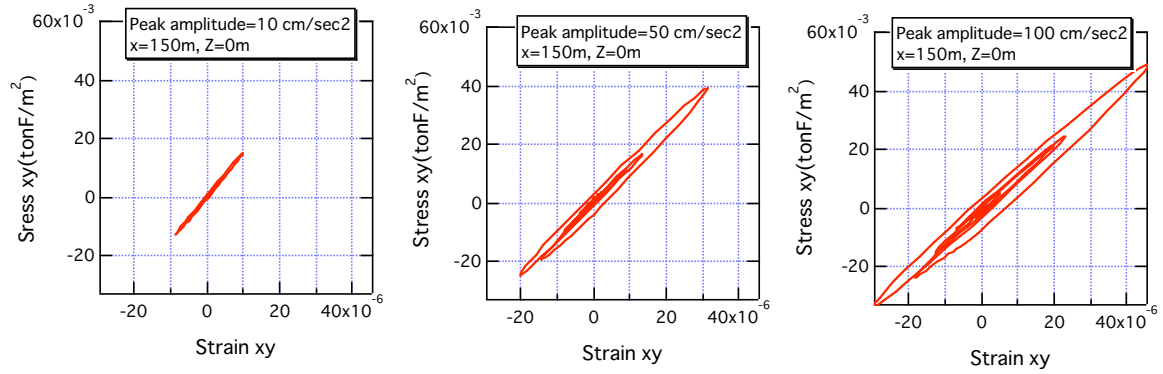


Figure 11. Stress-strain relations of nonlinear responses for different amplitudes of incident waves

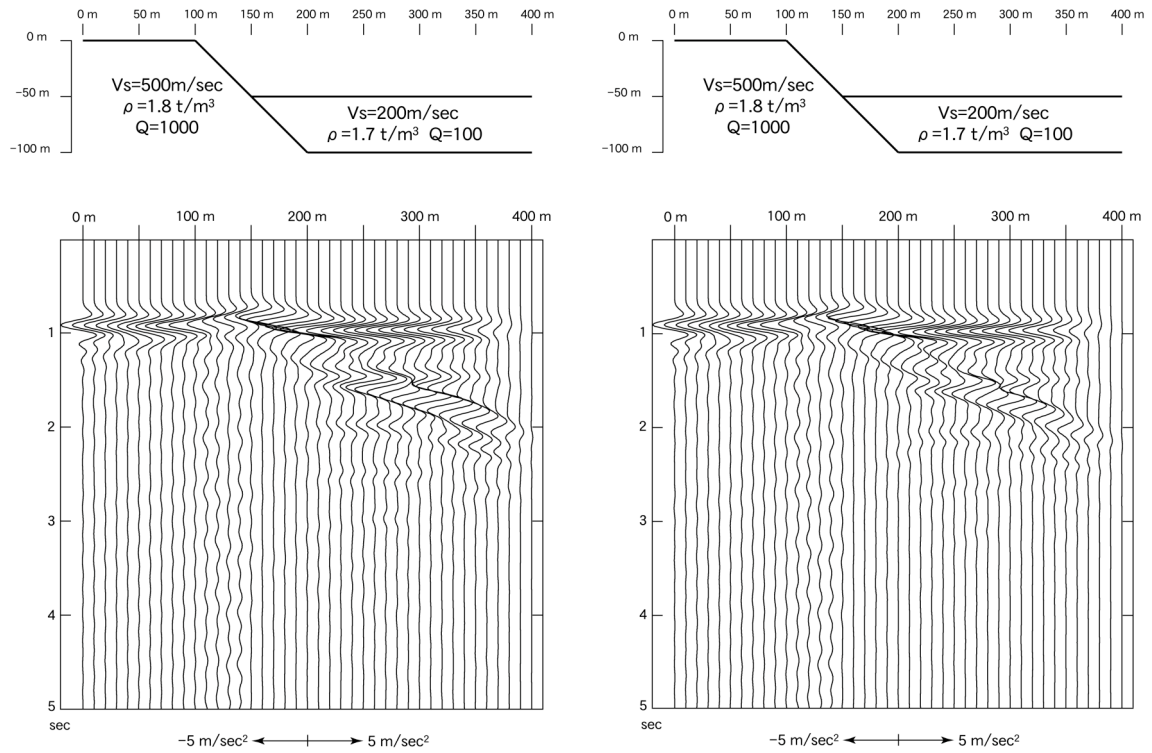


Figure 12. Response results for No.2 ground (left: linear response, right: nonlinear response)

CONCLUDING REMARKS

We presented a new method for analyzing nonlinear responses of irregularly structured grounds. This is based on the applicability of the Pseudo-Spectral Method. We can analyze easily nonlinear behaviors of any irregularly structured grounds using this method. In this paper, two types of model

grounds were dealt with to demonstrate the availability of this method. The analyzed results show that the secondary waves due to the irregularity of ground structure are especially affected by the material nonlinearity of soils. The material nonlinearity generally diminishes the effects of irregularly structured grounds. In addition, nonlinear responses of irregular grounds in geometry depend on effective factors such as: the impedance ratio between the surface layers and bed layer; the strain-dependencies of rigidity and damping ratio of soils; the amplitudes of incident waves in the bed layer; and the irregular configuration of grounds.

REFERENCES

- Kamiyama M. and Matsukawa T. "A new method for estimating nonlinear response of ground," Proceedings of the 12th European Conference on Earthquake, 1-10, 2002.
- Schnabel PB, Lysmer J, Seed HB. "SHAKE A Computer program for earthquake response analysis of horizontally layered sites," Report No. EERC72-12, Berkeley, University of California, 1972.
- Furumoto Y., Sugito M. and Yashima A., "Frequency-dependent equivalent linearized technique for FEM responses analysis of ground", Proceedings of the 12th World Conference on Earthquake Engineering, 1-10, 2002.
- Kamiyama M, Satoh T, Suzuki T. "Engineering effects of short-period surface waves in laterally inhomogeneous ground," Journal of Structural Mechanics and Earthquake Engineering, Proceedings of Japan Society of Civil Engineers , 682, 225-243, 2001 (in Japanese).
- Graves RW. "Simulating seismic wave propagation in 3D elastic media using staggered-grid finite differences," Bull Seism Soc Am, 86, 1091-1106. 1996.