

## **FIXED HEAD KINEMATIC PILE BENDING MOMENT TIME HISTORY: ARTIFICIAL NEURAL NETWORK APPROACH**

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### **ABSTRACT**

This paper is aimed at predicting kinematic bending moment at fixed head of pile embedded in homogeneous soil deposit overlying bedrock. Artificial Neural Network (ANN) is used as a tool for function approximation to achieve this objective. The data generated for training and testing the ANN models is based on Beam on Dynamic Winkler Formulation (BDWF). Two ANN models are developed namely ANN1 and ANN2. ANN1 predicts the pile head bending moment when bedrock is excited by natural frequency of the overlying soil deposit. ANN2 is applicable to a range of excitations frequencies. The inputs to ANN1 are the length to diameter ratio of pile and ratio of the elastic modulus of pile to that of soil. The inputs to ANN2 also include the frequency ratios. The output of ANN models is the normalized pile head bending moment. To predict full time history bending moment response at fixed pile head, the ANN2 model is subjected to three earthquakes time histories using three different soil profiles. The results of ANN2 model are then compared with BDWF solution. A good agreement is found between the two methods for the full range of time history. The ANN models can be used as preliminary design tool to find bending moment at fixed head pile at a selected frequency, most importantly at the fundamental frequency of the soil deposit.

Keywords: pile, Winkler foundation, artificial neural network, kinematic interaction

### **INTRODUCTION**

The seismic waves vibrate the soil which imposes curvatures in the piles. The piles thus develop bending moments along its length. The magnitude of this moment can sometimes be quite high and shall be evaluated as part of any dynamic soil pile structure interaction problem (Ahmad I. & Akhtar N.K. 2006). For fixed head piles embedded in homogenous soils, the kinematic bending moment are high at the fixed end of pile, particularly at frequencies closed to the fundamental frequency of the soil deposit. There is no simple solution available to estimate the kinematic

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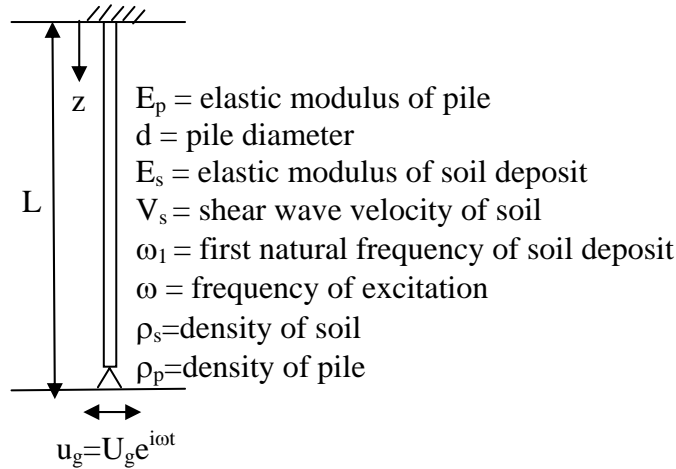
bending moment at the fixed head of pile. In this paper a simple solution is proposed based on feed forward back propagation Artificial Neural Network.

Two ANN models are developed in this paper to estimate normalized bending moment ( $M_n$ ) at the fixed head of pile. The normalized bending moment is  $M_n = M/(\rho_p a_{\text{rock}} d^4)$ , where  $M$  is the pile head bending moment,  $\rho_p$  is the pile density, and  $a_{\text{rock}}$  is acceleration at bedrock. The first model, ANN1 estimate  $M_n$  when the bedrock is excited with the fundamental frequency of the overlying soil deposit. The second model, ANN2 is applicable for a range of excitation frequencies. The data used for training and the testing the ANN models is based on Beam on Dynamic Winkler Formulation (BDWF). The BDWF is coded in MATLAB for data generation.

Three earthquake time histories are used to excite three different soil pile configurations, and the bending moment response at fixed pile head is determined through ANN2 model. The results of the ANN2 for full time history is in close agreement to the results obtained from BDWF method. The ANN models can, therefore, be employed as preliminary design tool to find bending moment at fixed pile head at selected frequencies, and can also be used to predict full time history response.

## PARAMETERS OF SEISMIC SOIL PILE INTERACTION

The layout of the pile soil system considered in this study is given in figure 1. The fixed head pile rests on rock formation, which is considered as a hinged support. The bedrock is excited by vertically propagating S-waves characterized by a harmonic displacement of  $u_g(t) = U_g e^{i\omega t}$ , where  $U_g$  is the ground displacement amplitude and  $\omega$  is the excitation circular frequency. The pile group effect is not considered as it plays a negligible role in kinematic interaction (Gazetas et al., 1992).



**Figure 1. Soil pile system**

The BDWF model is adopted in this paper to generate data for the training of ANN models. In all cases, soil damping ratio ( $\beta_s$ ) is considered to be 10 percent, which is compatible with the design level of strain in soil,  $\nu$  is considered to be 0.4 and  $\rho_p/\rho_s$  of 1.6. The ranges of input parameters selected to generate data for the ANN models and the corresponding output range are given in Tables 1. The considered ranges cover most of the practical situations.

**Table 1: Ranges of Parameters for ANN2 and ANN1 shown in parenthesis**

	$E_p/E_s$	$L/d$	Frequency ratio ( $a_0$ )	$\omega/\omega_1$	$M_n$
Minimum	100 (100)	10 (10)	0.01	0.0637	0.6641 (63.92)
Maximum	10000 (10000)	40 (40)	0.80	20.4	6083.2 (6133.9)

### BEAM-ON-DYNAMIC-WINKLER-FOUNDATION (BDWF) MODEL

Data for ANN models was generated by modeling the soil pile system as BDWF. The pile is connected to free field soil along its length through continuously distributed interface elements. The end of the interface element that is connected to the free field soil is excited by the corresponding free field displacement  $u_{ff}(z,t)$ .

#### Interface Element

Each interface element consists of a spring and a dashpot arranged in parallel. The spring stiffnesses ( $k_x$ ) is adopted from M. Kavvadas and G. Gazetas (1993). Thus  $k_x = \delta E_s$  where  $E_s$  soil modulus of elasticity of soil,  $\nu$  soil poisson's ratio, and

$$\delta = \frac{2}{1-\nu^2} \left( \frac{E_s d^4}{E_p I_p} \right) \left( \frac{L}{d} \right)^{1/8}$$

During soil pile interaction, the seismic energy is dissipated through hysteretic (material damping) and radiation (geometric damping). The former incorporates the internal energy dissipation in the soil, and is, thus related to soil damping ratio,  $\beta_s$  and the later is a geometric effect and represents the radiation of energy by waves spreading geometrically away from the pile soil interface. Hence the distributed dashpot/length of pile is,  $c_x = c_r + c_m$ , where  $c_r$ = distributed radiation dashpot coefficient and  $c_m$ = distributed material dashpot coefficient. In this study these coefficients are adopted from Gazetas, G. & Dobry, R. (1984a,b).

$$c_m = \frac{2k_x \beta_s}{\omega} \quad c_r = 2d\rho_s V_s \left( 1 + \left( \frac{V_c}{V_s} \right)^{\frac{5}{4}} \right) a_0^{-\frac{1}{4}}$$

where  $V_c$  is the apparent velocity of the extension compression waves taken as the Lysmer's analog velocity  $V_{La}$  and  $V_s$  is the shear wave velocity of soil under consideration.

$V_c = V_{La} = \frac{3.4V}{\pi(1-\nu)}$  at all depths except near the ground surface ( $z \leq 2.5d$ ), where three-dimensional effects arising from the stress-free boundary are better reproduced by use of  $V_c \equiv V_s$

### **Solution of differential equation governing kinematic response of piles**

The steady state pile displacement response to the harmonic excitation  $u_g(t)$  at the bedrock is governed by the following differential equation

$$U_{pp}''''(z) - \lambda^4 U_{pp}(z) = \alpha U_{ff}(z), \text{ where}$$

$$\lambda^4 = \frac{m_p \omega^2 - Sx}{E_p I_p}, \alpha = \frac{Sx}{E_p I_p}, Sx = K_x + ic_x \omega$$

$Sx$  is complex impedance function,  $m_p$  is the pile mass per unit length,  $I_p$  is the second moment of inertia of pile, and  $U_{ff}(z)$  and  $U_{pp}(z)$  displacement amplitudes of free field and pile at depth  $z$  respectively.

Differential equation has the general solution

$$U_{pp}(z) = [e^{-\lambda z} e^{\lambda z} e^{-i\lambda z} e^{i\lambda z}] \begin{Bmatrix} D1 \\ D2 \\ D3 \\ D4 \end{Bmatrix} + s U_{ff}(z)$$

$$s = \frac{\alpha}{q^{*4} - \lambda^4}, \text{ where } q^* \text{ is complex wave number} = \sqrt{\frac{\omega}{V_s + i\beta_s}}$$

$D_1, D_2, D_3$ , and  $D_4$  are arbitrary constants to be evaluated through pile boundary conditions.

### **Boundary Conditions**

The boundary conditions are: the rotation and shear at top of pile are zero i.e.  $\theta(0,t)=0$ ,  $V(0,t)=0$ , respectively and moment at pile base is zero,  $M(L,t)=0$ ; and displacement at pile base is equal to ground displacement at bedrock i.e.  $u_p(L,t)=u_g(t)$ .

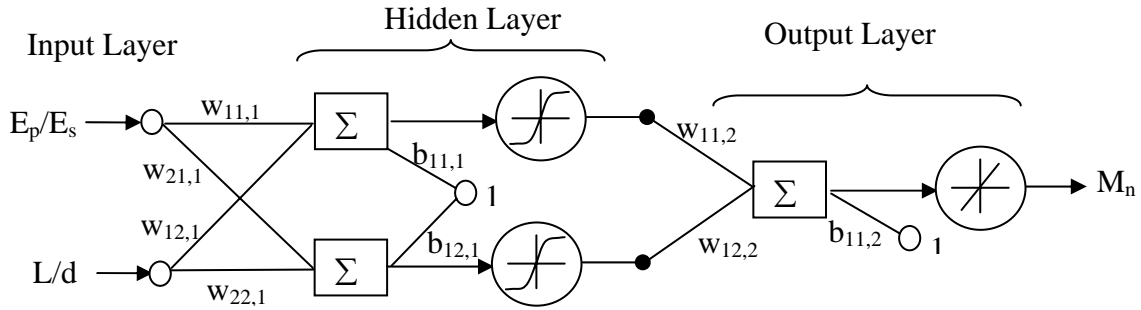
The solution of the differential equation is coded in MATLAB which first evaluates the free field displacements as a function of depth, their first, second, and third derivative. Determine four arbitrary constants through four boundary conditions and then evaluates normalized pile head bending moment  $M_n$ .

## **ARTIFICIAL NEURAL NETWORK (ANN) MODEL**

### **Architecture of ANN Model for Estimating Pile Head Moment**

Artificial neural network is a computational mechanism able to acquire, represent, and compute a mapping from one multivariate space of information to another, given a set of data representing that mapping (Garrett, 1994). The applications of the ANN can broadly be categorized as classification, data association, data conceptualization, and data filtering. One of the most common engineering applications of ANN is to use inputs to predict certain outputs. About 80% of neural network applications utilize back-propagation neural network for prediction. Back-

propagation ANN has been applied to various problems of civil engineering like predicting concrete shear capacity (Ahmad, 2005; Bohigas & Mari, 2004; Seleemah, 2005; El-Chabib, 2005), seismic liquefaction potential (Najjar and Ali, 1998), friction capacity of driven piles (Goh, 1995), overturning response of rigid block under near-fault type excitation (Gerolymos et al., 2005). A Levenberg–Marquardt back-propagation algorithm was used in this research. It is one of the fastest methods available for training moderate-sized feed-forward neural networks (Hagan et al. 1996). The architecture of ANN1 model consisting of an input layer of two input neurons, a hidden layer of two neurons, and an output layer consisting of one output neuron is shown in Figure 2. The symbols  $w$  and  $b$  in figure 2 represent connection and bias weights with subscripts representing the corresponding neurons between two layers.



**Figure 2: Artificial Neural Network Architecture for ANN1**

The design of a neural network may proceed as follow:

First, an appropriate architecture is selected for the neural network. In this study, two ANN models are developed i.e. ANN1, and ANN2. Each ANN model consists of an input layer, a hidden layer, and an output layer. The inputs to ANN1 model are  $E_p/E_s$  and  $L/d$  while ANN2 model uses four inputs  $E_p/E_s$ ,  $L/d$ ,  $a_o$ , and  $\omega/\omega_1$ . The input layer consists of nodes (neurons) equal in number to the input parameters. In the present case, the input layer of ANN1 model therefore consists of two input neurons, and that of ANN2 model consists of four input neurons. The output of the ANN models is  $M_n$  and therefore each ANN model has only one output neuron in the output layer. The output neuron is assigned a linear function (Figure 2). The hidden layer consists of hidden neurons. The hidden neurons have a nonlinear transfer function. The effectiveness of an ANN model to simulate highly nonlinear problems is attributed to the nonlinear transfer function. In this study a symmetric sigmoid (i.e. hyperbolic tangent) function is used in the hidden neurons. The number of hidden neurons plays a crucial role in ANN performance. In this study, two to ten neurons were considered for ANN models. Optimum numbers of neurons were found to be two and five respectively for ANN1 and ANN2. These neurons avoid underfitting i.e. large training and testing errors and prevent overfitting i.e. low training error but high testing error.

Second, a subset of examples is then used to train the network with the help of a suitable algorithm. In this study, the network is supplied with 50% of the total data for training, and the remaining 50% is reserved for testing the performance of the trained ANN (step 3 below). The network adjusts the connection weights and bias such as to reduce the error between the target,

i.e. known  $M_n$ , and the corresponding network output, using Levenberg–Marquardt back-propagation algorithm.

Third, the prediction performance of the trained network is tested with data not seen before (50% of the total data in this study). The network predicts the output using the connection weights and biases established in the training phase.

The data generated to train ANN1 and ANN2 are first randomized and then divided into two sets namely the training data set and the testing data set. The data was so divided so as to give comparable statistical properties for training and testing.

### Scaling of Training Data

Preprocessing of the training data is performed so that the processed data was in the range of -1 to +1. The training data sets (inputs and targets outputs) are scaled (preprocessed) according to

$$P_n = 2 \times \frac{(P - \min P)}{(\max P - \min P)} - 1 \quad (1)$$

$$T_n = 2 \times \frac{(T - \min T)}{(\max T - \min T)} - 1 \quad (2)$$

$P$  = matrix of the input vectors;  $T$  = matrix of the target output vectors;  $P_n$  = matrix of scaled input vectors;  $T_n$  = matrix of scaled target output vectors;  $\min P$  = vector containing minimum values of the original input;  $\max P$  = vector containing maximum values of the original input;  $\min T$  = vector containing the minimum value of the target output ( i.e. minimum value of  $M_n$  in the training dataset);  $\max T$  = vector containing the maximum value of the target output ( i.e. maximum value of  $M_n$  in the training dataset). The scaled data was then used to train the neural network. The data from the output neuron have to be postprocessed to convert the data back into unscaled units to get actual  $M_n$  value according to

$$T = 0.5 \times (T_n + 1)(\max T - \min T) + \min T \quad (3)$$

The maximum and minimum values of input and output vectors are given in table 1.

The training was carried out until the average sum squared error over all the training patterns was minimized. This occurred after about 1500 cycles of training. The connection and bias weights obtained after ANN training can be used to estimate the  $M_n$ . The weight and bias matrices obtained after the training phase of ANN models are  $W_1$  = weight matrix representing connection weights between the input layer neurons and hidden layer;  $W_2$  = weight matrix representing connection weights between the hidden layer neurons and the output neuron;  $B_1$  = bias vector for the hidden layer neurons;  $B_2$  = bias vector for the output layer neuron. These weight matrices for ANN1 and ANN2 models are given below.

*Weight and Bias Matrices for ANN1*

$$W_1 = \begin{bmatrix} -0.7452 & 0.0054 \\ 1.0848 & 0.0825 \end{bmatrix} \quad B_1 = \begin{bmatrix} -0.5887 \\ -0.9930 \end{bmatrix} \quad W_2 = [-1.2477 \quad 0.6479]$$

$$B_2 = [-0.1755]$$

*Weight and Bias Matrices for ANN2*

$$W_1 = \begin{bmatrix} -0.0466 & 0.6654 & -8.0697 \\ 1.0610 & -1.7342 & 2.6758 \\ -0.8282 & 1.9736 & -5.9451 \\ -1.1908 & 1.7099 & -2.6744 \\ 1.1245 & -1.7225 & 2.6786 \end{bmatrix} \quad B_1 = \begin{bmatrix} -13.0827 \\ 1.2521 \\ -0.4956 \\ -1.2072 \\ 1.2315 \end{bmatrix}$$

$$W_2 = [-470.7 \quad -765 \quad 1.5 \quad 755.8 \quad 1522.3] \quad B_2 = [-758.05]$$

### Procedure for estimating Pile Head Moment

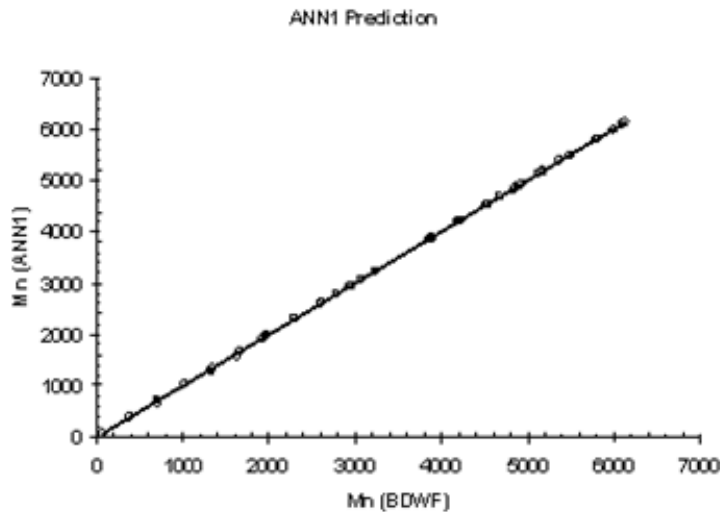
The ANN model described in this paper can be used to predict the  $M_n$ . The procedure can easily be programmed into a calculator capable of performing simple matrix operations. The input data is first preprocessed according to equation 1 to get scaled input vector  $P_n$ . The  $M_n$  is then obtained through the network as follows:

$$T_n = W_2 \{ \tanh(W_1 \times P_n + B_1) \} + B_2 \quad (4)$$

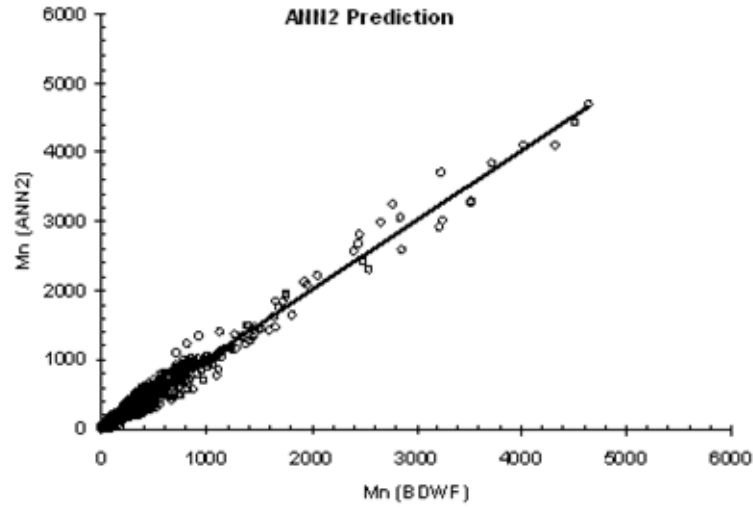
The scaled output  $T_n$  is then unscaled using equation 3 to obtain  $M_n$ .

### ANN Model Prediction at discrete frequencies

ANN models were presented with new data that was not part of the training data set, which form 50% of the corresponding total data, and the corresponding  $M_n$  calculated. Figure 3 and figure 4 show respectively the prediction of  $M_n$  by ANN1 and ANN2 plotted against the corresponding  $M_n$  calculated using BDWF. The correlation coefficients (R) for both ANN models are 0.99 for testing and training dataset.



**Figure 3 ANN1 Prediction**



**Figure 4. ANN2 Prediction**

#### **Time History response of ANN2 Model**

The performance of ANN2 model is evaluated for full time history range of earthquake signal. Three soil pile systems given in table 2 are subjected separately to three earthquake time histories which are the Tabas earthquake (Iran 1978-09-16), Northridge (1994-01-17) earthquake, and Loma Prieta (1989-10-18) earthquake. The bending moments at the fixed head of pile is compared with the corresponding BDWF solution in figure 5 to figure 7, which show close agreement between the two.

**Table 4: Soil Pile System**

Soil-Pile System	Soil Pile Parameters			
	L (m)	d (m)	$V_s$ (m/s)	$E_p$ (N/m <sup>2</sup> )
SP1	20	1.0	100	$30 \times 10^9$
SP2	40	1.5	80	$200 \times 10^9$
SP3	10	1.0	150	$200 \times 10^9$



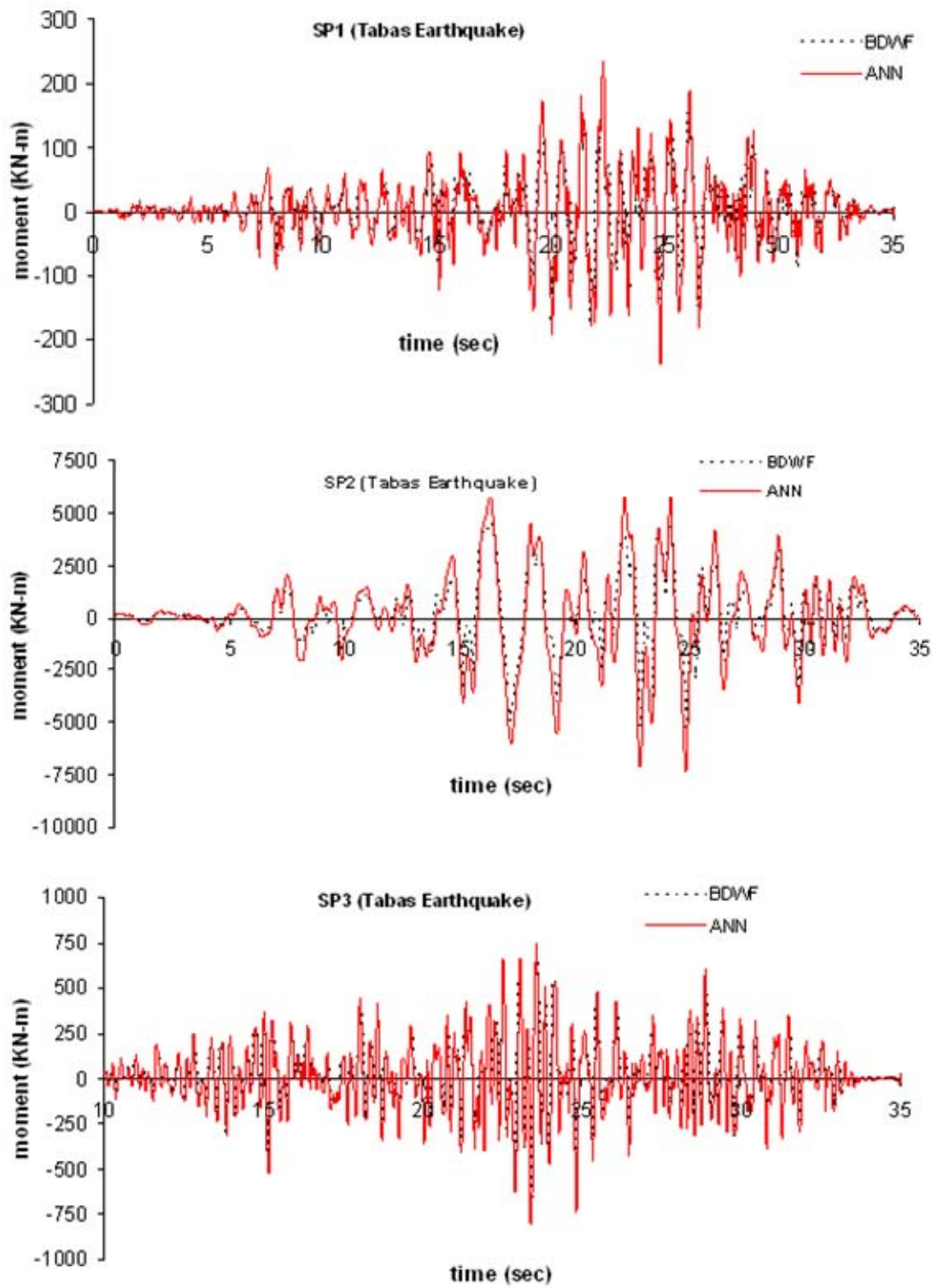


Figure 5: Comparison of bending moment time history of ANN with BDWF

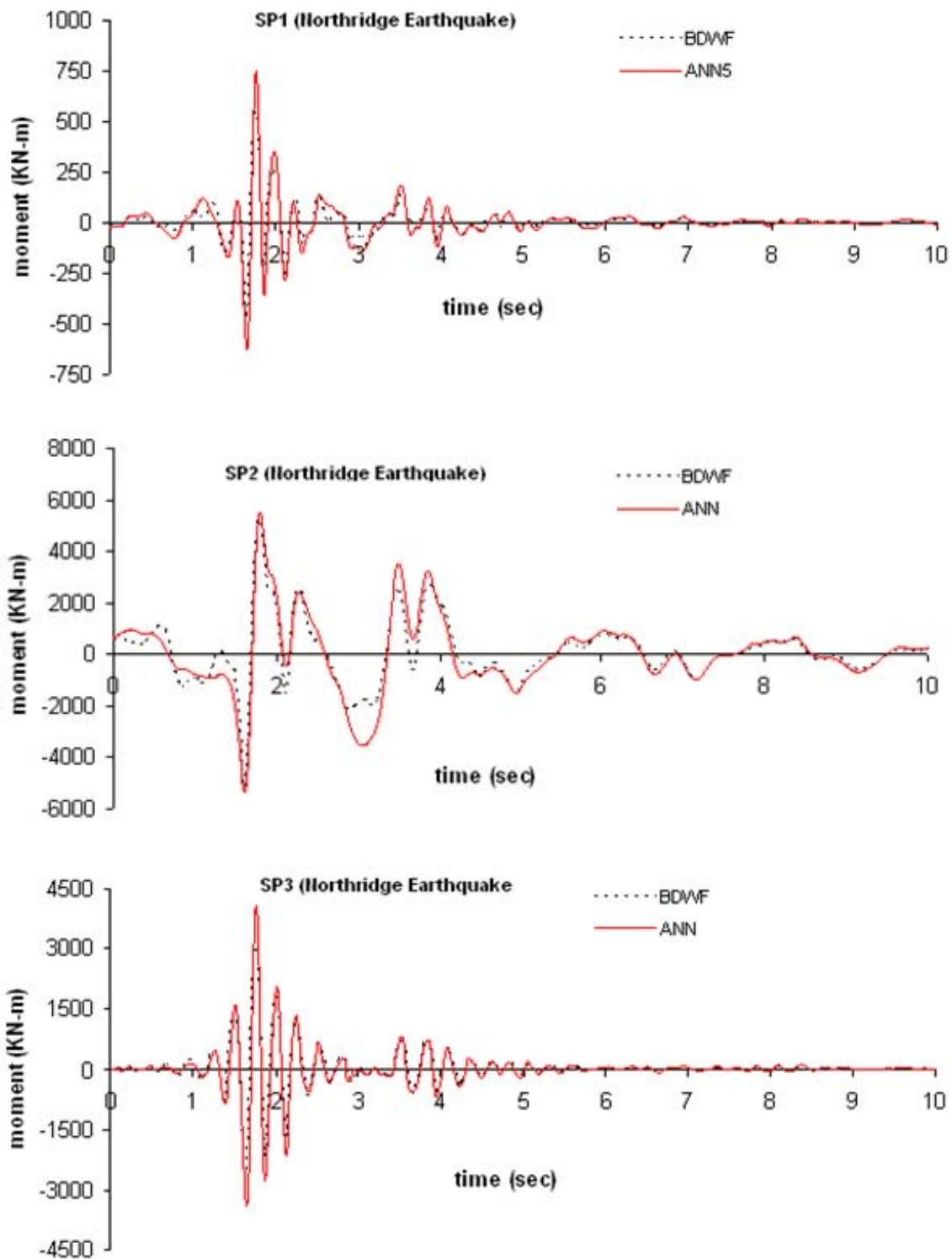


Figure 6: Comparison of bending moment time history of ANN with BDWF

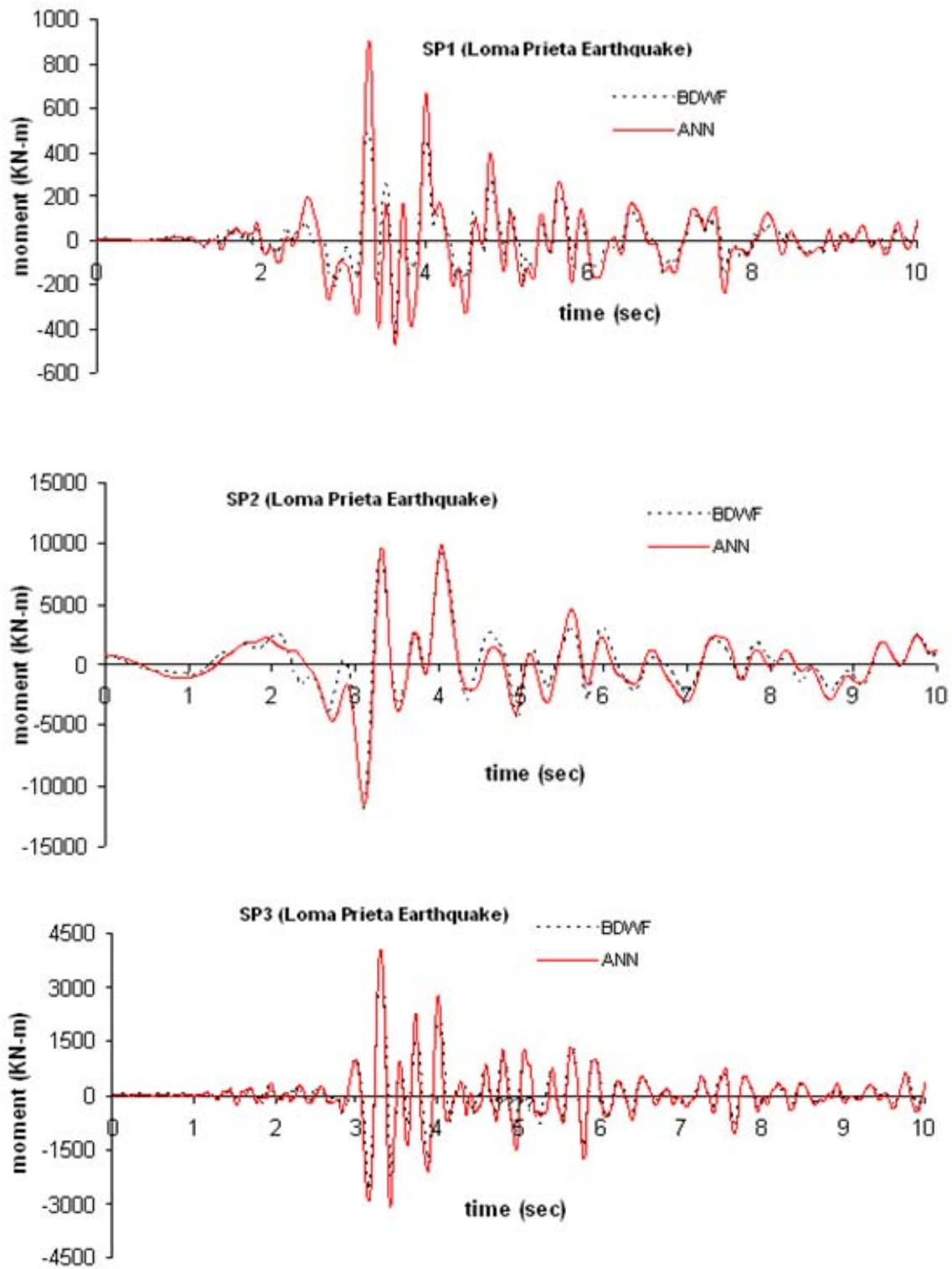


Figure 7: Comparison of bending moment time history of ANN with BDWF

## CONCLUSION

Two ANN models, namely, ANN1 and ANN2 are developed in this paper to predict kinematic bending moment at the fixed head of pile. The first model, the ANN1, estimates kinematic bending moment when the bedrock is excited with the fundamental frequency of soil deposit whereas the second model, the ANN2, is applicable to any frequency of excitation. The accuracy of both ANN models is confirmed on testing data which was not used in the ANN training. Furthermore, the performance of the ANN2 model is also validated for full time history response by comparing its results with that of BDWF solutions. The ANN models drastically reduce the amount of calculations involved and are found to be accurate enough for engineering use. They present solutions that can easily be implemented in simple hand calculator capable of matrix operation.

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