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APPLICATION OF THE CLM METHOD FOR THE SOLUTION OF THE INVERSE PROBLEM IN THE SASW METHOD

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ABSTRACT

The SASW method aims to determine the small strain dynamic soil characteristics of shallow soil layers. The method involves an in situ experiment, the determination of an experimental dispersion curve and the solution of an inverse problem, formulated as a non-linear least squares problem. The latter is usually solved with a gradient based local optimization method, which converges fast, but does not guarantee to find the global minimum of the objective function. The method of Coupled Local Minimizers combines the advantage of gradient based local algorithms with the global approach of genetic algorithms. A cooperative search mechanism is set up by performing simultaneously a number of local optimization runs, that are coupled by pairs of synchronization constraints.

A synthetic example demonstrates that the CLM method succeeds in finding the global minimum of an objective function with multiple minima and can be used successfully to solve the inverse problem in the SASW method. This is further illustrated by a second example, where the method is applied on experimental data obtained on a test site near the high speed train track in Lincent (Belgium). The obtained results show good agreement with the results obtained with other experimental techniques, as the Seismic Cone Penetration Test.

Keywords: Dynamic soil characteristics, SASW method, optimization algorithms, Coupled Local Minimizers.

INTRODUCTION

The Spectral Analysis of Surface Waves (SASW) method is a non-invasive geophysical prospection method to determine the dynamic soil characteristics (dynamic shear modulus and material damping ratio) of shallow soil layers and is based on the dispersive characteristics of surface waves in a layered medium (Nazarian & Desai, 1993; Yuan & Nazarian, 1993). The method is well developed for the determination of the shear strain modulus, and has recently been applied to the determination of the material damping ratio (Rix, et al., 2000; Lai, et al., 2002).

The SASW method consists of three steps. The first step involves an in situ experiment where vibrations are generated at the soil's surface using a falling weight, an instrumented impact hammer or a hydraulic shaker and measured with geophones or accelerometers up to a distance from the source of typically 50 m. A common source-receiver configuration is most often used. It is assumed that the response at sufficiently large distance from the source is dominated by dispersive surface waves. In the second step, an experimental dispersion curve is determined from the response at the surface. An inverse problem is finally formulated as an optimization problem where the objective function is defined as the squared difference between the experimental and a computed theoretical dispersion curve. The latter corresponds to the first mode of a layered halfspace or a combination of multiple modes in the case of large stiffness

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contrasts and/or inverse layering where stiff layers are underlain by softer layers (O'Neill & Matsuoka, 2005).

The (constrained) optimization problem is usually solved with a gradient based local optimization method, by iteratively changing the layer thicknesses and elastic characteristics of a layered soil profile. Local optimization methods, as the Gauss-Newton method, converge fast, but do not guarantee to find the global minimum of the objective function. Global optimization methods as genetic algorithms (Holland, 1975; Pezeshk & Zarrabi, 2005) and simulated annealing (Kirkpatrick, et al., 1983; Beaty, et al., 2002) are more robust and more likely to detect the global minimum. Their main drawback, however, is that they require a large number of function evaluations since they are based on probabilistic searching without the use of gradient information.

The method of Coupled Local Minimizers (CLM) offers a valuable alternative as it combines the advantage of the local gradient based algorithms (relatively fast convergence) with the global approach of genetic algorithms (parallel strategy and information exchange). The CLM method has been originally conceived by Suykens, et al. (2001) and Suykens & Vandewalle (2002) and was implemented with first order search methods, that use first order derivatives. In the context of finite element model updating and structural health monitoring, Teughels (2003) and Teughels, et al. (2003) have implemented the CLM method to solve global optimization problems with multiple local minima as an iterative optimization method that generates discrete steps in the design space instead of continuous time variations of the design variables. A cooperative search mechanism is set up by performing simultaneously a number of local optimization runs, that are coupled by pairs of synchronization constraints. The global search process is directed by the gradients in each search point. Moreover, the second order Newton based algorithm accelerates the convergence speed considerably and has a favourable effect on the convergence (Teughels et al., 2003). The CLM method implemented in this way is especially convenient for the solution of low dimensional optimization problems.

The SASW method is briefly reviewed, with emphasis on the definition of the inverse problem. The CLM method is introduced next and is developed in a way that can be used for the current optimization problem. A synthetic example demonstrates that gradient based local optimization methods may converge to a local minimum of the objective function, whereas the CLM method successfully converges to the global minimum of the objective function. Additional comments are made on the proper definition of the inverse problem. Finally, the CLM method is employed to determine the soil profile of a site in Lincent (Belgium); results are compared with results obtained with the Seismic Cone Penetration Test (SCPT).

INVERSE MODELING IN THE SASW METHOD

In the SASW method, an inverse problem is formulated as a non-linear least squares problem. The objective function $f(\mathbf{x})$ is defined as:

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{r}(\mathbf{x})^{\mathrm{T}}\mathbf{r}(\mathbf{x}) \tag{1}$$

where each element $r_i(\mathbf{x})$ $(i = 1, \ldots, m)$ of the m-dimensional residual vector $\mathbf{r}(\mathbf{x})$ is a smooth non-linear function of the n-dimensional vector $\mathbf{x}$ with soil parameters x_j $(j = 1, \ldots, n)$. At each frequency f_i $(i = 1, \ldots, m)$, the residue $r_i(\mathbf{x})$ is defined as the difference between the phase velocity C_i^E corresponding to an experimental dispersion curve and the computed phase velocity $C_i^T(\mathbf{x})$ of the soil profile with parameters $\mathbf{x}$:

$$r_i(\mathbf{x}) \quad = \quad C_i^E - C_i^T(\mathbf{x}) \quad i = 1, \ldots, m. \tag{2}$$

The theoretical dispersion curve $C_i^T(\mathbf{x})$ of the soil profile is usually calculated as the fundamental solution, corresponding to the first surface wave, of a transcendental eigenvalue problem, formulated using the direct stiffness method or an equivalent approach as the thin layer method (Kausel & Roësset, 1981). This methodology can be used if the layering of the soil is regular, in a sense that the stiffness of the layers increases with depth. It can lead to erroneous results, however, if the soil exhibits large stiffness contrasts and/or reversals between layers. In such cases, higher-mode Rayleigh waves may affect the surface displacements and the dispersion curve derived from the experimental data may differ from the fundamental Rayleigh wave dispersion curve (Gucunski & Woods, 1991; Tokimatsu, et al., 1992). Several authors have attempted to tackle this problem (Gabriels, et al., 1987; Gucunski & Woods, 1992; Beaty et al., 2002; Ganji, et al., 1998; Levshin, et al., 2005; O'Neill & Matsuoka, 2005). In the present paper, a methodology similar to the approach of Zomorodian & Hunaidi (2006) is followed. An effective theoretical dispersion curve, accounting for the dominance of higher modes, is calculated as $C_i^T(\mathbf{x}) = \omega / k_i^T(\mathbf{x})$, where $k_i^T(\mathbf{x})$ is the horizontal wavenumber k_r where the modulus of the Green's function $\tilde{u}_{zz}^G(k_r, \omega_i)$ at the frequency $\omega_i = 2\pi f_i$ reaches its absolute maximum. The Green's function $\tilde{u}_{zz}^G(k_r, \omega)$ represents the vertical displacements due to a vertical harmonic point load at the soil's surface and is calculated in the frequency-wavenumber domain by means of the direct stiffness method.

The non-linear least squares problem aims to minimize the objective function $f(\mathbf{x})$ in equation (1) for the soil parameters $\mathbf{x}$, where the dimension m of the residual vector $\mathbf{r}(\mathbf{x})$ is larger than the dimension n of the vector $\mathbf{x}$. This problem can be viewed as an optimization problem and iterative methods are applied to solve it.

The Newton-based trust region method is a local optimization method where a model function $m_k(\mathbf{p})$ is constructed of the objective function $f(\mathbf{x})$ in the vicinity of the current point $\mathbf{x}_k$. A candidate for the new iterate is computed by approximately minimizing $m_k(\mathbf{p})$ in the trust region:

$$\min_{\mathbf{p}} \; m_k(\mathbf{p}), \qquad \text{such that } \| \mathbf{p} \| \leq \Delta_k \tag{3}$$

where $\mathbf{p}$ is the step vector from $\mathbf{x}_k$ and Δ_k denotes the trust region radius at the current iteration k. The model $m_k(\mathbf{p})$ is usually defined as a quadratic function:

$$m_k(\mathbf{p}) \;=\; f(\mathbf{x}_k) + \mathbf{p}^{\mathrm{T}} \nabla f(\mathbf{x}_k) + \frac{1}{2} \mathbf{p}^{\mathrm{T}} \mathbf{B}_k \mathbf{p} \tag{4}$$

where $f(\mathbf{x}_k)$ and $\nabla f(\mathbf{x}_k)$ are the objective function value and gradient at the current iterate $\mathbf{x}_k$, respectively, and $\mathbf{B}_k$ is equal to the Hessian $\nabla^2 f(\mathbf{x}_k)$ or an approximation to it.

For large-scale problems, the Matlab Optimization Toolbox applies an approximate strategy to solve the optimization problem defined by equations (3) and (4) that is known as the two-dimensional subspace minimization method. The trust region subproblem is restricted to a two-dimensional subspace, spanned by the direction of the gradient $\nabla f(\mathbf{x}_k)$ and by either the Newton direction $-\mathbf{B}_k^{-1} \nabla f(\mathbf{x}_k)$ of the quadratic model (when $\mathbf{B}_k$ is positive definite) or a direction of negative curvature (when $\mathbf{B}_k$ is indefinite). The latter is calculated approximately by a truncated preconditioned conjugate gradient method. In the former case, the trust region subproblem becomes:

$$\min_{\mathbf{p}} \; m_k(\mathbf{p}), \quad \text{such that} \qquad \| \mathbf{p} \| \leq \Delta_k, \quad \mathbf{p} \in \text{span}[\nabla f(\mathbf{x}_k), \mathbf{B}_k^{-1} \nabla f(\mathbf{x}_k)]. \tag{5}$$

Local optimization methods, as the Newton-based trust region method, converge fast, but do not guarantee to find the global minimum of the objective function, as will be demonstrated in a numerical example. In the following section, it is outlined that the CLM method offers a valuable alternative as it combines the advantage of the local gradient based algorithms (relatively fast convergence) with the global approach of genetic algorithms (parallel strategy and information exchange).

THE CLM METHOD

Outline of the method

Consider the minimization of an objective function $f(\mathbf{x})$ with multiple local minima, among which the global minimum has to be found. In the Coupled Local Minimizers (CLM) method, a population of t search points is used, at which the following average objective function $\langle f \rangle$ (and its derivatives) is calculated and minimized:

$$\langle f \rangle = \frac{1}{t} \sum_{l=1}^{t} f(\mathbf{x}^{(l)}) \tag{6}$$

where $\mathbf{x}^{(l)}$ denotes the l-th ($l = 1, \ldots, t$) search point in the population.

Instead of performing separate, independent searches from each of the search points, as would be the case in a multi-start local optimization, the set of optimizers are coupled in order to create an interaction so that the population generates a minimum that is better than the best result that would be obtained from all individual local processes. Hence, a cooperative search mechanism is set up that is realized by minimizing the average objective function $\langle f \rangle$. During the minimization process the search points are pairwisely coupled by the following synchronization constraints that enforce them to end in the same final point:

$$\mathbf{x}^{(l)} - \mathbf{x}^{(l+1)} \quad = \quad 0, \quad l = 1, 2, \ldots, t \tag{7}$$

resulting in the following equality constrained minimization problem:

$$\min_{\mathbf{x}^{(l)} \in \mathbb{R}^n} \langle f \rangle \quad \text{with} \quad \mathbf{x}^{(l)} - \mathbf{x}^{(l+1)} = 0, \quad l = 1, 2, \ldots, t. \tag{8}$$

The boundary condition $\mathbf{x}^{(t+1)} = \mathbf{x}^{(1)}$ is applied on the average objective function $\langle f \rangle$.

The CLM technique is implemented with the augmented Lagrangian method (Rao, 1996) in which the augmented Lagrangian function $\mathcal{L}_A$ is defined by the average objective function $\langle f \rangle$ of the population together with synchronization constraints between the individual local minimizers:

$$\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda}, \gamma, \eta) \quad = \quad \frac{\eta}{t} \sum_{l=1}^{t} f(\mathbf{x}^{(l)}) + \sum_{l=1}^{t} [\boldsymbol{\lambda}^{(l)}]^{\mathrm{T}} [\mathbf{x}^{(l)} - \mathbf{x}^{(l+1)}] + \frac{\gamma}{2} \sum_{l=1}^{t} \| \, \mathbf{x}^{(l)} - \mathbf{x}^{(l+1)} \, \|^2 \, . \tag{9}$$

The vector $\mathbf{x} = [\mathbf{x}^{(1)}; \ldots; \mathbf{x}^{(t)}]$ collects all search vectors $\mathbf{x}^{(l)}$, while the vector $\boldsymbol{\Lambda} = [\boldsymbol{\lambda}^{(1)}; \ldots; \boldsymbol{\lambda}^{(t)}]$ contains all vectors $\boldsymbol{\lambda}^{(l)}$ with Lagrange multipliers. η is a weighting factor on the average objective function, γ is the penalty parameter and $\| \cdot \|$ denotes the Euclidean vector norm. The augmented Lagrange method is related to the quadratic penalty method, but it reduces the possibility of ill-conditioning of the subproblem (due to very high values of the penalty parameter γ) by introducing explicit Lagrange multiplier estimates at each step into the penalty function. Constraints are imposed on multiple search points to reach the same final position. If the parameters η and γ are chosen appropriately, an improved solution is obtained which in the best case corresponds to the global minimum.

Augmented Lagrange methods are based on successive (approximate) minimization of $\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda}, \gamma, \eta)$ with respect to $\mathbf{x}$ only, with updates of the Lagrange multipliers $\boldsymbol{\Lambda}_k$ (and, eventually, the penalty parameter γ) between the main iterations. The minimizer $\mathbf{x}_k^*$ (or a good estimate of it) in the main iteration k is obtained by minimizing $\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda}_k, \gamma)$ with the Lagrange multipliers $\boldsymbol{\Lambda}_k$ and the penalty parameter γ kept fixed (the latter should not approach infinity). The Lagrange multipliers $\boldsymbol{\Lambda}_k$ are updated before starting the next iteration:

$$\boldsymbol{\lambda}_{k+1}^{(l)} \quad = \quad \boldsymbol{\lambda}_k^{(l)} + \gamma [\mathbf{x}_k^{*(l)} - \mathbf{x}_k^{*(l+1)}] \tag{10}$$

In this way the Lagrange multipliers $\boldsymbol{\Lambda}_k$ become a reasonable estimate of the optimum $\boldsymbol{\Lambda}^*$, at which the minimum of $\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda} = \boldsymbol{\Lambda}^*, \gamma)$ with respect to **x** generates the global minimum of $\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda}, \gamma, \eta)$ with respect to **x** and $\boldsymbol{\Lambda}$.

Implementation with a trust region Newton method

In the implementation of the CLM algorithm (Teughels et al., 2003), the trust region Newton method (Nocedal & Wright, 1999) is used to minimize $\mathcal{L}_A(\mathbf{x}, \boldsymbol{\Lambda}, \gamma, \eta)$ with respect to **x**. In each sub-iteration s, a quadratic approximation $m(\mathbf{p})$ of $\mathcal{L}_A$ at the current population $\mathbf{x}_s$ has to be minimized within a trust region Δ_s. The quadratic model $m(\mathbf{p})$ is defined by the truncated Taylor series of $\mathcal{L}_A$:

$$\min_{\mathbf{p}} \; m(\mathbf{p}) = \mathcal{L}_A + [\nabla \mathcal{L}_A]^{\mathrm{T}} \mathbf{p} + \frac{1}{2} \mathbf{p}^{\mathrm{T}} [\nabla^2 \mathcal{L}_A] \mathbf{p} \tag{11}$$

where the step vector **p** from $\mathbf{x}_s$ is subject to $\| \mathbf{p} \| \leq \Delta_s$ and where $\nabla \mathcal{L}_A$ and $\nabla^2 \mathcal{L}_A$ are the gradient and the Hessian of $\mathcal{L}_A$ at $\mathbf{x}_s$.

Assuming that the value of the objective function in each search point is independent of all other search points, the following expressions are valid for $l = 1, \ldots, t$ (Teughels, 2003):

$$\begin{aligned} \nabla_{\mathbf{x}^{(l)}} \mathcal{L}_A &= \frac{\eta}{t} \nabla f(\mathbf{x}^{(l)}) - \boldsymbol{\lambda}^{(l-1)} + \boldsymbol{\lambda}^{(l)} - \gamma[\mathbf{x}^{(l-1)} - \mathbf{x}^{(l)}] + \gamma[\mathbf{x}^{(l)} - \mathbf{x}^{(l+1)}] && (12) \\ \nabla^2_{\mathbf{x}^{(l)}\mathbf{x}^{(l)}} \mathcal{L}_A &= \frac{\eta}{t} \nabla^2 f(\mathbf{x}^{(l)}) + 2\gamma \mathbf{I} && (13) \\ \nabla^2_{\mathbf{x}^{(l)}\mathbf{x}^{(l-1)}} \mathcal{L}_A &= -\gamma \mathbf{I} = \nabla^2_{\mathbf{x}^{(l)}\mathbf{x}^{(l+1)}} \mathcal{L}_A && (14) \end{aligned}$$

where **I** denotes the n by n identity matrix. These expressions need to be included in the gradient vector and the band-structured Hessian matrix. The boundary constraints are $\mathbf{x}^{(0)} = \mathbf{x}^{(t)}$ and $\mathbf{x}^{(t+1)} = \mathbf{x}^{(1)}$.

As a Newton-based method is used, the search process in the CLM method has a high convergence rate. Convergence is enforced by the use of a trust region strategy. The CLM algorithm, as described above, is implemented in the Optimization Toolbox of Matlab, by specifying the proper equations for $\mathcal{L}_A, \nabla \mathcal{L}_A$ and $\nabla^2 \mathcal{L}_A$. The minimization problem in each main iteration k is solved with the trust region Newton method.

Choice of the CLM parameters and normalization

The number of search points t needed to achieve a good performance depends on the shape of the objective function and influences the cost of the computation. If t is too small, however, the global minimum may be missed.

The search process is also influenced by the tuning parameters γ and η. Increasing γ gives more weight to the quadratic penalty terms in $\mathcal{L}_A$. If γ is too high, the search points will first approach each other and then search together for the nearest local minimum. A low value of γ results in a wide exploration of the search domain by each search point, which can be useful when few search points are used. If the relative weight of the constraints is too low, the optimization process may not converge. The parameter η is used to tune the weight of the average objective function term to the Lagrange terms in $\mathcal{L}_A$.

It is difficult to give general guidelines for the a priori determination of the parameters t, γ and η as they are problem dependent. The objective function and the synchronization constraints in $\mathcal{L}_A$ are normalized as follows:

$$\overline{f} = \frac{f + T}{R_f} \qquad \text{such that} \qquad 0 \leq \overline{f} \leq 1 \tag{15}$$

$$\overline{\Delta x_j^{(l)}} = \frac{\Delta x_j{}^{(l)}}{R_{cj}} \qquad \text{such that} \qquad 0 \leq |\overline{\Delta x_j^{(l)}}| \leq 1. \tag{16}$$

The inequalities (15) and (16) should only hold on that part of the search space that will be explored. The translation parameter T and the factors R_f and R_{cj} are not unique and can only be estimated. As only their order of magnitude is important, the following estimations can be used: $T = |\min(0, f^{\min})|$, $R_f = f^{\max} + T$ and $R_{cj} = |x_j^{\max} - x_j^{\min}|$, with $f^{\min}$ and $f^{\max}$ the minimum and maximum value of the average objective function and $x_j^{\max}$ and $x_j^{\min}$ the upper and lower boundary of the design variable x_j. As $f^{\min}$ and $f^{\max}$ are not known in advance, they are determined iteratively. Due to the normalization, the relative weights of the different terms in $\mathcal{L}_A$ are less dependent on the characteristics of each particular minimization problem.

The initial estimates of the Lagrange multipliers $\lambda_j^{(l)}$ are randomly chosen in the interval $[-1;1]$ in order to promote a wide exploration of the search space. Similarly, the initial population of search points $\mathbf{x}_{k=0}^{s=0} = [\mathbf{x}^{(1)}; \ldots; \mathbf{x}^{(t)}]_{k=0}^{s=0}$ is well spread in the search domain within the range of physical meaningful values.

The strength of the CLM method is that it is likely to find the global minimum in relatively few iterations. The method requires the evaluation of the gradient and Hessian, however. Consequently, it is appropriate for low-dimensional functions for which the computation of the derivatives in the current search points only represents a marginal cost in comparison with the evaluation of the function in additional search points.

SYNTHETIC EXAMPLE

Problem outline

A soil profile 1 is considered that consists of three layers on a halfspace (table 1). Each layer is characterized by the thickness d, the shear wave velocity C_s, the longitudinal wave velocity C_p, the density ρ, and the hysteretic material damping ratio β.

Table 1: Dynamic characteristics of the soil profile 1.

Layer	d [m]	C_s [m/s]	C_p [m/s]	ρ [kg/m^3]	β [-]
1	3	160	320	1750	0.025
2	4	185	370	1750	0.025
3	8	200	400	1750	0.025
4	∞	400	800	1750	0.025

Figure 1 shows the dispersion curves of the lowest seven modes, calculated with a direct stiffness formulation. The first mode varies from the Rayleigh wave velocity of the underlying halfspace to the Rayleigh wave velocity of the top layer. Higher modes start at discrete frequencies and have a phase velocity that varies between the shear wave velocity of the halfspace and the top layer. Superimposed on the same graph is the effective dispersion curve, which is calculated using a methodology that is similar to the approach of Zomorodian & Hunaidi (2006) and based on the maximum of the Green's function $\tilde{u}_{zz}^G(k_r, \omega)$ of the layered halfspace. The effective dispersion curve follows the fundamental mode, demonstrating that the surface response is indeed dominated by the first mode for a soil profile where the stiffness increases with depth.

Local optimization

A synthetic inverse problem is considered where the shear wave velocities C_{s1} and C_{s2} (or the Young's moduli) of the two top layers are selected as the unknown design variables and collected in a vector $\mathbf{x}$ with dimension $n = 2$, while all other material characteristics are constant with values as summarized in table 1. The residual vector $\mathbf{r}(\mathbf{x})$ is defined at frequencies ranging from 5 to 30 Hz with a step of 5

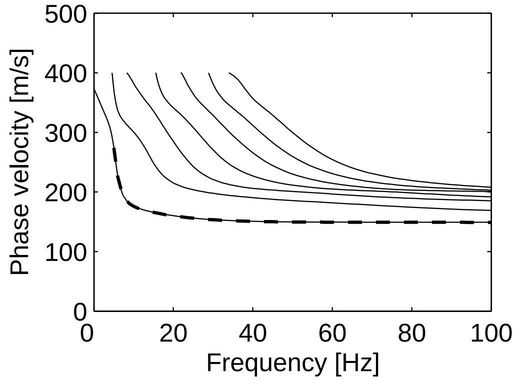

Figure 1: Dispersion curves of soil profile 1 (solid lines) and effective dispersion curve (dashed line).

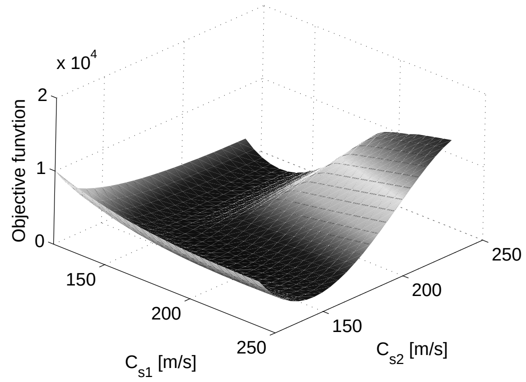

Figure 2: Objective function based on the fundamental surface mode.

Hz and from 40 to 100 Hz with a step of 10 Hz, so that $m = 13$. Figure 2 shows the objective function $f(\mathbf{x})$ as a function of the shear wave velocities C_{s1} and C_{s2} of both top layers. Apart from the global minimum at (160;185) m/s, the objective function also has a local minimum at (183;146.5) m/s.

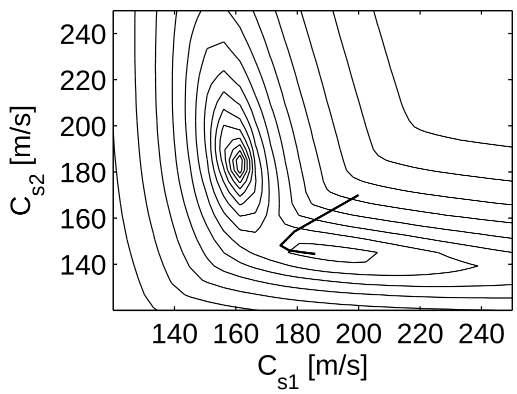

Figure 3: Local optimization starting from the point (200;170) m/s.

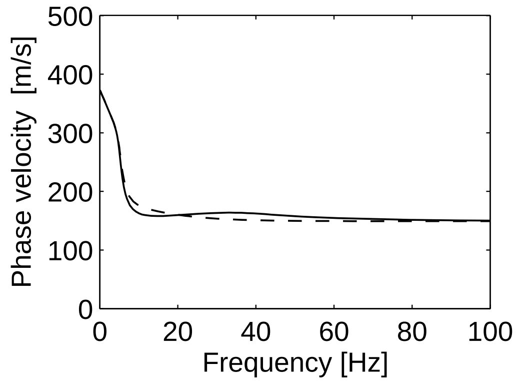

Figure 4: Dispersion curve of the first surface mode corresponding to the global (dashed line) and the local (solid line) minimum of the objective function.

Table 2: Dynamic characteristics of the soil profile 2.

Layer	d [m]	C_s [m/s]	C_p [m/s]	ρ [kg/m^3]	β [-]
1	3	183	266	1750	0.025
2	4	146.5	293	1750	0.025
3	8	200	400	1750	0.025
4	∞	400	800	1750	0.025

A local optimization algorithm, as the Newton-based trust region method, may iterate to this local minimum. This is illustrated in figure 3, where the algorithm iterates from the starting point (200;170) m/s to the local minimum. The latter corresponds to a dispersion curve that is different from the dispersion curve of the first surface wave curve, as shown in figure 4.

The local minimum at (183;146.5) m/s corresponds to a soil profile 2 where the second layer is softer than the top layer (table 2). In such a medium, the assumption that the surface response is dominated by the first surface wave is not valid, as will be discussed in a following subsection. First, we will apply the CLM method to the inverse problem defined with the fundamental surface mode.

Global optimization

The CLM method is applied to the optimization problem as explained above. Three ($t = 3$) two-dimensional starting vectors ($n = 2$) are considered, corresponding to the points (180;220) m/s, (200;170) m/s and (220;200) m/s. The translation factor T on the average objective function in equation (15) is equal to zero (as the objective function is positive), while a scaling factor $R_f = 10000$ is used, as to fulfill the inequality (16). The weighting factor η on the average objective function is equal to 5.0. The normalization factors R_{cj} on the synchronization constraints for both shear wave velocities are equal to 179.28 m/s, corresponding to a Young's modulus of 150 MPa, which is a good estimate of the maximum variation of the shear wave velocity in the search domain. The initial estimates of the Lagrange multipliers $\lambda_j^{(l)}$ are randomly chosen in the interval $[-1; 1]$ in order to promote a wide exploration of the search space. A penalty factor $\gamma = 0.5$ results in a good compromise between a wide exploration of the search space and computation time.

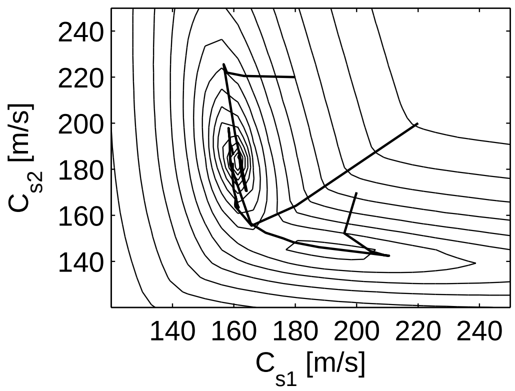

Figure 5: CLM optimization starting from the points (180;220) m/s, (200;170) m/s and (220;200) m/s.

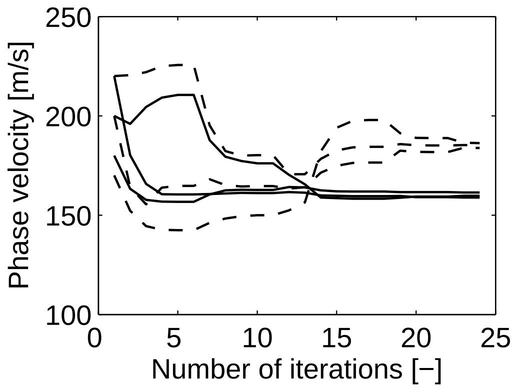

Figure 6: History of the shear wave velocity in the top layer (solid line) and the second layer (dashed line) for the three search points in the population.

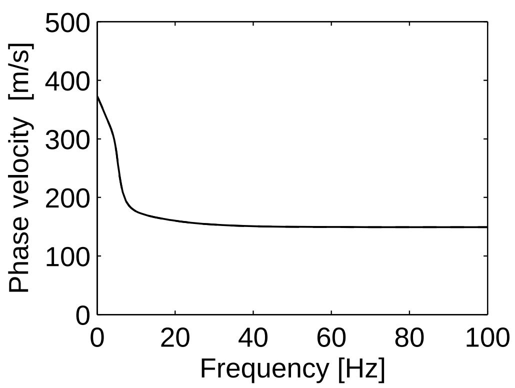

Figure 7: Dispersion curve of the first surface mode corresponding to the global minimum of the objective function (dashed line) and after optimization with the CLM method (solid line).

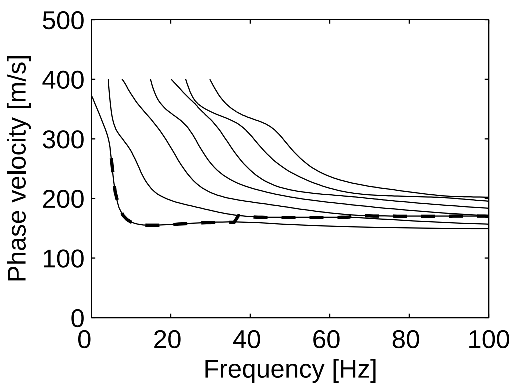

Figure 8: Dispersion curves of soil profile 2 (solid lines) and effective dispersion curve (dashed line).

Figure 5 illustrates how the three search points explore the search domain but finally arrive in the global minimum of the objective function, whereas figure 6 shows how the shear wave velocity in the top and the second layer converge to values of 160 m/s and 185 m/s, respectively, for the three search points in the population. The resulting dispersion curve almost perfectly matches the dispersion curve of the first surface mode of the soil profile, as shown in figure 7.

It is concluded that the CLM method is a versatile gradient based method that can find the global minimum of objective functions exhibiting local minima in relatively few iterations. It is therefore very useful to solve problems of higher dimension that exhibit local minima, which are not as easy to visualize as in the two-dimensional problem considered.

It must be stressed, however, that the local minimum in the previous example corresponds to a soil profile with a soft layer overlain by a stiffer layer. That solution could have been avoided if the effective dispersion curve in stead of the fundamental mode would have been used in the definition of the inverse problem, as discussed in the following subsection.

Alternative objective function

Consider again the solution of the optimization problem, obtained with the Newton-based trust region method, that was denoted as soil profile 2 (table 2) and identified as a local minimum on figure 3. Figure 8 shows the dispersion curves of soil profile 2, together with the effective dispersion curve. It is apparent that the effective dispersion curve does not correspond to the first surface mode in the full frequency range, but also follows dispersion curves of higher modes. This demonstrates that the surface response is also affected by higher Rayleigh modes.

If a SASW test would have been performed on a site with soil profile 2, the experimental dispersion curve would correspond to the effective dispersion curve rather than the first surface mode, that has been employed in the definition of the objective function. This implies that the solution corresponding to soil profile 2 is not feasible in practice as it corresponds to a soil profile for which the effective dispersion curve is different from the first surface mode.

This problem can be solved by proper reformulation of the inverse problem. A first choice is to use the first surface mode together with a constraint on both shear wave velocities, excluding soil profiles where soft layers are overlain by stiff layers. A second choice is to use the effective dispersion curve in the formulation of the inverse problem. The latter solution allows to account for soil profiles where soft layers are overlain by stiffer layers.

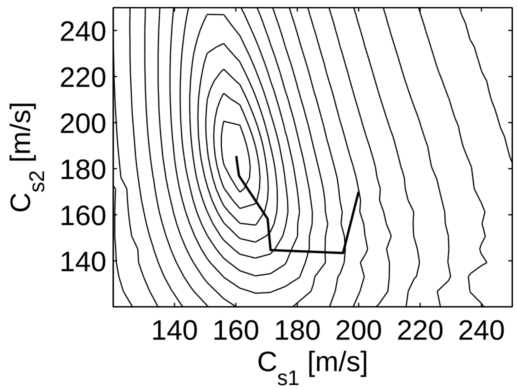

Figure 9: Objective function based on the effective theoretical dispersion curve.

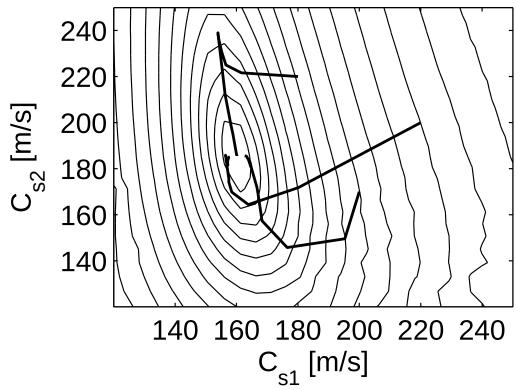

Figure 10: Objective function based on the effective theoretical dispersion curve.

Figure 9 shows a contour plot of the objective function, using the effective dispersion curve in stead of the first surface mode. The objective function now exhibits a single (global) minimum at (160;185) m/s, corresponding to soil profile 1. Superimposed on this graph is the path followed by the Newton-based trust region method from the starting point (200;170) m/s to the global minimum. The CLM method remains useful in practice, as it can never be excluded that the objective function has local minima (this is certainly the case in optimization problems of higher dimension); figure 10 illustrates that the solution corresponding to soil profile 1 is also found with the CLM method using three two-dimensional starting vectors, corresponding to the points (180;220) m/s, (200;170) m/s and (220;200) m/s.

EXAMPLE BASED ON IN SITU EXPERIMENTS

The CLM method described above is employed to invert an experimental dispersion curve obtained at a test site near the high speed train track in Lincent (Belgium). The experimental dispersion curve for this site is determined using Nazarian's method (Nazarian & Desai, 1993) (figure 11). The computational time necessary to invert the dispersion curve is not more than some minutes on a PC for both the LS and the CLM method.

In the inversion procedure, the soil is modeled using 3 layers on a halfspace. The design variables are the layer thicknesses and shear wave velocities, as well as the shear wave velocity of the halfspace ($n = 7$). Two ($t = 2$) seven-dimensional starting vectors are considered. The translation factor T on the average objective function in equation (15) is equal to zero, while a scaling factor $R_f = 10000$ is used. The weighting and penalty factors η and γ are equal to 5.0 and 0.5 respectively. The normalization factors R_{cj} on the synchronization constraints for all shear wave velocities are equal to 179.28 m/s. The initial estimates of the Lagrange multipliers $\lambda_j^{(l)}$ are randomly chosen in the interval $[-1;1]$. The other characteristics of the profile are derived from the results of earlier tests. For all layers, a Poisson's ratio $\nu = 1/3$, a density $\rho = 1800\,\mathrm{kg/m^3}$ and a damping ratio $\beta = 0.03$ is used.

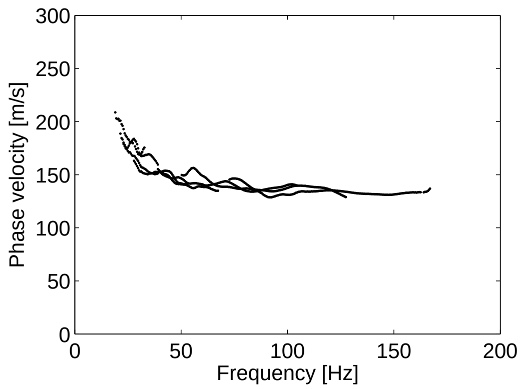

Figure 11: Experimental dispersion curve at the site in Lincent.

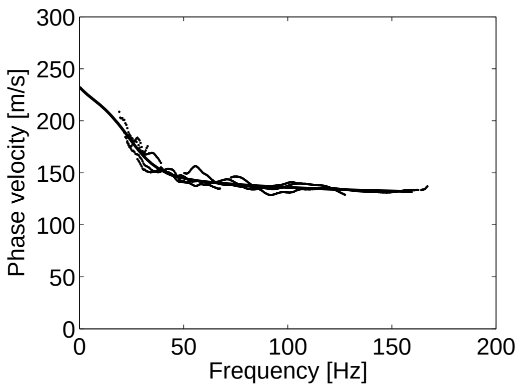

Figure 12: Experimental dispersion curve and the dispersion curve of the converged solution.

Table 3: Dynamic characteristics of the soil profile in Lincent.

Layer	d [m]	C_s [m/s]	C_p [m/s]	ρ [kg/m^3]	β [-]
1	0.53	139	278	1800	0.03
2	1.54	154	308	1800	0.03
3	2.68	224	448	1800	0.03
4	∞	249	498	1800	0.03

Figure 12 represents a good agreement between the experimental dispersion curve and the dispersion curve of the converged solution. The resulting dynamic soil characteristics are shown in table 3, while the shear wave velocity profile is compared with the results of the SCPT in figure 13. It can be observed that the SASW inversion procedure using a CLM algorithm results in a relatively good estimation of the shear wave velocities. The discrepancy between the soil profiles determined by the SASW method and the SCPT method is partially due to the assumptions made in the forward model used in the SASW method (horizontally layered isotropic halfspace, and assumption that the first surface mode dominates the response), the accuracy of the SCPT results, which is principally related to the resolution of the accelerometers used, and the different expected spatial resolution of both methods.

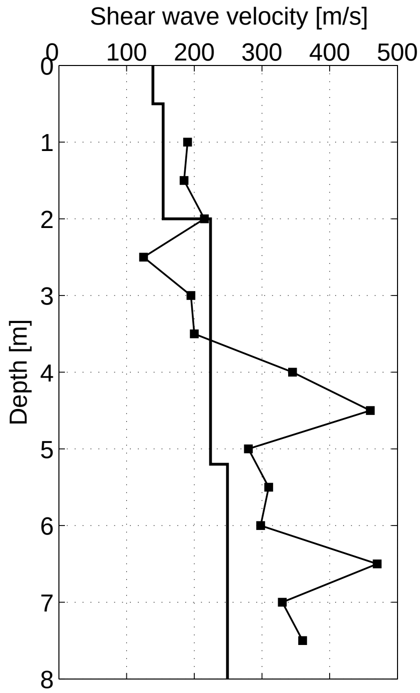

Figure 13: Shear wave velocity profile for the site in Lincent obtained with the SASW test (solid line) and the SCPT test (squares).

CONCLUSION

The method of Coupled Local Minimizers has been successfully applied to the solution of the inverse problem as it is formulated in the third step of the SASW method. This has been demonstrated by considering a synthetic example of a layered halfspace with three layers on a halfspace, for which a two-dimensional inverse problem is solved. The example demonstrates that gradient based local optimization methods may converge to a local minimum of the objective function. This problem can be solved by proper reformulation of the inverse problem using appropriate constraints or using the effective theoretical dispersion curve in stead of the fundamental mode. In all cases, the CLM method offers a valuable alternative to local gradient based algorithms as it can find the global minimum of objective functions exhibiting local minima in relatively few iterations. The CLM method is further applied to an inverse problem of higher dimension, based on an experimental dispersion curve that has been measured at a site in Lincent (Belgium). The shear wave velocity profile has been determined and compares well with SCPT results.

REFERENCES

K. Beaty, et al. (2002). ‘Simulated annealing inversion of multimode Rayleigh wave dispersion curves for geological structure’. *Geophysical Journal International* **151**:622–631.

P. Gabriels, et al. (1987). ‘In situ measurements of shear-wave velocity in sediments with higher-mode Rayleigh waves’. *Geophysical Prospecting* **35**:187–196.

V. Ganji, et al. (1998). ‘Automated inversion procedure for Spectral Analysis of Surface Waves’. *Journal of Geotechnical and Geoenvironmental Engineering, Proceedings of the ASCE* **124**(8):757–770.

N. Gucunski & R. Woods (1991). 'Inversion of Rayleigh wave dispersion curve for SASW test'. In *Proceedings of the 5th international conference on soil dynamics and earthquake engineering*, pp. 127–138, Karlsruhe.

N. Gucunski & R. Woods (1992). 'Numerical simulation of the SASW test'. *Soil Dynamics and Earthquake Engineering* **11**:213–227.

J. Holland (1975). *Adaptation in natural and artificial systems*. University of Michigan Press, Ann Arbor, Michigan.

E. Kausel & J. Roësset (1981). 'Stiffness matrices for layered soils'. *Bulletin of the Seismological Society of America* **71**(6):1743–1761.

S. Kirkpatrick, et al. (1983). 'Optimization by simulated annealing'. *Science* **220**:671–680.

C. Lai, et al. (2002). 'Simultaneous measurement and inversion of surface wave dispersion and attenuation curves'. *Soil Dynamics and Earthquake Engineering* **22**(9-12):923–930.

A. Levshin, et al. (2005). 'The use of crustal higher modes to constrain crustal structure across Central Asia'. *Geophysical Journal International* **160**:961–972.

S. Nazarian & M. Desai (1993). 'Automated surface wave method: field testing'. *Journal of Geotechnical Engineering, Proceedings of the ASCE* **119**(7):1094–1111.

J. Nocedal & S. Wright (1999). *Numerical Optimization*. Springer, New York, USA.

A. O'Neill & T. Matsuoka (2005). 'Dominant Higher Surface-wave Modes and Possible Inversion Pitfalls'. *Journal of Environmental and Engineering Geophysics* **10**(2):185–201.

S. Pezeshk & M. Zarrabi (2005). 'A new inversion procedure for Spectral Analysis of Surface Waves using a genetic algorithm'. *Bulletin of the Seismological Society of America* **95**(5):1801–1808.

S. Rao (1996). *Engineering Optimization - Theory and Practice*. John Wiley & Sons, New York, third edn.

G. Rix, et al. (2000). 'In situ measurement of damping ratio using surface waves'. *Journal of Geotechnical and Geoenvironmental Engineering, Proceedings of the ASCE* **126**(5):472–480.

J. Suykens & J. Vandewalle (2002). 'Coupled Local Minimizers: Alternative formulations and extensions'. In *2002 World Congress on Computational Intelligence - International Joint Conference on Neural Networks IJCNN 2002*, pp. 2039–2043, Honolulu, USA.

J. Suykens, et al. (2001). 'Intelligence and cooperative search by Coupled Local Minimizers'. *International Journal of Bifurcation and Chaos* **11**(8):2133–2144.

A. Teughels (2003). *Inverse modelling of civil engineering structures based on operational modal data*. Ph.D. thesis, Department of Civil Engineering, K.U.Leuven.

A. Teughels, et al. (2003). 'Global optimization by Coupled Local Minimizers and its application to FE Model Updating'. *Computers and Structures* **81**(24-25):2337–2351.

K. Tokimatsu, et al. (1992). 'Effects of multiple modes on Rayleigh wave dispersion characteristics'. *Journal of Geotechnical Engineering, Proceedings of the ASCE* **118**(10):1529–1143.

D. Yuan & S. Nazarian (1993). 'Automated surface wave method: inversion technique'. *Journal of Geotechnical Engineering, Proceedings of the ASCE* **119**(7):1112–1126.

S. Zomorodian & O. Hunaidi (2006). 'Inversion of SASW dispersion curves based on maximum flexibility coefficients in the wave number domain'. *Soil Dynamics and Earthquake Engineering* **26**:735–752.