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HORIZONTAL COMPLIANCE FUNCTION OF ADJACENT SURFACE RIGID FOOTINGS IN HOMOGENEOUS SOIL LIMITED BY A SUBSTRATUM

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ABSTRACT

This article studies the dynamic interaction between surface rigid footings in a homogeneous viscoelastic soil limited by a substratum. The vibrations come from a rigid footings placed in soil layer and subjected to horizontal harmonic load. The required dynamic response of rigid surface footings constitutes the solution of the waves equations obtained by taking into account the conditions of interaction. The solution is formulated in 3-D frequency domain Boundary Element Method in conjunction with the Kausel-Peek Green's function for a layered stratum and quadrilateral constant element to study the dynamic interaction between adjacent footings.This approach allows the establishment of a mathematical model enabling us to determine the horizontal compliance function of adjacent footings according to theirs different separations, depth of substratum and the frequency of excitation.

Keywords: Dynamic foundation-soil-foundation interaction, homogeneous soil, boundary element method, Green's function, 3-D analysis.

INTRODUCTION

The dynamic interaction phenomenon foundation-soil-foundation has long been recognized as an important factor in the seismic and machine vibration response of structures, closely spaced structures and portions of a structure. Rational analysis of the phenomenon requires taking into account the dynamic nature of the interaction between the soil and the foundation. This is essentially a wave propagation problem with mixed boundary conditions (i.e. rigid body displacement under the foundations and none traction elsewhere).
Although a solution of a foundation-soil-foundation interaction problem in most cases involves a straightforward application of any of the well established soil-structure interaction methods, a relatively small number of limited 3-D investigations have appeared in the related literature. Furthermore, this is probably due to the substantial computational effort required by the Finite Element Method and the usual straightforward Boundary Element method formulations. Furthermore, there is a noticeable absence of simplified discrete models which is due perhaps, to the general lack of rigorous results which would be used for the verification and the calibration of such models.

Generally, the complicated geometries, loadings and soil conditions have discouraged the development of analytical solutions. The case of a rectangular foundation (square) embedded or not in a semi-infinite
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(e.g. Wong and Trifunac, 1975) developed an analytical solution based on the exact infinite series solution to study two-dimensional, antiplane, building-soil-building interaction for two or more buildings. The Finite Element Method has also been applied by (e.g. Kausel et al, 1975; Kausel and Roesset, 1975) in determining the behavior of rigid foundations resting on or embedded in a stratum over bedrock. However, (e.g. Gonzalez, 1974; Lin and Tassoulas, 1986; Lin et al, 1987) used the Finite Element Method in conjunction with consistent boundaries to analysis the cross-interaction between two foundations subjected to harmonic forces.A frequency domain Boundary Element Method formulation has been developed to treat wave propagation problems, soil-structure problems and structure-soil-structure problems which limit the discretization at the soil foundations interface. In this approach the field displacement is formulated as an integral equation in terms of Green's functions (e.g. Beskos, 1987). Using this method in conjunction with constant element and half-space Green's function of (e.g. Luco and Apsel, 1987; Wong and Luco, 1986) studied the cross-interaction between two rigid square surface foundations in layered viscoelastic half-space and subjected to external harmonic forces. (e.g. Wang et al, 1991) used the Boundary Element Method in conjunction with the constant elements and the half-plane Green's function for study the cross-interaction of two rigid or flexible surface or embedded strip foundations. (e.g. Qian and Beskos,1996; Qian et al, 1998) studied the harmonic wave response of two 3-D rigid surface foundations by Boundary Element Method in conjunction with isoparametric elements and the half-space Green's functions. Using the boundary element method in conjunction with constant element, full-space and relaxed boundary conditions, (e.g. Karabalis and Mohammadi, 1998; Mohammadi, 1992) studied the 3-D dynamic foundation-soil-foundation interaction on layered soil. (e.g.Wang and Schmid,1992) analysed the dynamic interaction between 3-D structures by coupling Finite Element and Boundary Elements Methods. However, (e.g. Qian et al, 1996) study the dynamic cross-interaction between flexible surface footings by combined Boundary Element and Finite Element Methods. Using a semi-analytical formulation (e.g. Gazetas and Roesset , 1979) analysed the 2-D problem of strip foundations on a layered half-space. Whereas, (e.g. Triantafyllidis and Prange, 1987, 1989; Liou, 1994) proposed a semi-analytical solution for the interaction study between small numbers of footings with regular shapes. (e.g. Waas, 1972 ; Kausel, 1974) developed a semi-discrete analytic method to model the far-field with homogenous boundary conditions for 2-D and axisymmetric problems (Thin Layer Theory). With the aid of the Thin Layer Theory, (e.g. Kausel and Peek, 1982) obtained the Green's functions for multilayer soil. This semi-discrete analytical model is then combined with the Finite Element Model or Boundary Element one of the near-field to solve the soil-structure interaction problems in layered media. Using this approach combined with Boundary Element Model (e.g. Boumekik et al, 1986; Boumekik, 1985) studied the 3-D problem of embedded foundations on a layered substratum, (e.g. Sbartai, 2002; Sbartai and Boumekik, 2002, 2003, 2003, 2004, 2005) analysed the transmission of the surface wave in the soil layer caused by machine foundation in one time and the dynamic foundation-soil-cavity interaction or foundation-soil-foundation on layered soil limited by a rigid bedrock in second time. In this work, the solution is formulated with Boundary Element method in frequency domain, this in conjunction with the (e.g. Kausel and Peek, 1982) Green's functions for a layered stratum using constant elements to study the dynamic interaction between adjacent foundations, where the foundation-soil-foundation interface and the free surface between adjacent foundations are discretized. Within the discretized medium, the Green's functions (displacement of the i^{th} element due to harmonic unit force applied on the j^{th} element) are calculated. Through the numerical studies, the effects of a set of parameters are considered, presented and discussed.

BASIC EQUATIONS

Model of calculation

The model of calculation is represented on the Fig.1. The two considered foundations are supposed to be rigid, of rectangular form (square) and placed at the surface or partially embedded in homogeneous soil limited by a substratum. The soil at height H, is supposed to be viscoelastic, linear and characterized by its mass density ρ, shear modulus G, its damping coefficient β and Poisson's ratio ν. The first footings are subjected to horizontal harmonic external force Px. It is assumed that the time dependence of the excitation is of the type $e^{i\omega t}$ for which ω denotes the angular frequency. For brevity, this time factor will be omitted in the sequel. The goal being to obtain the horizontal compliance functions of the two footings.

Displacements in an unspecified point "α "of the soil may be obtained from the solution of the wave equation:

$$((C_p^2 - C_s^2)u_{j,ik} + C_s^2 u_{k,ji} + C_p^2 \omega^2 u_k)\rho = 0 \qquad (1)$$

where

C_s, C_P are the celerities of the shearing and compression waves respectively, ω the angular frequency of excitation and ρ the mass density of the soil. The solution of equation (1) can be formulated by the following boundary integral equation:

$$u_\alpha = \int_S U_{\alpha\beta} . t_\beta . dS_\beta \qquad (2)$$

with

$U_{\alpha\beta}$ are the Green's functions which represent displacements in a point α due to a unit harmonic load (vertical and horizontal) applied in another point β of the soil and t_β represent a harmonic load distributed on a surface of the soil dS_β. As long as the medium is continuous, this last relation remains very difficult to evaluate. However, if the solid mass of the soil is discretized in an adapted way, this relation becomes algebraic and the displacement can then be calculated.

Discretization of the model

In this approach, the principle of the discretization of the soil solid mass is represented on the Fig.5. It is based on two types of discretization: the first one is horizontal and the second one is vertical. The horizontal discretization consists in subdividing any horizontal section of the soil solid mass in square elements of S_k sections. The average displacement of the element is replaced by the displacement of its center and for which the distribution of the constraints is supposed to be uniform. However the vertical discretization consists to subdivide the soil solid mass in under layers (Infinite Elements in the horizontal direction). Knowing that the thickness of each under layer is lower than the wavelength of Rayleigh (λ/10) it is possible to linearize the displacement of an under layer with another. In the discretized model, the Eq. (2) is expressed in algebraic form as follows:

$$d_\alpha = \sum_{\beta=1}^{NRT} \int_S U_{\alpha\beta} . t_\beta . dS_\beta \qquad (3)$$

where

NRT represents the total number of elements discretizing the soil between two foundations and the interface soil-foundations

Displacement matrix of discretized soil

The total matrix displacement of the soil is obtained by successive application of the loads units distributed on the constituent elements of the solid mass of the discretized soil. This matrix includes the terms of flexibilities of the soil which will be occupied by the first foundation (medium1), the soil which will be occupied by the second foundation (medium 2) and those of the coupling between the two mediums which can be written as follows:

$$[\mathrm{F_t}] = \begin{bmatrix} F_1 & F_{12} \\ & \\ F_{21} & F_2 \end{bmatrix} \tag{4}$$

where
F_1, ($3N_1 x 3N_1$) is the flexibility matrix of medium 1. F_2, ($3N_2$ x $3N_2$) is the flexibility matrix of medium 2. F_{12}, ($3N_1$ x $3N_2$) is the flexibility matrix of coupling of medium 1 on the medium 2. F_{21}, ($3N_2$ x $3N_1$) is the flexibility matrix of coupling of medium 2 on the medium 1. N_1 and N_2 are respectively the number of elements discretizing medium 1 and medium 2.
The Displacements in the two mediums are expressed then by:

$$\{\mathrm{d}_1\} = [F_1]\{t_1\} + [F_{12}]\{t_2\} \tag{5}$$

$$\{\mathrm{d}_2\} = [F_2]\{t_2\} + [F_{21}]\{t_1\} \tag{6}$$

where
$\{\mathrm{t}_1\}$ represent the vector charges of the medium 1. $\{\mathrm{t}_2\}$ represent the vector charges of the medium 2.
$\{\mathrm{d}_1\}$ represent the vector displacements of the medium 1. $\{\mathrm{d}_2\}$ represent the vector displacements of the medium 2 representing the same characteristics as the vector $\{\mathrm{d}_1\}$.

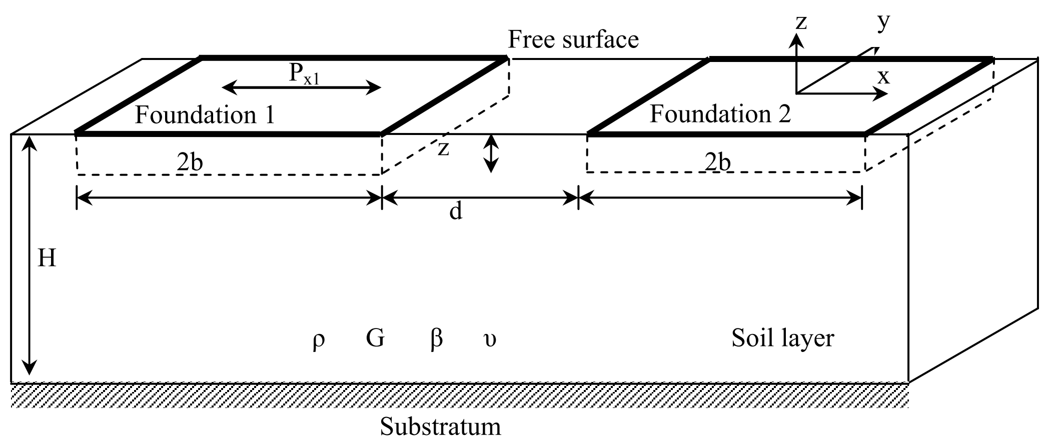

Figure 1. Model of calcul

Condition of compatibility and equilibrium
When the two footingss are in place, they impose their displacements on the various sections which will be constrained to move like a rigid body. For all the elements of the model, one can write the following relations:

$$\{\mathrm{d}_1\} = [R_1]\{D_1\} \tag{7}$$

$$\{\mathrm{d}_2\} = [R_2]\{D_2\} \tag{8}$$

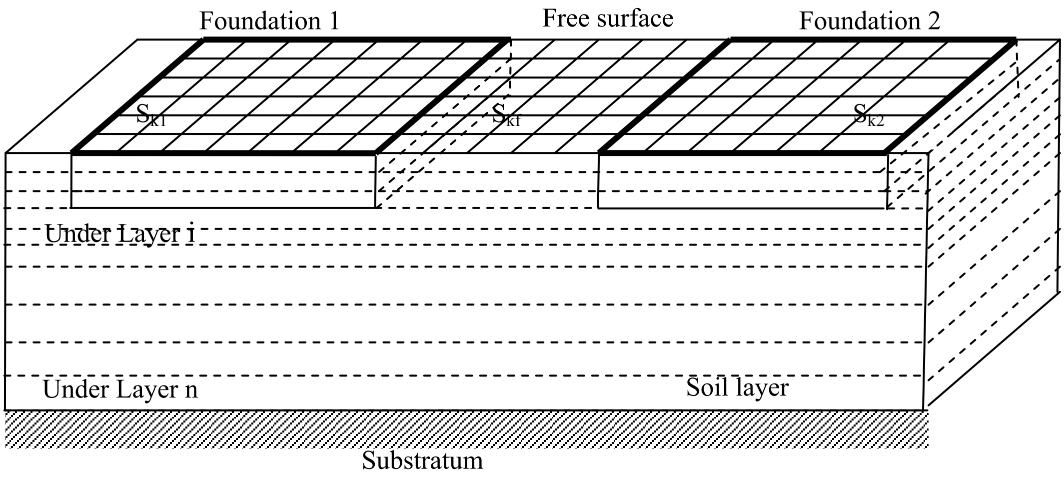

Figure 2. Horizontal and vertical discretization

$\{D_1\}=\{\Delta_x,\Delta_y,\Delta_z,\varphi_x,\varphi_y,\varphi_z\}_1^t$ the vector displacement of the first foundation for the 6 degrees of freedom considered; $\{D_2\}=\{\Delta_x,\Delta_y,\Delta_z,\varphi_x,\varphi_y,\varphi_z\}_2^t$ the vector displacement of the second foundation (6 degrees of freedom) considered and $[R_1]=[R_1,R_2,......,R_k,......,R_{N_1}]^t$ is a matrix of transformation of dimension ($3N_1$ x 6), depending only on the geometrical characteristics of the discretized volume of the soil of the first foundation where under matrix is given by:

$$[R_1]_k=\begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}_k \tag{9}$$

in which x_k , y_k and z_k are the co-ordinates of the element k compared to the center of the foundation.
$[R_2]=[R_1,R_2,......,R_k,......,R_{N_2}]^t$ is also a matrix of transformation of dimension ($3N_2$ x 6), depending only on the geometrical characteristics of the discretized volume of the soil of the second foundation where under matrix $[R_2]_j$ is similar to that of the relation (9) in which x_j, y_j and z_j are the co-ordinates of the element j compared to the center of the second foundation. If one notes P_{i1} and M_{i1} the components of the vector charges applied to the first foundation, the equilibrium between the latter and the forces distributed on the elements discretizing the volume of the foundation are expressed for the loads of translations and rotations by:

$$P_{i1}=\sum_{k=1}^{N1} t_k=\sum_{k=1}^{N1}\begin{pmatrix} h \\ t \\ n \end{pmatrix}_k \qquad (i = x,\ y,\ z) \tag{10}$$

$$M_{i1}=\sum_{k=1}^{N_1}\begin{pmatrix} y\cdot n - z\cdot t \\ z\cdot h - x\cdot n \\ x\cdot t - y\cdot h \end{pmatrix}_k \qquad (i = x,\ y,\ z) \tag{11}$$

These two last expressions can be put in the following matrix form:

$$\{P_1\} = [R_1]\cdot\{t_1\} \tag{12}$$

Same manner, one can write the same thing for the second foundation one has then:

$$P_{i2} = \sum_{j=1}^{N2} t_j = \sum_{j=1}^{N2} \begin{pmatrix} h \\ t \\ n \end{pmatrix}_j \qquad (i = x,\ y,\ z) \tag{13}$$

$$M_{i2} = \sum_{j=1}^{N2} \begin{pmatrix} y\cdot n - z\cdot t \\ z\cdot h - x\cdot n \\ x\cdot t - y\cdot h \end{pmatrix}_j \qquad (i = x,\ y,\ z) \tag{14}$$

These two last expressions can be put in the following matrix form:

$$\{P_2\} = [R_2]\cdot\{t_2\} \tag{15}$$

Response of the model

The relation binding the vector directly charges external $\{P_1\}$ applied to the centre of gravity of the footing with the vectors displacements $\{D_1\}$ and $\{D_2\}$ can be expressed starting from the relations (5), (6), (7), (8) and (12) by:

$$\{P_1\} = [K_1]\cdot\{D_1\} + [K_{12}]\{D_2\} \tag{16}$$

The relation binding the vector charges external applied to the centre of gravity of the second footing to the vectors displacements starting from the relations (5), (6), (7), (8) and (15) by:

$$\{P_2\} = [K_2]\cdot\{D_2\} + [K_{21}]\{D_1\} \tag{17}$$

If the second foundation is unloaded ($P_2 = 0$), the Eqs. (16) and (17) becomes:

$$\{P_1\} = [K_1]\cdot\{D_1\} + [K_{12}]\cdot\{D_2\} \tag{18}$$

$$\{0\} = [K_2]\cdot\{D_2\} + [K_{21}]\cdot\{D_1\} \tag{19}$$

From there system one can write:

$$[C_{11}] = \left[[K_1] - [K_{12}][K_2]^{-1}\cdot[K_{21}]\right]^{-1} \tag{20}$$

is the compliance matrix of the loaded foundation and

$$[C_{12}] = -[K_2]^{-1}\cdot[K_{21}][C_{11}] \tag{21}$$

is the coupling compliance matrix of the unloaded foundation.
where
K_1 is the dynamic stiffness matrix of the loaded footing (1).
K_2 is the dynamic stiffness matrix of the unloaded footing (2).
K_{12} is the coupling matrix of the loaded footing on the unloaded footing.

In the following, these two last relations are used to analyses the dynamic interaction between two surface square rigid footings placed on homogenous soil where only one footing is loaded.

VALIDATION FOR ADJACENT FOUNDATIONS

The results of this work will be validated while comparing results them obtained by the present study at those obtained by the 3-D frequency domain BEM formulation of Karabalis et al [20]. The comparison relates to the case of a square foundations placed at the surface of a viscoelastic and isotropic semi-infinite soil having the following characteristics: $\rho = 1$; $G = 1$; $Cs = 1$; $\nu = 0.333$; , $\beta = 0.05$, $H/b = 16$ (to approach the semi-infinite one) with $b = 1/2$ is the half wide of the foundations. Only the first footing is loaded with the unit vertical force $P_z = 1$, however the second footing is unloaded. The dimensionless vertical compliance C_v is defined as:

$$C_{11} = Gb \cdot K^{-1} \tag{22}$$

$$C_{12} = Gb^2 \cdot K^{-1} \tag{23}$$

The soil is discretized horizontally in 36 quadrilateral constant elements on the soil-footings interfaces and 36 quadrilateral constant elements on the free surfaces between the footings. For the vertical discretization the depth of the substratum will be subdivided in 10 under layers. The compliances are calculated at relative distance $d/b = 2$ between two footings versus different dimensionless frequency a_0. The results thus presented on (fig.3) are practically comparable and the maximum errors are observed only in static case.

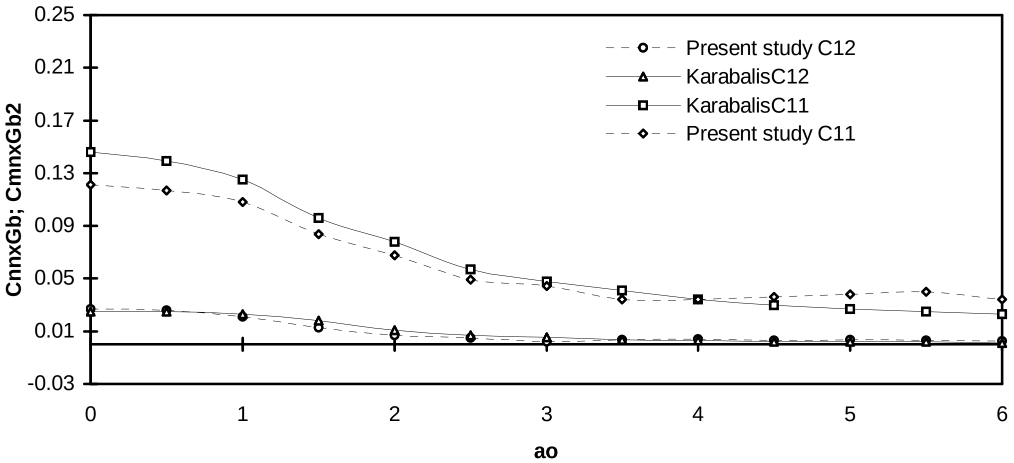

Figure 3. Validation of vertical compliance of two adjacent square foundations on half-space C_{11} and C_{12} for varying frequency with $d/b = 2$ and $\beta = 0.05$

PARAMETRIC ANALYSES

In this application, the considered foundation is subjected to unit horizontal force $P_x = 1$ for different dimensionless frequency $a_0 = \omega b/2C_s$. The soil is discretized of the same manner that used in the preceding paragraph (validation) and is characterised by $\rho = 1$, $G = 1$, $\nu = 0.333$ and $\beta = 0.05$. For this, the horizontal compliance $C_{11}xGb$ of loaded foundation and the horizontal coupling compliance $C_{12}xGb^2$ of the unloaded one have been studied for different cases of relative depth layer stratum (H/b = 2, 4, 8 semi-infinite) according to relative frequency a_o. In Figs. (4) and (5) the effect of the soil layer depth according to the frequencies is examined while the foundations are massless, the distance ration between foundations is $d/b = 2$, and the damping level is kept constant at $\beta = 0.05$. The dimensionless horizontal compliance $C_{mn}(\omega)$ indicates the horizontal compliance of the foundation "n" when the foundation "m" is loaded with horizontal force. While varying the depth of the substratum according to the frequencies we note:

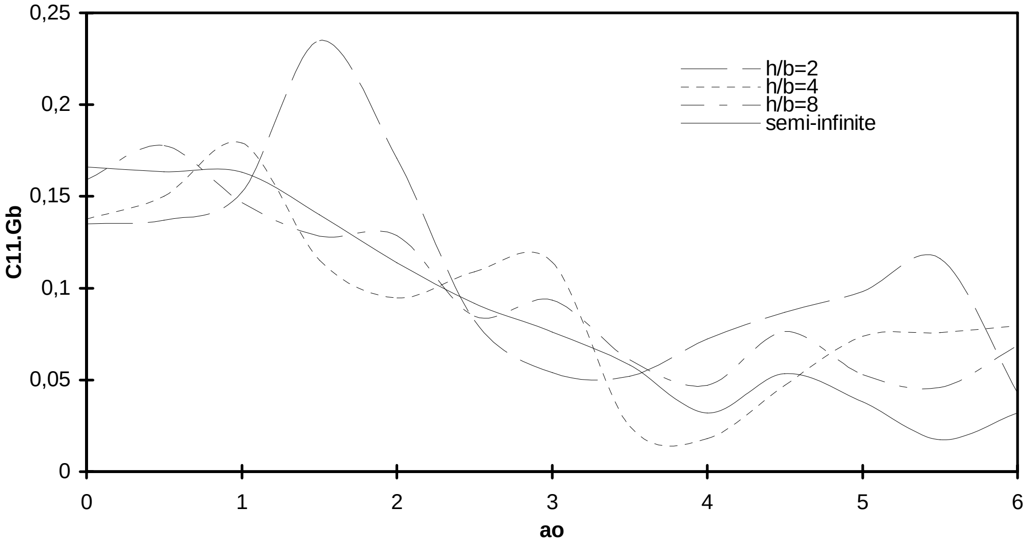

Figure 4. Horizontal compliance C_{11} of two adjacent square foundations on stratum over rigid bedrock for varying depths of the stratum with d/b = 2 and β = 0.05

- The static response increases when the layer depth increases.
- The response of the foundation on the stratum approaches the semi-infinite solution when layer depth increases ($h/b \geq 8$).
- A remarkable shift in the resonant frequencies.
- A variation in the peaks of resonance.
- An important variation in the magnitude of the resonant frequencies when the substratum is not very deep.

The behaviour of the unloaded foundation is similar to that described above for the loaded foundation, the only difference being that the magnitude of the resonant peak increases when the soil layer depth increases. We noted that the values of horizontal C_{12} are more important than for the vertical one. That means than the unloaded foundation is more affected by interaction phenomenon for the horizontal mode than for the vertical one.

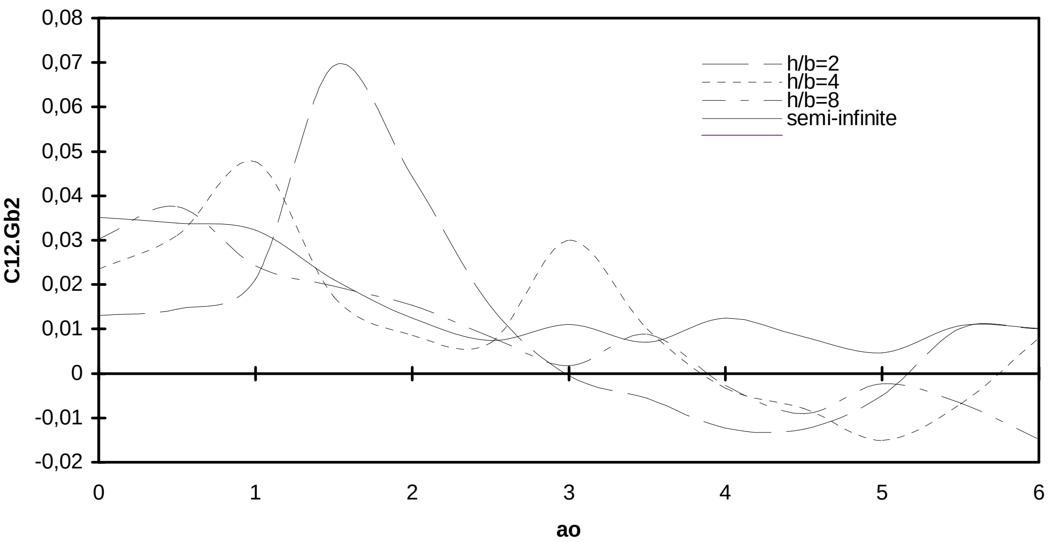

Figure 5. Horizontal compliance C_{12} of two adjacent square foundations on stratum over rigid bedrock for varying depths of stratum with d/b =2 and $\beta = 0.05$

CONCLUSIONS

In this paper, the dynamic interaction between two surface rigid footings resting on homogeneous viscoelastic soil subjected to horizontal harmonic external force excitation has been developed and fully tested. The solution is formulated in frequency domain Boundary Element Method in conjunction with the Kausel-Peek Green's function for a layered stratum and quadrilateral constant element to study the dynamic interaction between adjacent footings with which the parameters of interaction structure-soil-structure in a soil layer profile will be numerically given. The advantage of the method used lies in limited the enough number of elements used in the discretization of the model on the one hand and the taking into account of the heterogeneity of the soil on the other hand. This study shows well us the great importance of the interaction foundation-soil-foundation which proves to be different from the interaction soil-foundation (single foundation). To this end, we recommend to take into account this phenomenon in account for the study of any structure.

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