

## LABORATORY STUDY OF DYNAMIC PROPERTIES OF BABOL SAR AND TOYOURA SANDS UNDER VARIOUS CONFINING PRESSURES WITH BENDER ELEMENT TESTS

Fardin JAFARZADEH<sup>1</sup>, and Nima ALA<sup>2</sup>

### ABSTRACT

Small strains shear modulus of soils ( $G_{max}$  or  $G_0$ ) is a key parameter in both static and dynamic analysis. This parameter with the state of stress and shear strain amplitude is used for predicting the dynamic behavior of soils and dynamic response of geotechnical structures. To evaluate the maximum shear modulus from laboratory tests, propagation of seismic waves by means of piezoceramic transducers has been widely adopted in the last decade, termed simply “bender elements”, housed in triaxial apparatus. The bender element method is a fast, simple and useful technique to obtain the shear modulus at small strain levels ( $G_{max}$ ) in soils by measuring shear wave velocity through a specimen. This paper presents a typical evaluation of the shear modulus for two various granular materials along isotropic stress paths. The effect of the mean effective stress on the elastic properties of unbound granular materials is a well-known experimental result. Power laws between the mean effective stress and the shear modulus of two natural sands are established for isotropic stress paths using bender elements. The coefficients of such power laws that relate the maximum shear moduli of a sandy soil to the confining pressure and its initial void ratio are proposed for both of sands in dry and fully saturated conditions. These observations seem to be a typical feature of the behavior of granular materials. This study made an attempt to provide the reference information for the natural sand commonly find in Northern Iran, so called Babolsar Sand and the Japanese Standard Sand, Toyoura. A series of isotropic consolidation triaxial tests were conducted on reconstituted Babolsar and Toyoura Sands under dry and fully saturated conditions with various densities. Concurrent shear wave velocities were measured using bender elements with various pulsating frequencies in order to study the effects of specimen density, confining pressure, and the transmitting frequency of bender element blades in the  $G_{max}$  of the Soil. Similar tests were, finally, performed on Toyoura Sand in order to verify the accuracy and reliability of the results. All the mentioned results are presented in this paper.

Keywords: Bender element tests, Babolsar sand, Toyoura sand, Small strains, Shear modulus

### INTRODUCTION

The design of foundations and excavations requires the prediction of ground movements and hence a knowledge of the relevant stress-strain properties (Mair, 1993). It is now appreciated that under working conditions the strains in the ground surrounding foundations or excavations may be relatively small. Therefore it is important to measure stiffness at small strain levels  $< 0.1\%$ , since stress-strain relationships are non-linear (Burland, 1989; Atkinson, 2000). Figure 1 is an idealization of soil stiffness over a large range of strain, from very small to large, and approximately distinguishes strain

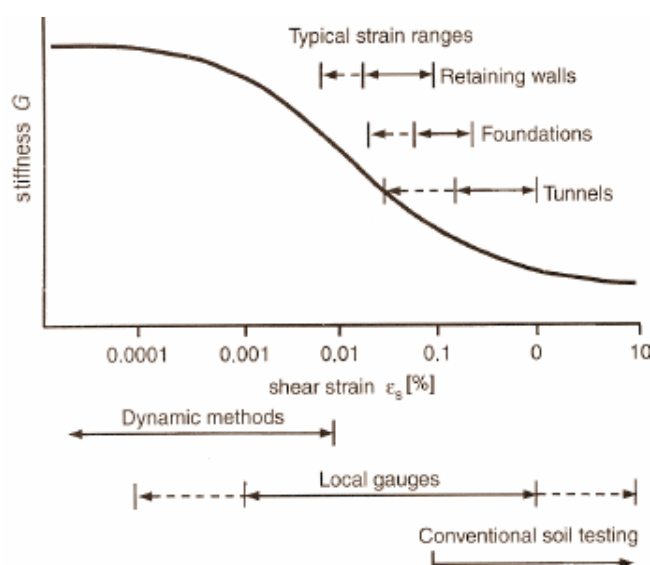
---

<sup>1</sup> Assistant Professor, Department of Civil Engineering, Sharif University of Technology, Tehran, Iran, [fardin@sharif.edu](mailto:fardin@sharif.edu)

<sup>2</sup> M.Sc. Student of Geotechnical Engineering, Sharif University of Technology, Tehran, Iran, [nimaala@mehr.sharif.edu](mailto:nimaala@mehr.sharif.edu)

ranges. At very small strains the shear modulus ( $G$ ) reaches a nearly constant limiting value  $G_0$ . This figure summarizes the current understanding of the variation of soil stiffness.

The “bender elements” technique has had an ongoing progress in the last two decades. It is now frequently associated with the triaxial apparatus in order to determine the elastic properties of soils such as sands, gravels, clays, etc. For instance, many theoretical works (Bates, 1989; Viggiani & Atkinson, 1995; Brignoli *et al.*, 1996; Jovicic *et al.*, 1996; Arulnathan *et al.*, 1998; Blewett *et al.* 2000) have led to a more accurate measurement of the velocity of the shear waves generated by these piezoelectric transducers. A large quantity of experimental results was also published in the literature. This additional contribution demonstrates a typical evolution of the maximum shear modulus for two granular materials along isotropic stress paths, Babolsar sand and Toyoura sand.



**Figure 1. Characteristic Stiffness-Strain behavior of soils with typical strain ranges for laboratory tests and structures (Atkinson & Sallfors, 1991; Mair, 1993)**

## MATERIALS

Two granular materials were used in this experimental study:

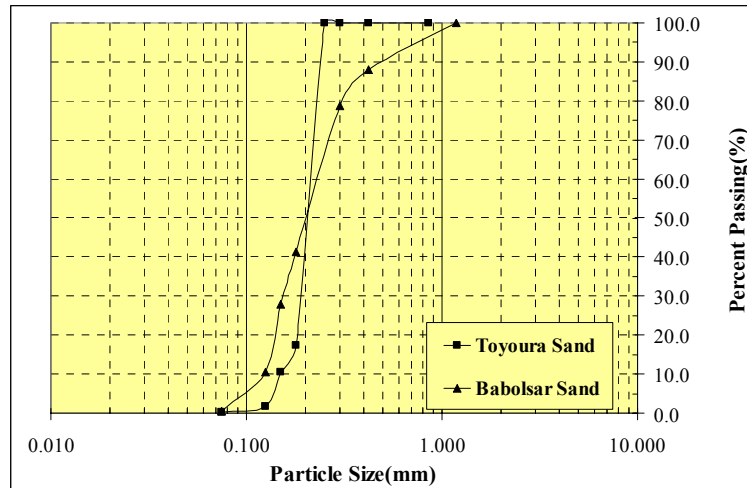
- A fine and siliceous sand named Babolsar Sand; is natural sand obtained from northern Iran
- Toyoura Sand; is a well-known Japanese standard sand

The basic properties of the soils (Index Properties) are given in Table 1.

**Table 1. Properties of the granular materials**

Granular Material	$G_s$	$e_{\max}$	$e_{\min}$	$D_{10}$	$D_{50}$	$C_u$	$C_c$
Babolsar Sand	2.777	0.815	0.578	0.13	0.21	1.77	0.97
Toyourea Sand	2.660	0.914	0.673	0.15	0.21	1.47	0.98

Particle Size Distribution Curve for both sands has been shown in Figure 2. These sands are classified as poorly graded sand, SP, according to the Unified Soil Classification System.



**Figure 2. Particle size distribution curve of Babolsar and Toyoura Sands**

## LABORATORY TEST EQUIPMENT

A Bender Element system mounted on a triaxial apparatus made by *WFI (Wykeham Farrance International)* was used for this study with a special triaxial cell (Figure 3). The apparatus has a maximum capacity of 50 *kN* for the vertical load and 2 *MPa* for the pressure cell. The cell structure is very stiff and consists of two platens connected by three tie rods located outside a Perspex pressure cell. The device is equipped with the following equipments:

- A pair of piezoceramic bender element transducers mounted on the top cap and base pedestal;
- A TGA1241 Signal Generator which when used with the Bender Elements produces user defined pulsed signals to the Bender Receiver;
- A ADC-212 Oscilloscope with its PC parallel connector and mains adaptor;
- A pair of LVDT, one for local measurement of the axial strain, and the next with the Volume Change device for measuring the Volume Change;
- A load cell located inside the pressure cell;
- A pressure transducer to measure the pore pressure at the bottom of the specimen;
- A 16 Channel Data-Logger
- A pressure gage to measure the cell pressure;
- De-aired Water Tank and its Vacuum Pump and De-aired Water Panel;
- Distribution Panel and a pair of Bladders;
- Volume Change Device
- A 50 *kN* loading frame.

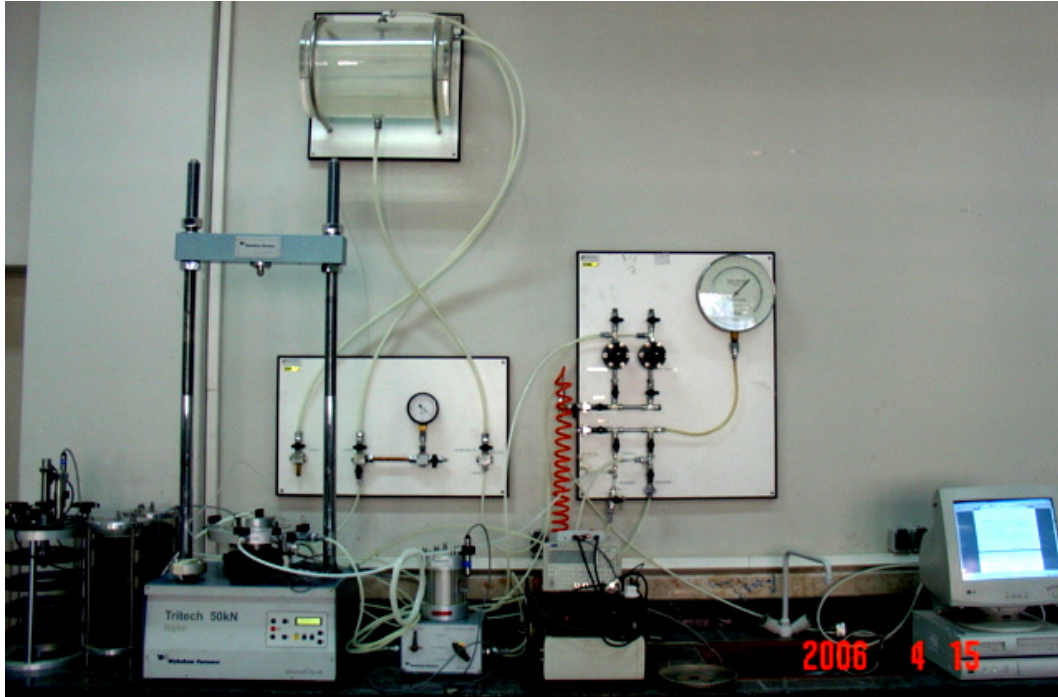


Figure 3. General view of the triaxial apparatus with bender elements of Sharif University of Technology

## TEST METHOD AND SAMPLE PREPERATION

Consolidated drained triaxial tests were carried out in the laboratory. Cylindrical specimens were 100 mm in diameter and 200 mm in height. The preparation procedure was not changed by the pair of bender elements embedded in the lower and upper pedestals of the triaxial cell. Loose, medium-dense, and dense samples of Babolsar and Toyoura sands were reconstituted with dry deposition method. The relative densities ( $D_r$ ) of the specimens were 20%, 50%, and 80% respectively. Medium-dense and dense specimens were deposited in six layers and were compacted by tamping. Because the specimens are not cohesive, after a specimen was formed in a split mold, a 30 kPa negative pressure, which will not consolidate the specimen, was applied to support the specimen. After the mold was removed and the pressure chamber and other components were installed, a 10 kPa cell pressure was applied and the negative pressure was removed. Then the required full confining pressure was applied. As the minimum confining pressure in the tests was 10 kPa, this procedure prevented overconsolidation prior to testing.

The saturation process comprised three stages: gaseous carbon dioxide percolation, de-aired water percolation, and step by step application of back pressure up to 100 kN/m<sup>2</sup> which guaranteed B-values of at least 0.95. The direction of de-aired water percolation was from bottom to top of the specimen.

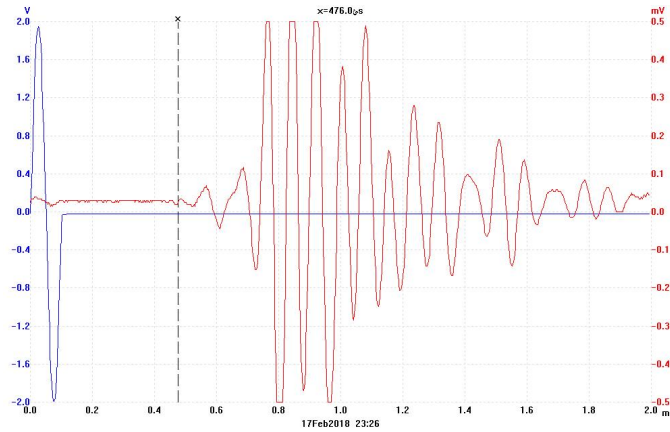
The bender elements, which are piezoelectric transducers, provide a non destructive and punctual measure of the shear modulus. Application of an impulse is converted into tangential vibrations of the transmitter, which cause a shear wave to be propagated through the sample. This phenomenon is reversible, and then a mechanical force on the receiver produces an electric field. The input and output electric signals were recorded on an oscilloscope on which the propagation time,  $T_w$ , of the shear wave could be determined, according to the instructions. According to relevant literature, results of numerical analysis of the bender element tests demonstrate that the travel time should not be taken as the time corresponding to the first deflection of the received signal, which may lead to overestimation of the shear modulus of up to about 14%, and can be taken as the point of the first inversion of the received signal (Viggiani *et al.*, 1995). An example of the transmitted and received signals which were

seen on the PC screen is shown in Figure 4. The distance of propagation  $L_w$  is the length between the tips of the bender elements (180 mm). The velocity of the shear wave is therefore:

$$V_w = L_w / T_w \quad (1)$$

Owing to the direction and the polarization of the shear wave, only the component  $G_{\max}$  of the small strain stiffness matrix can be identified, assuming a linear cross-anisotropic elastic behavior. The shear modulus is related to the unit mass  $\rho$  of the material and to the shear wave velocity  $V_w$ :

$$G_{\max} = \rho \times [L_w / T_w]^2 \quad (2)$$



**Figure 4. Travel Time diagram for dry Toyoura sand specimen using 10 kHz sinusoidal transmitted frequency at the 250 kPa confining pressure,  $D_r=50\%$**

## TESTING PROGRAM

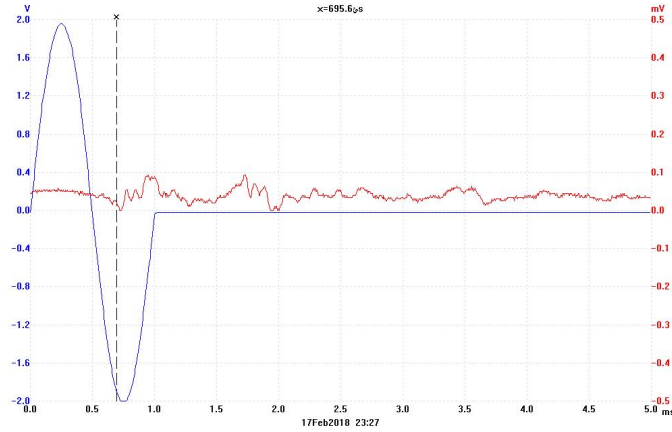
Travel Time Measurements were carried out on specimens with the Relative Densities ( $D_r$ ) equal to 20%, 50%, and 80%. Tests were performed on dry and fully saturated samples of both Sands. Specimens were consolidated isotropically with effective confining pressures increasing gradually from 10 kPa to 750 kPa in the steps equal to 10, 20, 50, 100, 250, 500, and 750 kPa. In order to examine the effect of the transmitted signal frequency and verify the Nyquist Rate, 7 different frequencies equal to 1, 5, 10, 15, 30, 40, and 50 kHz were applied to the bender element transmitter in each stress level. The effect of wave shape also inspected by means of two wave forms: Sinusoidal and Square. The total number of tested specimens was 21 which consist of six Toyoura sand specimens and fifteen Babolsar sand specimens. Five of Babolsar sand tests were repeated in order to examine the repetition of tests.

According to sampling theorem, an analog signal can be exactly reconstructed from its samples if the sampling rate is at least twice the highest frequency component present in the signal. This means, if the signal contains the highest frequency component of 1 kHz, the sampling rate must be at least 2 Kilo Samples/Second. This sampling rate is also known as Nyquist Rate (Oppenheim, 1997). So the rate is equal to 2 in theory, but should take at least 5 in practice. Violation of the sampling theorem means sampling the signal at a sample rate less than Nyquist rate. This leads to unacceptable distortion in the signal. This distortion is known as aliasing.

In this research the triggered rate of the input and output signals was adjusted to 2 ms (0.002 s). So the sampling rate is 2 Kilo Samples/Second. According to the theory, thus, the transmitted signal frequency should be at least 1 kHz but not less than 2.5 kHz in practice.

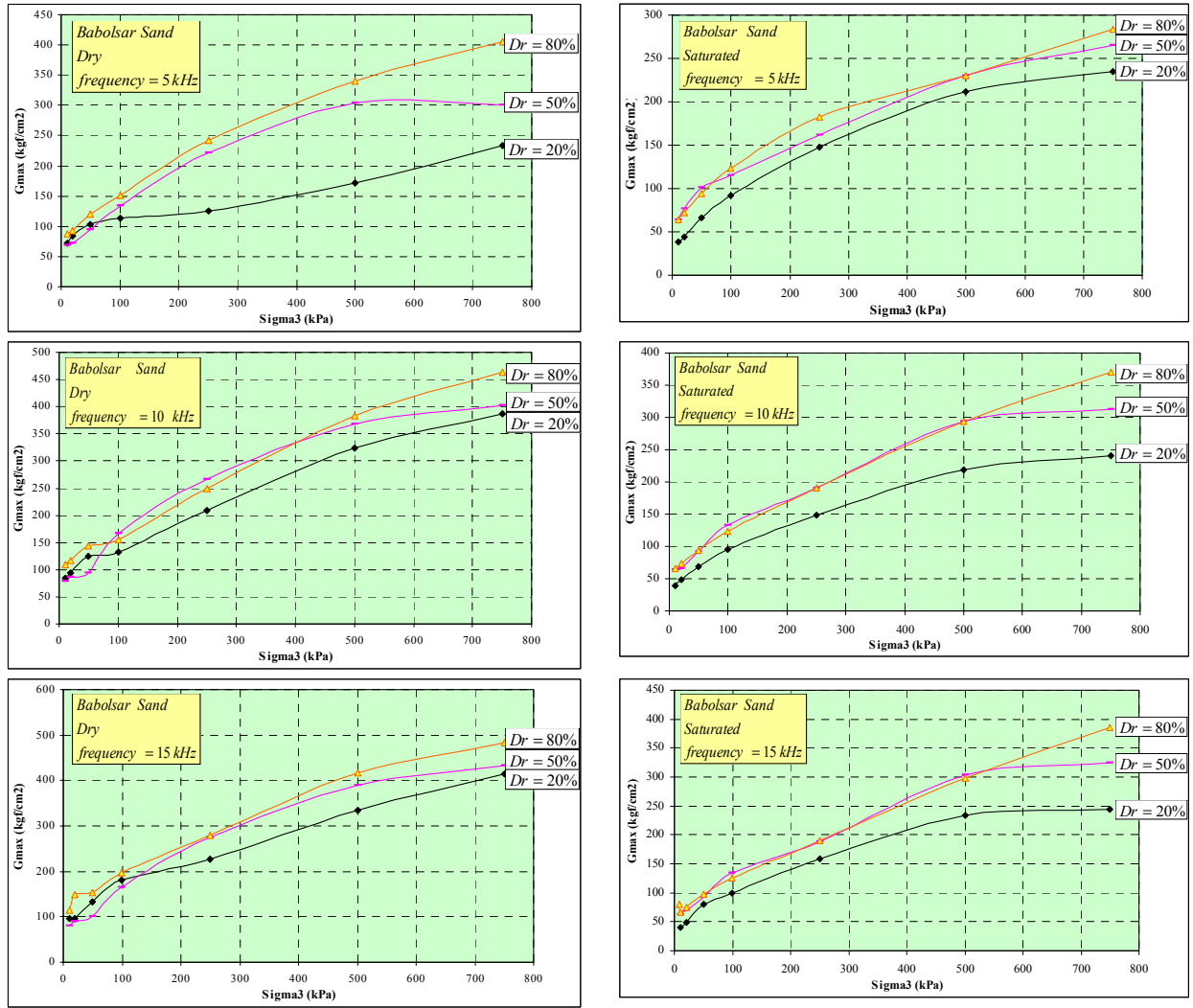
## TEST RESULTS AND ANALYSIS

According to practical Nyquist Rate, all of the travel times measured by means of application of transmitted signal equal to 1 kHz was distorted and not acceptable. An example of the aliased received signal is shown in Figure 5. Additionally, application of the higher frequencies, such as 30, 40, and 50 kHz, are lost of contrast. The data, therefore, presented in this paper are those related to the transmitted frequencies equal to 5, 10, and 15 kHz since the best resolution in received signal is attained via these frequencies.



**Figure 5. An example of the aliased received signal corresponding to 1 kHz sinusoidal transmitted signal on Toyoura dry specimen at the 250 kPa confining pressure,  $D_r=50\%$**

The original test data for 5, 10, and 15 kHz transmitted signals of  $G_{max}$  versus the effective confining pressure,  $p'$ , and the relative density,  $D_r$ , of the Babolsar sand are plotted in Figure 6 as a surface. Figure 7 shows the same data for Toyoura sand. From the figures it can be inferred that the values of  $G_{max}$  increase with the increase of confining pressure.



**Figure 6. Dependence of  $G_{max}$  to the effective confining pressure,  $p'$ , and the relative density,  $D_r$  (Babolsar Sand)**



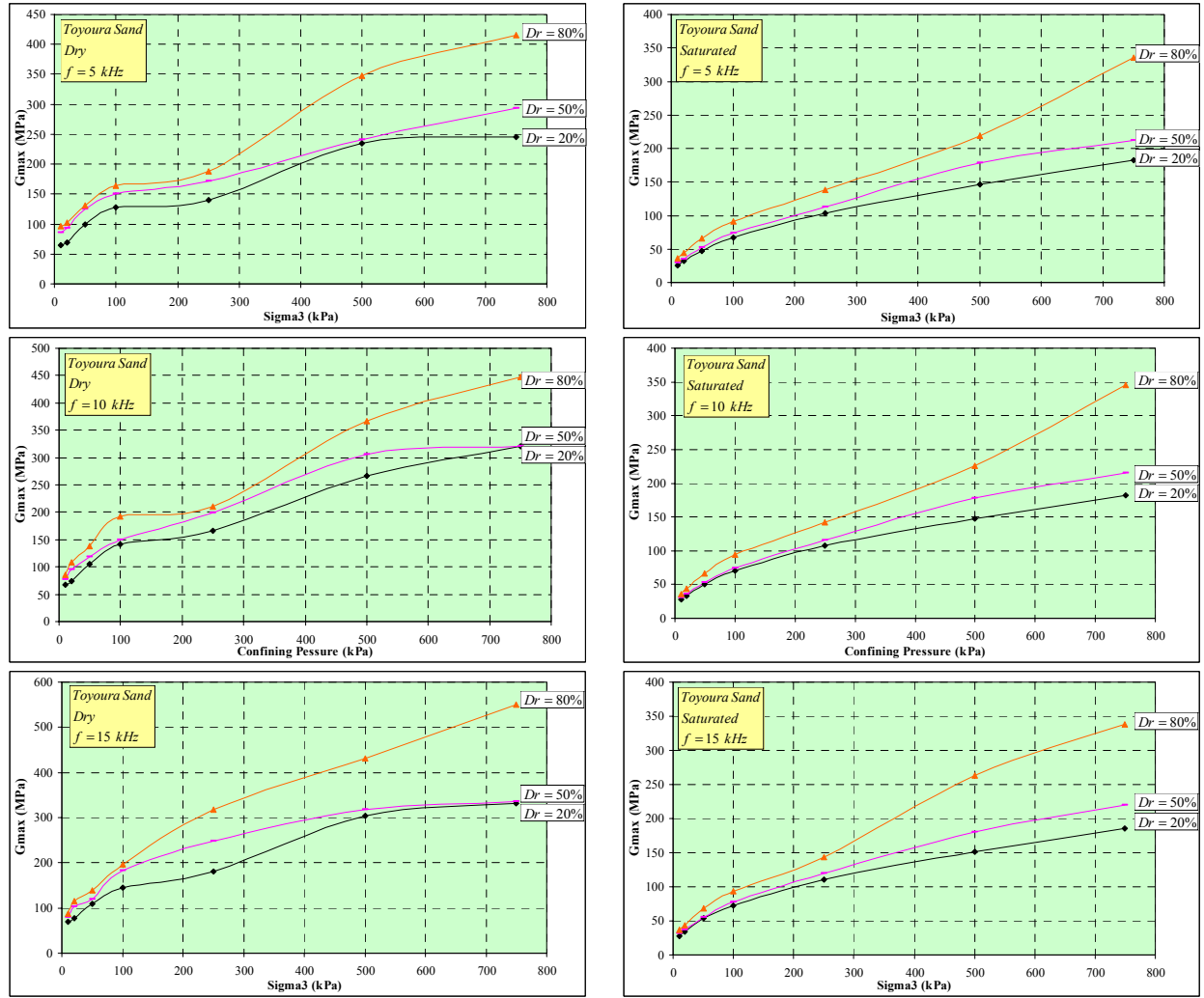


Figure 7. Dependence of  $G_{\max}$  to the effective confining pressure,  $p'$ , and the relative density,  $Dr$  (Toyouura Sand)

## EVOLUTION OF THE SHEAR MODULUS ALONG ISOTROPIC STRESS PATHS

A critical review of the literature shows that the shear modulus of unbound granular materials mainly depends on the void ratio,  $e$ , and on the effective principal stresses  $\sigma'_1$  and  $\sigma'_3$ . However, during triaxial tests, the minor principal stress  $\sigma'_3$  is kept constant. Therefore, the shear modulus  $G_{\max}$  is expressed as a function of the void ratio and of the mean effective stress  $p'$  such as (Hardin & Richart, 1963; Iwasaki & Tatsuoka, 1977):

$$G_{\max} (kN/m^2) = A \times f(e) \times p'^n (kN/m^2) \quad (3)$$

Where  $A$  and  $n$  are material constants and  $f(e)$  a function of the void ratio (Table 2). In contrast with previous works, two separate series of parameters are proposed in this article for dry and saturated specimens. The values of the two parameters  $A$  and  $n$  proposed by the authors previously mentioned are clarified in Table 2. These values are determined by application of curve fitting techniques to all experimental data. All the references can be found in Dano (2002).



**Table 2. Relations between  $G_{max}$ ,  $e$  and  $p'$  (Kokusho, 1987)**

Constants in proposed empirical equations on small strain modulus:  $G_0 = A.F(e)(\sigma'_0)^n$  (Kokusho, 1987)

	Reference	A	F(e)	n	Soil material	Test Method
Sand	Hardin-Richart (1963)	7000	$(2.17-e)^2/(1+e)$	0.5	Round grained Ottawa sand	Resonant Column
		3300	$(2.97-e)^2/(1+e)$	0.5	Angular grained crushed quartz	Resonant Column
	Shibata-Soelarno (1975)	42000	$0.67-e/(1+e)$	0.5	Three kinds of clean sand	Ultrasonic Pulse
	Iwasaki et al. (1978)	9000	$(2.17-e)^2/(1+e)$	0.38	Eleven Kinds of clean sand	Resonant Column
	Kokusho (1980)	8400	$(2.17-e)^2/(1+e)$	0.5	Toyoura sand	Cyclic Triaxial
	Yu-Richart (1984)	7000	$(2.17-e)^2/(1+e)$	0.5	Three kinds of clean sand	Resonant Column
Clay	Hardin-Black (1968)	3300	$(2.97-e)^2/(1+e)$	0.5	Kaolinite, etc.	Resonant Column
	Marcuson-Wahls (1972)	4500	$(2.97-e)^2/(1+e)$	0.5	Kaolinite, I* <sub>p</sub> =35	Resonant Column
		450	$(4.4-e)^2/(1+e)$	0.5	Bentonite, I <sub>p</sub> =60	Resonant Column
	Zen-Umehara (1978)	2000~4000	$(2.97-e)^2/(1+e)$	0.5	Remolded clay, I <sub>p</sub> =0~50	Resonant Column
	Kokusho et al. (1982)	141	$(7.32-e)^2/(1+e)$	0.6	Undisturbed clays, I <sub>p</sub> =40~85	Cyclic Triaxial

\*  $\sigma'_0$ : kPa,  $G_0$ : kPa, \*\*  $I_p$ : Plasticity Index

Four distinct power laws, which give evidence of the fabric change due to increase in confining pressure and also the effect of saturation, fit the experimental values of the shear modulus  $G_{max}$  of this research. The values of the parameters  $A$  and  $n$  of the Equation (3) are indicated in Table 3 for the sands presented in this research. In this table  $e$  is the initial void ratio; the power  $n$ , which characterizes the quantity of grain-to-grain contacts, was close to 0.5 for dry specimens but was closer to 0.4 for saturated specimen. The deviation from the predictions of the Hertz's theory ( $n = 1/3$ ) can be explained by the nature of the contacts, not necessarily punctual.

**Table 3. Experimental values of  $A$  and  $n$** 

$G_{max} (kPa) = A \frac{(x-e)^2}{1+e} p_0'^n (kPa)$				
Granular Material	Moisture Condition	A	x	n
Babolsar Sand	Dry	4350	4	0.398
	Saturated	21550	1.93	0.416
Toyoura Sand	Dry	38000	2	0.37
	Saturated	12900	1.94	0.476

## CONCLUSIONS

Triaxial tests under isotropic confinement were carried out on two unbound granular materials, Babolsar sand and Toyoura sand. The shear modulus was continuously measured using piezoelectric transducers. The following conclusions can be derived from the results of triaxial compression bender element tests on dry and saturated specimens of both sands:

- Shear moduli ( $G_{max}$ ) of sands increases with increase in confinement
- Travel times measured by means of application of transmitted signal less than the practical Nyquist Rate are distorted and not acceptable.
- The best resolutions in received signal are attained by the Nyquist Rate between 10 and 20.
- Application of the higher Nyquist Rate, such as 20 or 25 are lost of contrast.
- Shear moduli of dry specimens of sand were higher than shear moduli of saturated specimens which may be due to the particle lubrication effects of water.

- The effect of increase in relative density of soils on shear moduli is increase of the  $G_{\max}$  but the cause is less than the effect of confinement.
- Shear moduli determined by use of square waves was greater than ones determined via sinusoidal waves. So it can be inferred that usage of sinusoidal wave shapes are better since the results are more in conformity with the literature.

## REFERENCES

- Arulnathan, R., R.W. Boulanger, and M.F. Riemer (1998), “*Analysis of bender element tests*,” Geotechnical Testing Journal, 21(2), 120-131.
- Atkinson J. H. and Sallfors G. ( 1991)—*Experimental determination of soil properties*. General Report to Session 1. Proc. 10th ECSMFE, Florence, 3: 915–956.
- Atkinson J. H. (2000) — *Non-linear soil stiffness in routine design*. Geotechnique, 50 (5): 487–508.
- Bates, C.R. (1989), “*Dynamic soil property measurements during triaxial testing*,” Geotechnique, 39(4), 721-726.
- Blewett, J., I.J. Blewett, and P.K. Woodward (2000), “*Phase and amplitude responses associated with the measurement of shear-wave velocity in sand by bender element*,” Canadian Geotechnical Journal, 37, 1348-1357.
- Brignoli, E.G.M., M. Gotti, and K.H. Stokoe (1996), “*Measurement of shear waves in laboratory specimens by means of piezoelectric transducers*,” Geotechnical Testing Journal, 19(4), 384–397.
- Burland J. (1989) — *Small is beautiful: the stiffness of soils at small strains*. Can. Geotech. J., 26 (4): 499–516.
- Dano, Christophe, and Hicher, Pierre-Yves (2002), “*Evolution of Elastic Shear Modulus in Granular Materials Along Isotropic and Deviatoric Stress Paths*,” 15th ASCE Engineering Mechanics Conference, June 2-5, 2002, Columbia University, New York
- Hardin, B.O. and F.E. Richart (1963), “*Elastic wave velocities in granular soils*,” Journal of the Geotechnical Engineering Division, ASCE, 89(1), 33-65.
- Iwasaki, T. and F. Tatsuoka (1977), “*Effects of grain size and grading on dynamic shear moduli of sands*,” Soils and Foundations, 17(3), 19-35.
- Jovicic, V., M.R. Coop, and M. Simic (1996), “*Objective criteria for determining  $G_{\max}$  from bender element tests*,” Geotechnique, 46(2), 357-362.
- Kokusho T. (1987). “*Cyclic triaxial test of dynamic soil properties for wide strain range*,” Soils Foundations, 45-60
- Mair R. J. (1993)—*Developments in geotechnical engineering research: Application to tunnels and deep excavations*. Proc. Instn. Civ. Engrs. London, 3: 27–41.
- Oppenheim, Alan V., Willsky, Alan S., and Nawab, Syed Hamid (1997). “*Signals & Systems*,” Prentice Hall, Chapter Seven
- Viggiani, G. and J.H. Atkinson (1995), “*Interpretation of bender element tests*,” Geotechnique, 45(1), 149–154.