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EFFECT OF NON-PRISMATIC 3-D CANYON ON THE SCATTERING OF SEISMIC P- AND SV-WAVES

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ABSTRACT

The scattering of seismic waves by topographic irregularities can amplify ground motion and intensify damage such as the activation of landslides and rock failures during earthquakes. The scattering of waves by topographies causes spatial variation of the earthquake ground motion and should be considered in the analysis and design of structures located at these sites. It is common for topographic effects such as canyons to be analyzed using 2-D models that assume the canyons to be prismatic (i.e., have uniform cross sections). This paper investigates the effect of a non-prismatic 3-D canyon on the scattering of P- and SV-waves. The analysis uses an extended 3-D multi-domain boundary element method. A comparison of results for prismatic and non-prismatic canyons shows that the non-prismatic canyon can strongly affect the amplification and the variation of ground motion along the canyon. The significance of these effects depends on the pattern of cross-section variation along the canyon length, as well as the incident wave direction. It is shown that non-prismatic effect for the canyon widening toward ends amplifies the topographic effect in respect to the prismatic solution. However the canyon narrowing toward ends de-amplify the topographic amplification. Thus it is concluded that for non-prismatic canyons a 3-D solution is essential in order to get an accurate perspective of the phenomena of topography.

Keywords: Scattering, Seismic Wave, Boundary Element, Topographic Amplification, Non-Prismatic Canyon

INTRODUCTION

The importance of local site effects on strong ground motion has been well recognized. The analysis of scattering of elastic waves by surface irregularities such as canyons, alluvial valleys and sedimentary basins has been the subject of numerous experimental and theoretical studies e.g., Trifunac [1973], Celebi [1987], Geli [1988], Luco [1990], Zhang [1991], Zhao [1992], Sanchez-Sesma [1995], Athanasopoulos [1999], Paolocci [2002], Dravinski [2003], Hail [2004] and Assimaki[2004]. These studies show that amplification and de-amplification of ground motion occurs due to the particular shape of surface topography. Geli et al. [1988] successively analyzed more detailed configuration consisting of a layered profile and introduced nearby ridge effects, arriving at results similar to those of the aforementioned researchers. In addition, they concluded that the presence of surface topographic irregularities might have a greater effect on site response than soil layering and flexibility. As a general rule, for incident harmonic SH-waves, the maximum steady-state horizontal acceleration at the slope crest takes values about twice the acceleration at the free field beyond the crest, for wavelengths of the same order of magnitude as the geometrical

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characteristics of the cliff [Assimaki et al., 2004]. Along the slope surface, there exist both amplification and de-amplification of the input motion. A similar behavior governs the response in the case of incident SV waves, with the maximum amplification values being slightly higher. In addition to theoretical studies, evidences from observations have also confirmed the influences of surface topography [Vahdani etal., 2002, Athanasopoulos et al., 1999, Brambati, 1980 and Sirovich, 1982].

The boundary element method is one of the most powerful numerical techniques for analyzing these kinds of problems. While a great deal of work has been done on the two dimensional elastic response of an isotropic medium very little has been published on three dimensional analyses. In this research the multi-domain boundary element method proposed by Ahmad and Banerjee [1988] is used to study the amplification of elastic waves by a three dimensional canyon of arbitrary shape. Incident plane harmonic P- and SV-waves are considered. The accuracy of the method is tested through comparison with results of some earlier studies.

TOPOGRAPHY

An arbitrary shape canyon of finite length is considered. Seismic body waves arrive from an arbitrary direction with angles of θ_h and θ_v with respect to horizontal and vertical axes, respectively. The half-space is characterized by the P and S wave velocities c_p and c_s, respectively. To compute the total displacement at the canyon site due to incident body waves, the four following steps are performed:

1- Determine the ground motion for free field conditions (u_{ff}) for the half-space without the canyon (see Fig.2a)

2- Determine the tractions corresponding to the displacements in the previous step (see Fig.2b)

3- Apply the opposite of the computed tractions in step 2 at the base of the canyon as a traction boundary condition and determine the displacements (see Fig.2c)

4- Determine the total displacements by superposition of the two displacements obtained in steps (1) and (3), i.e. $u_{total} = u_{ff} + u_s$

In the first step the closed-form solution for the half-space displacements due to incident body waves is used to determine u_{ff}.

In the second step u_s is determined using the multi-domain boundary element technique as discussed in the following sections.

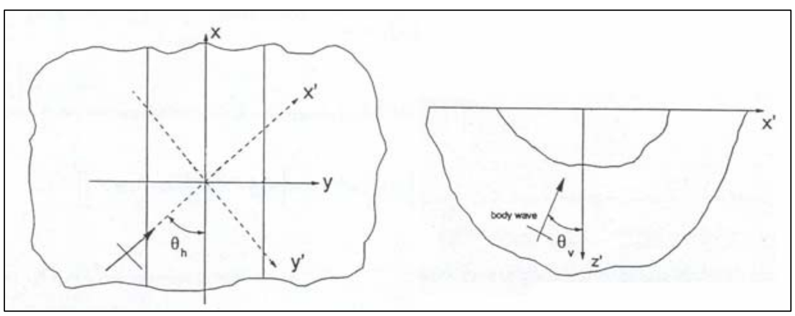

Figure 1: The topographic system considered is an arbitrarily shaped canyon of finite length with the incident waves arriving from an arbitrary direction

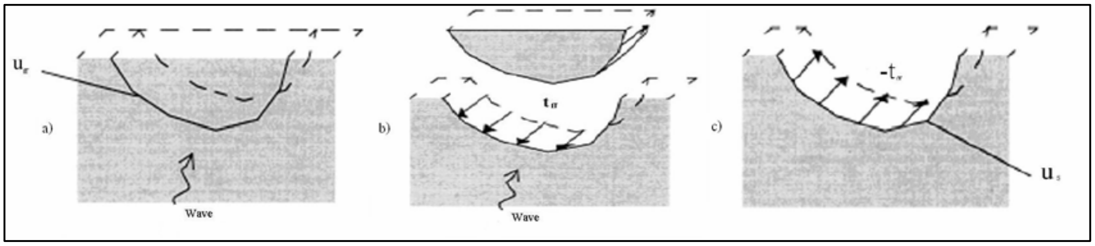

Figure 2: Superposition principle for wave scattering problem in a canyon: a) free-field without canyon; b) tractions induced by free-field displacements; and c) displacements due to reversed free-field tractions on the canyon surface

BACKGROUND THEORY AND BOUNDARY ELEMENT METHOD

The governing wave equation for an elastic, isotropic and homogeneous body is:

$$c_1^2 \nabla (\nabla . u) - c_2^2 \nabla \times \nabla \times u - \frac{\partial^2 u}{\partial t^2} = -b \tag{1}$$

in which *u* denotes the displacement vector, *b* denotes the body force vector, and c_1 and c_2 are the propagation velocities of compression (P) and shear (S) of the waves, respectively. The velocities are related to the properties of the medium through:

$$c_1 = (\lambda + 2\mu / \rho)^{0.5}, \ c_2 = (\mu / \rho)^{0.5} \tag{2}$$

where λ and μ are the Lame constants and ρ is the mass density [Dominguez, 1993, Manolis and Beskos, 1988].

The boundary element method (BEM) is very effective when dealing with wave propagation problems in infinite media with geometrical irregularities. The main advantage of this method is that discretization is only applied at the boundaries of the physical domain thus reducing the number of unknown variables significantly in comparison to other methods such as finite element and finite difference techniques. Since the fundamental solutions automatically satisfy the far-field condition, the BEM is especially well-suited to problems involving infinite domains such as a canyon located on a half-space [Dominguez, 1993]. The corresponding governing boundary equation for an elastic, isotropic, homogenous body can be obtained using the well-known dynamic reciprocal theorem as:

$$c^i u^i + \int_\Gamma p^* u d\Gamma = \int_\Gamma u^* p d\Gamma \tag{3}$$

where p^* and u^* are the fundamental solution for traction and displacement respectively, at a point *x* when a unit Dirac Delta load is applied at point *i*. In the BEM the variables *u* and *p* are discretized into the values at the so-called *collocation nodes*.

FUNDAMENTAL SOLUTIONS

The fundamental solution in the frequency domain for the displacement is the solution to Eq. (1) for a harmonic point force with unit amplitude applied at the point *y* in the *l* direction i.e.,

$$\rho b = \delta(r) e \tag{7}$$

Here *r* is the distance between the *observation point x* and the *source point* y and δ is the Delta Dirac function. In order to find the fundamental displacement solution, the principle of Helmholtz decomposition is used. The fundamental solution expressions are adopted from Dominguez [1993], and Manolis and Beskos [1988] as the following expressions.

$$u_{lk}^* = \frac{1}{\alpha \pi \rho c_2^2} \left[\psi \delta_{lk} - \chi r_{,l} r_{,k} \right] \tag{8-a}$$

$$\psi = \frac{\exp(-k_2 r)}{r} + (\frac{1}{k_2^2 r^2} + \frac{1}{k_2 r})\frac{\exp(-k_2 r)}{r} - \frac{c_2^2}{c_1^2}(\frac{1}{k_1^2 r^2} + \frac{1}{k_1 r})\frac{\exp(-k_1 r)}{r} \tag{8-b}$$

$$\chi = (\frac{3}{k_2^2 r^2} + \frac{3}{k_2 r} + 1)\frac{\exp(-k_2 r)}{r} - \frac{c_2^2}{c_1^2}(\frac{3}{k_2^2 r^2} + \frac{3}{k_2 r} + 1)\frac{\exp(-k_1 r)}{r} \tag{8-c}$$

in which $\alpha = 4$, δ_{ij} is Delta Dirac function and, $k_1 = \frac{i\omega}{c_1}$ and $k_2 = \frac{i\omega}{c_2}$ denote the wave numbers for compression and shear waves, respectively. The subscript *l* will only be used to indicate the coordinate direction of the point load.

The fundamental solution for free-field surface tractions in the frequency domain is:

$$\begin{aligned} p_{lk}^* = \frac{1}{\alpha\pi}[(\frac{d\psi}{dr} - \frac{1}{r}\chi)(\delta_{lk}\frac{\partial r}{\partial n} + r_{,k} n_l) - \frac{2}{r}\chi(n_k r_{,l} - 2 r_{,l} r_{,k}\frac{\partial r}{\partial n}) - \\ 2\frac{d\chi}{dr} r_{,l} r_{,k}\frac{\partial r}{\partial n} + (\frac{c_1^2}{c_2^2} - 2)(\frac{d\psi}{dr} - \frac{d\chi}{dr} - \frac{\alpha}{2r}\chi) r_{,l} n_k] \end{aligned} \tag{9-a}$$

$$\frac{d\psi}{dr} = (-\frac{2}{r} - k_2 - \frac{3}{k_2 r^2} - \frac{3}{k_2^2 r^3})\frac{\exp(-k_2 r)}{r} + \frac{c_2^2}{c_1^2}(\frac{1}{r} + \frac{3}{k_1 r^2} + \frac{3}{k_1^2 r^3})\frac{\exp(-k_1 r)}{r} \tag{9-b}$$

$$\frac{d\chi}{dr} = (-\frac{4}{r} - k_2 - \frac{9}{k_2 r^2} - \frac{9}{k_2^2 r^3})\frac{\exp(-k_2 r)}{r} + \frac{c_2^2}{c_1^2}(\frac{4}{r} + k_1 + \frac{9}{k_1 r^2} + \frac{9}{k_1^2 r^3})\frac{\exp(-k_1 r)}{r} \tag{9-c}$$

TREATMENT OF SINGULARITIES IN FUNDAMENTAL SOLUTION

The solution for the free-field displacement and subsequently the matrix *G,* contain singularities because of the *1/r* term. A weak singularity is encountered when the collocation node coincides with the integration node. To perform the numerical integration using standard Gauss-Legendre quadrature over an element where the collocation node is one of the element nodes, a method proposed by Lachat [Dominguez 1993] is used. In this method the element is divided into a number of triangles, each having one of the corners at the collocation node and the integration is performed over each of the triangles using a standard Gauss-Legendre quadrature rule over an equivalent collapsed quadratic element.

In the collapsed quadrilateral elements two of the corner nodes coincide so that one of the element sides has a length of zero. When performing the numerical integration over an element with such geometry, the Jacobian tends to zero as $r \to 0$ and cancels out the *1/r* singularity. Subsequently the accuracy of the Gauss-Legendre quadrature is satisfactory.

The singularities of the free-field traction solution, and thus of the matrix *H*, are of the $1/r^2$ kind when the observation point coincides with the integration point. Therefore, the method proposed by Lachat cannot be used to compute the diagonal terms of the matrix *H*. In this work a numerical method used to evaluate singular integrals for rigid body motion is adopted. This method normally applies only to closed domains. Ahmad and Banerjee [1988] generalized the method for the 2-D case to cover open domain boundaries where parts of the boundary are not discretized using the *enclosing elements technique.* Their method is extended to the 3-D case. The diagonal block of matrix H contains the tensor c_{ij} as well as the Cauchy principal value of the traction kernel integral, i.e.,

$$\overline{D}_{ij} = c_{ij} + \int_{S_1} \overline{F}_{ij} N_1 dS \tag{10}$$

in which N_1 is the shape function for the singular node and S_1 is the area of the singular element. The corresponding equation for the static problem is:

$$D_{ij}^s = c_{ij} + \int_{S_1} F_{ij}^s N_1 dS \tag{11}$$

From Equations 10 and 11 we can write:

$$\overline{D}_{ij} = D_{ij}^s + \int_{S_1} (\overline{F}_{ij} - F_{ij}^s) N_1 dS \tag{12}$$

In Equation 12, the diagonal blocks D_{ij}^s, which are the coefficients of the traction matrix for the static problem having the same geometry, can be obtained by using the rigid body motion, i.e.,

$$D_{ij}^s = c_{ij} + \int_{S_1} F_{ij}^s N_1 dS = -[\sum_{\alpha=2}^{A} \int_{S_1} F_{ij}^s N_\alpha dS + \sum_{q=2}^{Q} \sum_{\alpha=1}^{A} \int_{S_q} F_{ij}^s N_\alpha dS \tag{13}$$

Using this scheme, the diagonal blocks D_{ij}^s of the H matrix are obtained by the summation of non-singular integrations of the static traction kernel over all the elements of the modeled boundary as well as enclosing elements, i.e.,

$$D_{ij}^s = -[\sum_{\alpha=2}^{A} \int_{S_1} F_{ij}^s N_\alpha dS + \sum_{q=2}^{Q} \sum_{\alpha=1}^{A} \int_{S_q} F_{ij}^s N_\alpha dS + \sum_{e=1}^{L} \sum_{\alpha=1}^{A} \int_{S_e} F_{ij}^s N_\alpha dS] \tag{14}$$

The third summation in equation 14 corresponds to the L enclosing elements. Once D_{ij}^s is evaluated, the diagonal blocks $\overline{D}_{ij}$ related to the dynamic problem can be easily found by using Equation 13.

NUMERICAL RESULTS AND DISCUSSION

A general purpose 3-D computer program was developed to implement the boundary element procedures for an incident plane wave with circular frequency ω. The propagation direction of the wave was defined by angles θ_h and θ_v corresponding to the angle of the ray (normal to wave front) to the horizontal *x*- and vertical *z*-axes. The program can be used for canyons of arbitrary shape as well as closed or open domains. The numerical examples of this section are designed to demonstrate the accuracy and efficiency of the method for different cases.

COMPARISON WITH PREVIOUS RESULTS

In order to control the errors in the present method the discritization should be fine enough. Numerical results have demonstrated that the displacements on the canyon surface are not very sensitive to length of free-field that modeled in both sides of the canyon. Furthermore the authors studies lead to the fact that for obtaining accurate results, the element size should be smaller than one-fourth of the shear wavelength. To establish the numerical accuracy of the method, scattering of harmonic plane P- and SV-waves was tested by solving problems for which results are available from previous works. In all cases a semi-circular cross section of radius *R* cut in a homogenous half space is considered. The semi-cylindrical canyon and a length *1.5R* of the free-field on each sides of the canyon are discretized as shown in Figure 3. The model has *180* nine node boundary elements with *777* nodes along the main boundary. A length *1.5R* of the free-field on each sides of the canyon and a length *5R* along the canyon are modeled. An extensive sensitivity analysis (not shown here) proved that one does not need to go beyond such extension. The results presented in Figure 4 shows the displacement amplitudes around the canyon for the $\theta_h = 90$ and $\theta_v = 0^\circ$ for unit dimensionless frequency ($\Omega = \omega R / \pi c_2$) of the incident SV-wave. A similar comparison is presented in Figure 5 for an incident P-wave with $\theta_v = \theta_h = 45^\circ$. For these two test cases the numerical result obtained by Zhang and Chopra [1991] is used for the comparison. The comparisons in Figures 4 and 5 indicate that There is good agreement between the results obtained by the BE method and the previous results for all three displacement components. The results presented by this method are more accurate than the results presented by Zhang and Chopra due to this fact that Qusi-3D method is used by the latter authors. The finite length of the canyon has only a small effect on the computed displacements.

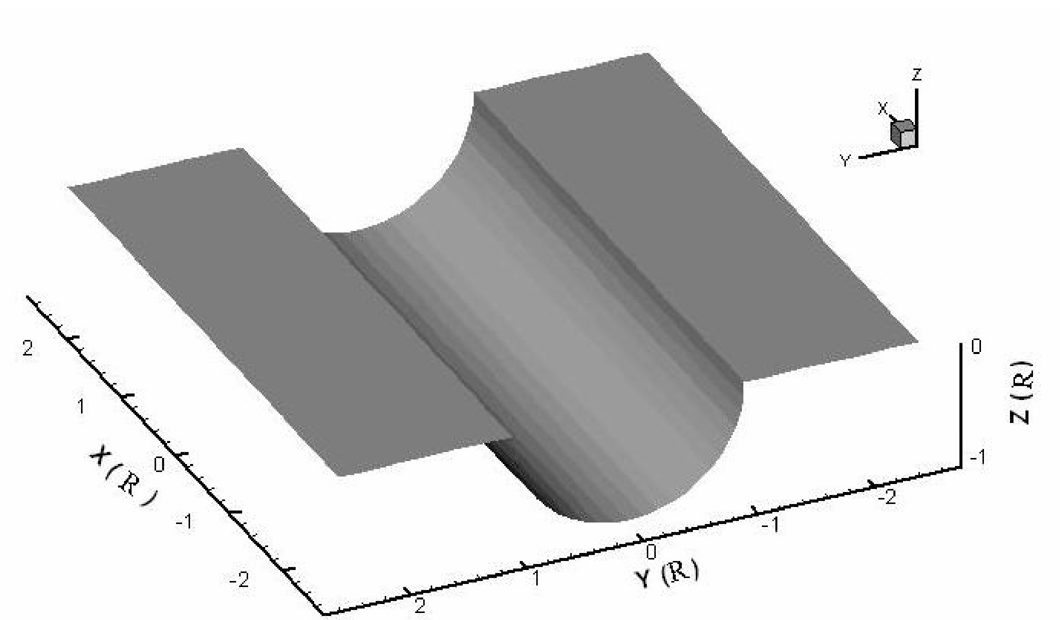

Figure 3: The semi-cylindrical canyon

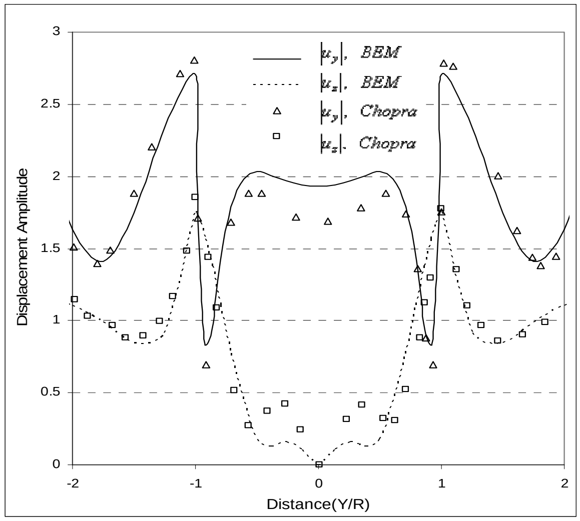

Figure 4: Displacement amplitudes obtained by the BE method and Zhang and Chopra [1991] for Incident SV-Wave with $\theta_h = 90$, $\theta_v = 0$

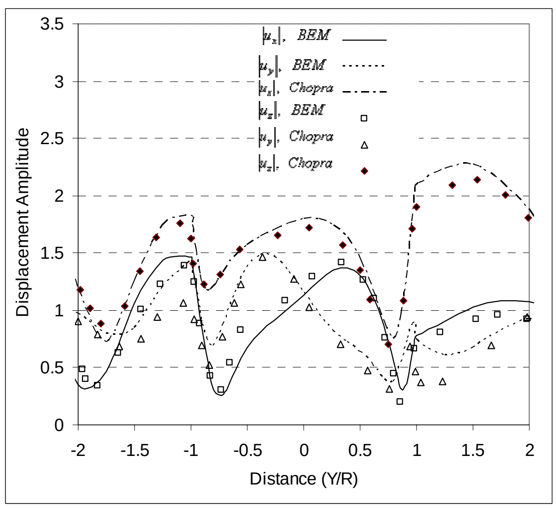

Figure 5: Displacement amplitudes obtained by the BE method and Zhang and Chopra [1991] for Incident P-Wave with $\theta_h = 45$, $\theta_v = 45$

EFFECTS OF NON-PRISMATIC CANYON CROSS SECTION

Two simple models are analyzed in order to investigate the effect of non-prismatic canyon cross sections. In the first model a canyon with a semi-circular cross section widens toward the ends and in the second model the canyon narrows toward the ends. The radius of the cross section is assumed to be a linear function of the canyon length as shown in Figure 6. The results for two different incident waves are shown in Figures 7 to 10. All these results show the displacement amplitude in the central cross-section of non-prismatic canyon with unit radius. Comparison of these figures to Figs. 4 and 5 indicate that the response of the non-prismatic canyons is significantly different from that of the prismatic canyon. The maximum displacement amplitude for the canyon widening toward the ends is larger than that for the prismatic canyon for both P-and SV- Waves. However for the canyon narrowing toward the ends the maximum displacement amplitude is smaller than that for the prismatic canyon. Therefore, the non-prismatic effect depends on how the variation of cross-section along the canyon is for different kind of seismic waves and so assuming a prismatic shape for the canyon when it is truly non-prismatic can yield non-conservative results.

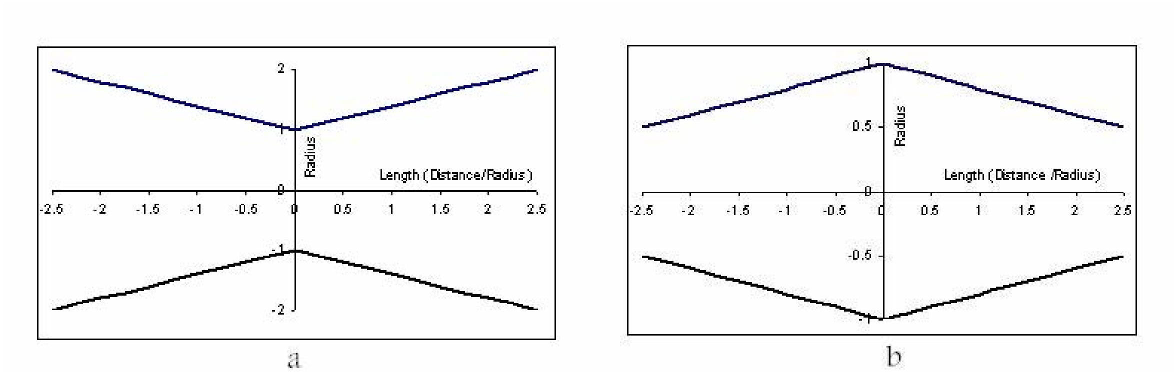

Figure 6: Radius variation for non-prismatic semi-circular canyons a: widening and b: narrowing toward the ends

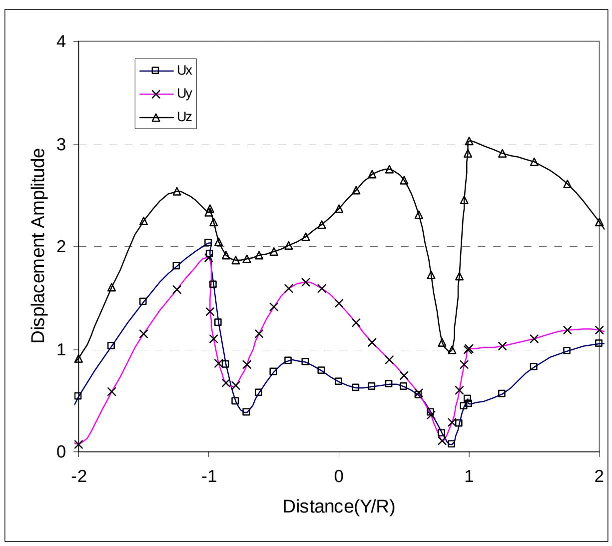

Figure 7: Displacement amplitudes for non-prismatic semi-circular canyon widening toward ends due to incident P-wave with $\theta_v = \theta_h = 45^\circ$

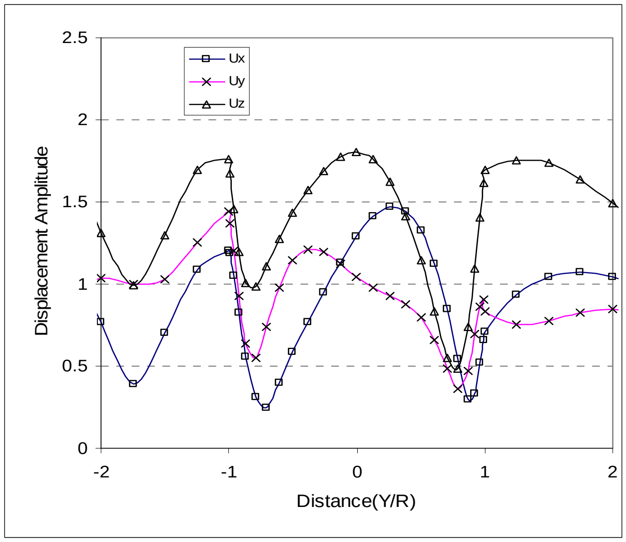

Figure 8: Displacement amplitudes for non-prismatic semi-circular canyon narrowing toward ends due to incident P-wave with $\theta_v = \theta_h = 45^\circ$

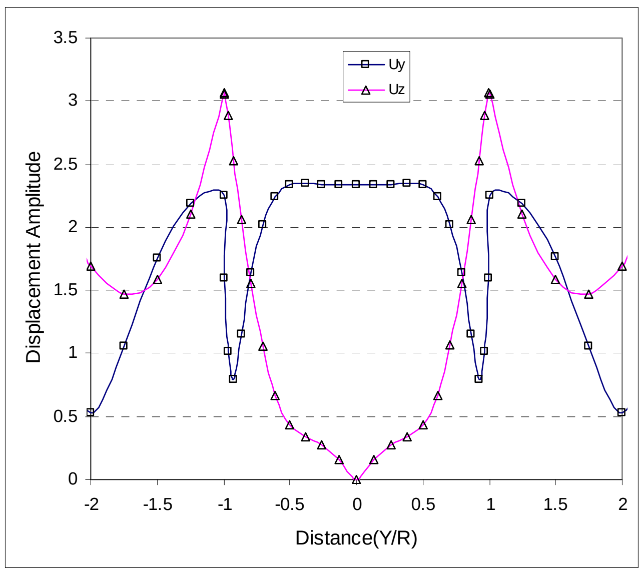

Figure 9: Displacement amplitudes for non-prismatic semi-circular canyon widening toward ends due to Incident SV-Wave with $\theta_h = 90$ **,** $\theta_v = 0$

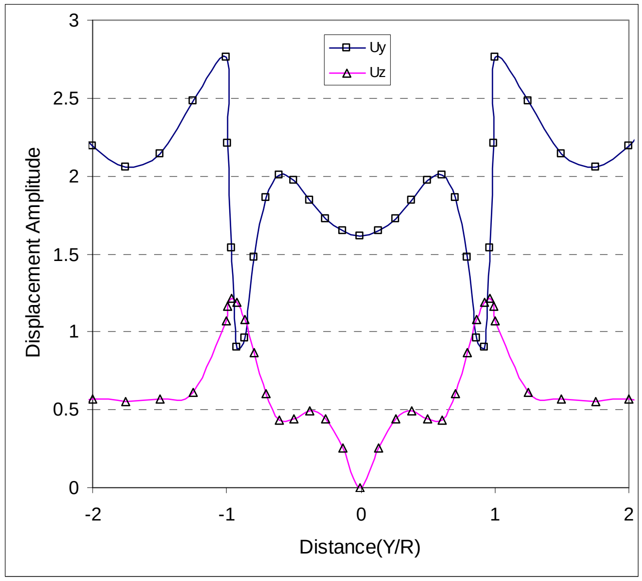

Figure 10: Displacement amplitudes for non-prismatic semi-circular canyon narrowing toward ends due to Incident SV-Wave with $\theta_h = 90$, $\theta_v = 0$

CONCLUSIONS

The multi-domain boundary element method proposed by [Ahmad and Banerjee, 1988] is extended to three dimensions. This method yields good accuracy in comparison to other methods used previously.

This method is one of the most general and can be used in conjunction with a sub-structuring technique to solve not only the problems of layered media and soil-structure interactions, but also any problem of three-dimensional canyons with complex topographies and therefore it can be used for real world problems.

It is shown that assuming a prismatic shape along the canyon can underestimate topographic amplification if the canyon is truly non-prismatic for both P- and SV-waves incident at certain angles.

It is shown that topographic amplification for the canyons widening toward the ends and narrowing toward the ends respectively is greater and smaller than for the prismatic canyon and physically it is acceptable.

Effect of non-prismatic depends on how the variation of the cross sections along the canyon is. The steep and fast cross section variations along the canyon results big and considerable effects on the topographic amplification.

Usually Concrete dams especially Arch dams are constructed in the narrowed cross section of canyon. Thus it is concluded that for non-prismatic canyons a 3-D solution is essential in order to get an accurate perspective of the phenomena of topography.

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