

TRACTION TRANSIENT FUNDAMENTAL SOLUTIONS FOR 3D SATURATED POROELASTIC MEDIA WITH INCOMPRESSIBLE CONSTITUENTS

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ABSTRACT

This paper presents simple time domain traction fundamental solutions for the three-dimensional well known u - p formulation of saturated porous media, neglecting the compressibility of fluid and solid particles. At first, the explicit Laplace transform fundamental solutions are obtained for the governing differential equations which are established in terms of solid displacements and fluid pressure. Subsequently, the closed form time domain traction and fluid flow fundamental solutions are derived by analytical inversion of the Laplace transform solutions. Finally, a set of numerical results are presented which demonstrate the accuracies and some salient features of the derived analytical transient traction fundamental solutions.

Keywords: Boundary element, Dynamic poroelasticity, Transient traction, Fundamental solution

INTRODUCTION

Wave propagation in saturated porous media and the dynamic response of such media are of great interest in geophysics, acoustics, soil dynamics and many earthquake engineering problems. From a macroscopical point of view, saturated soil is a two-phase medium constituted of solid skeleton and fluid. Dynamic behaviors of each phase as well as that of the whole mixture are governed by the basic principles of continuum mechanics. In phenomena with medium speeds, such as earthquake problems, it is reasonable to neglect the fluid inertial effects, and to reduce the complete dynamic governing differential equations to the simple commonly called u - p formulation (Zienkiewicz et al., 1980). The governing differential equations could be further simplified by neglecting the compressibility of the solid particles and fluid, which could be reasonably assumed incompressible compared to the soil skeleton (Gatmiri & Enguyen, 2004 and Schanz & Pryl, 2004).

Various analytical and numerical techniques have been developed for solving wave propagation problems in poroelastic media, including domain type methods such as the finite element method (FEM), boundary type methods such as the boundary element methods (BEM) and hybrid type methods which combine the advantages of two or more methods such as the hybrid BE/FE method. The BEM is one of the most efficient numerical methods for solving wave propagation problems, because of its ability to deal with semi-infinite or infinite domain problems that has long been recognized. Boundary integral equations and proper displacement and traction fundamental solutions

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are the two key ingredients required for solving wave propagation problems in saturated porous media by the BEM.

Predeleanu (1984), Manolis & Beskos (1989) and Wiebe & Antes (1991) were among the firsts who developed boundary integral equations and fundamental solutions governing the dynamics of poroelastic media, in terms of solid skeleton displacement and fluid displacements components. Later, Cheng et al. (1991), Dominguez (1992), Chen & Dargush (1995) and recently Schanz (2001) developed another forms of boundary integral equations and fundamental solutions of dynamic poroelasticity in terms of less independent variables. But their algorithms were based on transformed domain fundamental solutions.

Obviously, time domain BEM for modeling the transient behaviour of media is preferred than the transformed domain BEM, because formulating the numerical procedure entirely in time domain and combining it with the FEM, provides the basis for solving nonlinear wave propagation problems. Considering the independent practical variables of solid skeleton displacement and fluid pressure, Kaynia (1992) was the first who presented approximate transient 3D displacement fundamental solutions for the special case of short-time. Chen (1994) proposed another approximate transient 2D and 3D displacement solutions for the special case of short time as well as the general case, which were too complicated to be applied in BE algorithms. Gatmiri & Kamalian (2002) showed that Chen's approximation could not be used in the simplified case of u-p formulation. They derived another approximate transient 2D displacement fundamental solutions for the u-p formulation which were still too complicated to be used in BE algorithms. Later Gatmiri & Nguyen (2004) proposed much less complicated transient 2D fundamental solutions for the u-p formulation of saturated porous media consisted of incompressible constituents. Recently Kamalian et al.(2006) derived the transient displacement fundamental solutions for the simplified u-p formulation of 3D poroelastic media with incompressible constituents. Presentation of time-domain traction fundamental solutions for the u-p formulation of 3D saturated porous media with incompressible constituents constitutes the main essence of this paper. Some numerical results are plotted that show the accuracies and some salient features of the proposed solutions.

GOVERNING EQUATIONS

Equations of dynamic poroelasticity were first derived by variational formulation (Biot, 1956) and later rederived by using more modern theories such as the theory of mixtures (Prevost, 1980) and the principles of continuum mechanics (Zienkiewicz et al, 1980,1984). The later approach has been adopted in the present study. Following the procedure outlined by Zienkiewicz & Shiomi (1984), one can write the equations expressing, respectively, the conservation of total momentum, the flow conservation for the fluid phase, the constitutive equation of a poroelastic solid and the generalized Darcy's law as follows:

Equilibrium equation:

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i + \rho_f \ddot{w}_i \quad (1)$$

Constitutive relation:

$$\sigma_{ij} = \lambda u_{k,k} \delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \alpha p \delta_{ij} \quad (2)$$

Flow conservation for fluid phase:

$$-\dot{w}_{i,i} + \gamma = \alpha \dot{u}_{i,i} + \frac{\dot{p}}{Q} \quad (3)$$

Generalized Darcy's law:

$$p_{,i} = -\frac{1}{k} \dot{w}_i - \rho_f \ddot{u}_i - m \ddot{w}_i \quad (4)$$

where

$$m = \rho_f / n, \quad \rho = (1 - n)\rho_s + n\rho_f, \quad 1/Q = n/K_f + (\alpha - n)/K_s, \quad \alpha = 1 - K/K_s$$

u represents the displacement of the solid skeleton, p denotes the excessive fluid pore pressure and w represents the average displacements of the fluid relative to the solid. σ represents the total stress, the elastic constants λ and μ denote the drained Lamé constants and $\kappa = k/\eta$ is the permeability coefficient, with η and k denoting the fluid dynamic viscosity and the intrinsic permeability of the solid skeleton, respectively. ρ_s is the solid density, ρ_f denotes the fluid density, ρ represents the density of solid-fluid mixture, m denotes the mass parameter and n is the porosity. In addition, Q and α are material parameters which describe the relative compressibility of the constituents. K_s and K_f denote the bulk modulus of the solid grains and the fluid while K represents the bulk modulus of the solid skeleton. Finally, f and γ denote the body force and the rate of fluid injection into the media, respectively.

Omitting all terms of fluid acceleration in equation (1) as well as all dynamic terms in equation (4) and eliminating w_i from equations (3) and (4), the well known u-p governing equations for poroelastic media with incompressible solid particles and fluid ($1/Q \approx 0, \alpha \approx 1$) could be easily obtained in the Laplace transform domain as follows:

$$\mu \tilde{u}_{i,jj} + (\lambda + \mu) \tilde{u}_{j,ji} - \rho s^2 \tilde{u}_i - \alpha \tilde{p}_{,i} + f_i = 0 \quad (5)$$

$$k \tilde{p}_{,ii} - \alpha \tilde{u}_{i,i} + \tilde{\gamma} = 0 \quad (6)$$

where the tilde denotes the Laplace transform and s is the Laplace transform parameter.

LAPLACE TRANSFORM DOMAIN FUNDAMENTAL SOLUTIONS

The explicit three dimensional Laplace transform domain fundamental solutions for equations (5) and (6), which involves the responses to the suddenly applied point force as well as to the supplementary scalar source with Heaviside step function in time, at a distance (r), have been obtained by (Kamalian et al, 2006) as follows:

$$\begin{aligned} \tilde{G}_{ij} = & \frac{2\beta A_{ij}}{\rho(s+2\beta)s^3} + \left(\frac{A_{ij}}{s^2 + 2\beta s} + \frac{B_{ij}}{v_d \sqrt{s^2 + 2\beta s}} + \frac{C_{ij}}{v_d^2} \right) \frac{1}{\rho s} \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) \\ & - \left(\frac{A_{ij}}{\rho s^2} + \frac{B_{ij}}{\rho s v_s} + \frac{C_{ij}}{\rho v_s^2} - D_{ij} \right) \frac{1}{s} \exp\left(-\frac{r}{v_s} s\right) \end{aligned} \quad (7)$$

$$\tilde{G}_{4i} = s\tilde{G}_{i4} = -\frac{\beta r_{,i}}{2\pi\alpha r} \left(\frac{1}{r(s^2 + 2\beta s)} + \frac{1}{v_d \sqrt{s^2 + 2\beta s}} \right) \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) + \frac{\beta r_{,i}}{2\pi\alpha r^2 (s^2 + 2\beta s)} \quad (8)$$

$$\tilde{G}_{44} = \frac{1}{4\pi k r} \left[\left(\frac{2\beta}{s^2 + 2\beta s} \right) \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) + \frac{1}{s + 2\beta} \right] \quad (9)$$

Where

$$\beta = \frac{\alpha^2}{2\rho k} \quad (10)$$

$$v_d = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad (11)$$

$$v_s = \sqrt{\frac{\mu}{\rho}} \quad (12)$$

$$A_{ij} = \frac{1}{4\pi r^3} (3r_{,i}r_{,j} - \delta_{ij}) \quad (13)$$

$$B_{ij} = \frac{1}{4\pi r^2} (3r_{,i}r_{,j} - \delta_{ij}) \quad (14)$$

$$C_{ij} = \frac{r_{,i}r_{,j}}{4\pi r} \quad (15)$$

$$D_{ij} = \frac{\delta_{ij}}{4\pi r \mu} \quad (16)$$

G_{ij} represents the displacement of the solid skeleton in the i -th direction and G_{4j} denotes the fluid pressure, due to the unit Heaviside point force in the j -th direction. Also G_{i4} represents the displacement of the solid skeleton in the i -th direction and G_{44} denotes the fluid pressure, due to the unit Heaviside rate of fluid injection.

The Laplace domain traction vector and the fluid flow, acting at a distance (r) on an arbitrary surface with an outer normal n_i , due to the suddenly applied point force with unit Heaviside step function in time and to the supplementary scalar source with Dirac delta function in time, i.e.

$$f(x, t) = \delta(x_1)\delta(x_2)\delta(x_3)H(t) \quad (17)$$

$$\gamma(x, t) = \delta(x_1)\delta(x_2)\delta(x_3)\delta(t) \quad (18)$$

can be calculated from equations (2) and (4) based on u-p formulation, as follows:

$$\tilde{F}_{ij} = (\lambda \tilde{G}_{kj,k} - \alpha \tilde{G}_{4j}) n_i + \mu (\tilde{G}_{ij,m} + \tilde{G}_{mj,i}) n_m \quad (19)$$

$$\tilde{F}_{4j} = -k (\tilde{G}_{4j,m}) n_m \quad (20)$$

$$\tilde{F}_{44} = -k (s \tilde{G}_{44,m}) n_m \quad (21)$$

where

$$\begin{aligned} \tilde{G}_{ij,m} = & \left(\frac{A_{ij,m}}{\rho s (s^2 + 2\beta s)} + \frac{(B_{ij,m} - A_{ij} r_m)}{\rho s v_d \sqrt{s^2 + 2\beta s}} + \frac{(C_{ij,m} - B_{ij} r_m)}{\rho s v_d^2} \right) \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) \\ & - \frac{C_{ij} r_m (s + 2\beta)}{\rho v_d^3 \sqrt{s^2 + 2\beta s}} \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) + \frac{2\beta A_{ij,m}}{\rho (s + 2\beta) s^3} \\ & - \left[\frac{A_{ij,m}}{\rho s^3} + \frac{(B_{ij,m} - A_{ij} r_m)}{\rho s^2 v_s} + \frac{(C_{ij,m} - B_{ij} r_m)}{\rho s v_s^2} - \frac{D_{ij,m}}{s} - \frac{C_{ij} r_m}{\rho v_s^3} + \frac{D_{ij} r_m}{v_s} \right] \exp\left(-\frac{r}{v_s} s\right) \end{aligned} \quad (22)$$

$$\begin{aligned} \tilde{G}_{4i,m} = & \frac{\beta}{2\pi\alpha r} \left(\frac{3r_i r_m - \delta_{im}}{r^2 (s^2 + 2\beta s)} + \frac{3r_i r_m - \delta_{im}}{r v_d \sqrt{s^2 + 2\beta s}} + \frac{r_i r_m}{v_d^2} \right) \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) \\ & + \frac{\beta \delta_{im} - 3\beta r_i r_m}{2\pi\alpha r^3 (s^2 + 2\beta s)} \end{aligned} \quad (23)$$

$$\tilde{G}_{44,m} = \frac{-r_m}{4\pi k r} \left[\left(\frac{2\beta}{r (s^2 + 2\beta s)} + \frac{2\beta}{v_d \sqrt{s^2 + 2\beta s}} \right) \exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) + \frac{1}{r (s + 2\beta)} \right] \quad (24)$$

F_{ij} represents the traction in the i -th direction and F_{4j} denotes the fluid flow, due to the unit Heaviside point force in the j -th direction. F_{44} denotes the fluid flow, due to the Dirac delta rate of fluid injection.

TIME DOMAIN TRACTION FUNDAMENTAL SOLUTIONS

Returning to the real time domain, requires inverting the exponential functions of equations (19) to (24) as well as their coefficients, employing the convolution theorem and adding the results. Using the simplified assumptions of u-p formulation as well as solid grain's and fluid's incompressibility, enables one to obtain the inverse Laplace transforms of the exponential functions in an exact and much less complicated manner compared to the extremely difficult case of complete dynamic formulation of poroelastic media. Denoting the inverse Laplace transforms of the exponential functions by:

$$L^{-1} \left[\exp\left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s}\right) \right] = g_1(r, t) H\left(t - \frac{r}{v_d}\right) \quad (25)$$

$$L^{-1} \left[\frac{\exp \left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s} \right)}{\sqrt{s^2 + 2\beta s}} \right] = g_2(r, t) H \left(t - \frac{r}{v_d} \right) \quad (26)$$

$$L^{-1} \left[\frac{\partial}{\partial s} \left(\exp \left(-\frac{r}{v_d} \sqrt{s^2 + 2\beta s} \right) \right) \right] = -t g_1(r, t) H \left(t - \frac{r}{v_d} \right) \quad (27)$$

in which:

$$g_1(r, t) = \exp(-\beta t) \left[\frac{\beta r/v_d}{\sqrt{t^2 - (r/v_d)^2}} I_1 \left(\beta \sqrt{t^2 - (r/v_d)^2} \right) + \delta \left(t - \frac{r}{v_d} \right) \right] \quad (28)$$

$$g_2(r, t) = \exp(-\beta t) \left[I_0 \left(\beta \sqrt{t^2 - (r/v_d)^2} \right) \right] \quad (29)$$

and where I_0 , I_1 , $H(t)$ and $\delta(t)$ denote the modified Bessel functions for the first kind of orders zero, one and the Heaviside step and Dirac delta functions, respectively, the explicit and simple closed form expressions of the time domain traction fundamental solutions could be obtained as follows:

$$F_{ij} = (\lambda G_{kj,k} - \alpha G_{4j}) n_i + \mu (G_{ij,m} + G_{mj,i}) n_m \quad (30)$$

$$F_{4j} = -k (G_{4j,m}) n_m \quad (31)$$

$$F_{44} = -k (\dot{G}_{44,m}) n_m \quad (32)$$

where

$$\begin{aligned} G_{ij,m} = & \left[A_{ij,m} e_1(r, t) + (B_{ij,m} - A_{ij} r_{,m}) e_2(r, t) + (C_{ij,m} - B_{ij} r_{,m}) e_3(r, t) - C_{ij} r_{,m} e_4(r, t) \right] H \left(t - \frac{r}{v_d} \right) \\ & + A_{ij,m} e_5(r, t) - \left[\frac{A_{ij,m} (t - r/v_s)^2}{2\rho} + \frac{(B_{ij,m} - A_{ij} r_{,m}) (t - r/v_s)}{\rho v_s} \right. \\ & \left. + \frac{(C_{ij,m} - B_{ij} r_{,m})}{\rho v_s^2} - D_{ij,m} + \left(\frac{D_{ij} r_{,m}}{v_s} - \frac{C_{ij} r_{,m}}{\rho v_s^3} \right) \delta \left(t - \frac{r}{v_s} \right) \right] H \left(t - \frac{r}{v_s} \right) \end{aligned} \quad (33)$$

$$\begin{aligned} G_{4i,m} = & \frac{1}{2\pi\alpha r} \left[\frac{3r_{,i} r_{,m} - \delta_{im}}{2r^2} e_6(r, t) + \frac{3\beta r_{,i} r_{,m} - \beta \delta_{im}}{r v_d} g_2(r, t) + \frac{\beta r_{,i} r_{,m}}{v_d^2} g_1(r, t) \right] H \left(t - \frac{r}{v_d} \right) \\ & \frac{\delta_{im} - 3r_{,i} r_{,m}}{4\pi\alpha r^3} [1 - \exp(-2\beta t)] \end{aligned} \quad (34)$$

$$\dot{G}_{44,m} = \frac{-\beta r_m}{2\pi k r} \left[\frac{1}{r} e_7(r,t) + \frac{t}{r} g_1(r,t) - \frac{\beta}{v_d} g_2(r,t) \right] H\left(t - \frac{r}{v_d}\right) - \frac{r_m}{4\pi r^2 k} [\delta(t) - 2\beta \exp(-2\beta t)] \quad (35)$$

The functions $e_i(r,t)$ are given in Appendix.

Equations (30) to (35) show that as expected, in the simplified u-p formulation of saturated porous media with incompressible solid grains and fluid, similar to the case of complete formulation of dynamic poroelasticity, there are three types of body waves propagating throughout the media: the pressure wave with a propagation velocity of infinity, the diffusive wave with a propagation velocity of v_d and the shear wave with a propagation velocity of v_s .

NUMERICAL RESULTS

A set of numerical results are presented in this section to demonstrate the accuracy of the proposed analytical time-domain tractions fundamental solutions. Values obtained by numerical inversion of Laplace transform domain solutions are compared with those calculated by the presented analytical closed form solutions. The INLAP-DINLAP subroutine, provided by the IMSL subroutine library (1994) and based on de Hoog et al. (1982) algorithm, was employed to perform the numerical inversion of the Laplace transform domain solutions. A saturated soft soil with incompressible solid grains and pore water is considered in which the material properties are defined in the metric system as follows: $\lambda=12.5$ MPa, $\mu=8.33$ MPa, $\rho=2120$ kg/m³, $\alpha=1$, $\kappa=1 \times 10^{-7}$ m⁴/Ns. The applied force (or fluid source) point is located at coordinate (0,0,0) and the receiver is located at coordinate (0.1,0.2,0.3). Figures 1-4 depict the accuracies of the presented analytical closed form solutions for F11, F12, F41 and F44 components on the surface perpendicular to the radius vector r , respectively. As can be seen, excellent agreements exist between the proposed time-domain tractions and fluid flow solutions and the numerical inversion of Laplace transform domain solutions. It is also interesting to note the arrival times of the pressure ($v_p=\infty$), diffusive ($v_d=117.3$ m/s) and shear ($v_s=62.7$ m/s) waves, which could be detected by sudden changes appearing in the tractions or fluid flows. The notable initial values of the components F44, F4j (and also Fi4) differ from zero, because the pressure wave with a wave propagation velocity of infinity arrives immediately and affects the media's response. As can be observed, absolute values of these components decrease gradually and move towards zero with time.

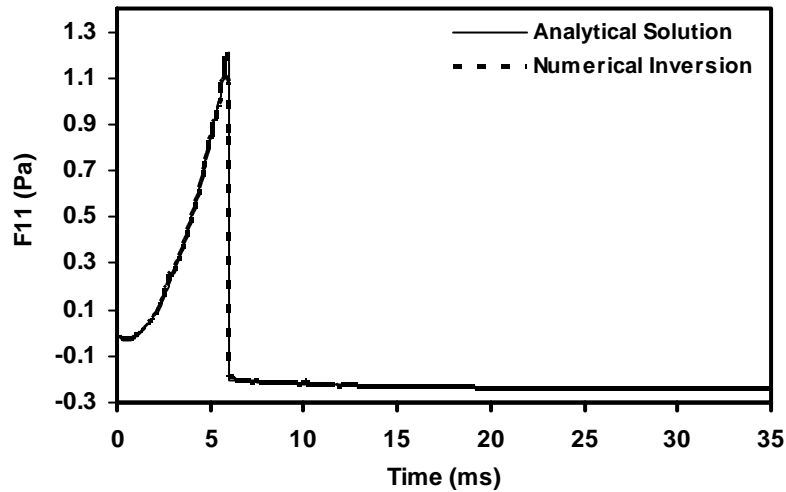


Figure 1. traction time history on a plane normal to (r) at (0.1,0.2,0.3) due to point force

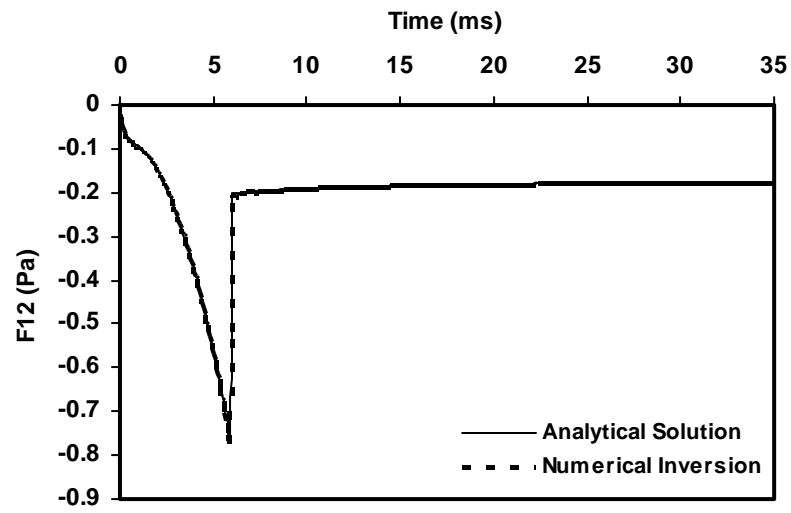


Figure 2. traction time history on a plane normal to (r) at $(0.1,0.2,0.3)$ due to point force

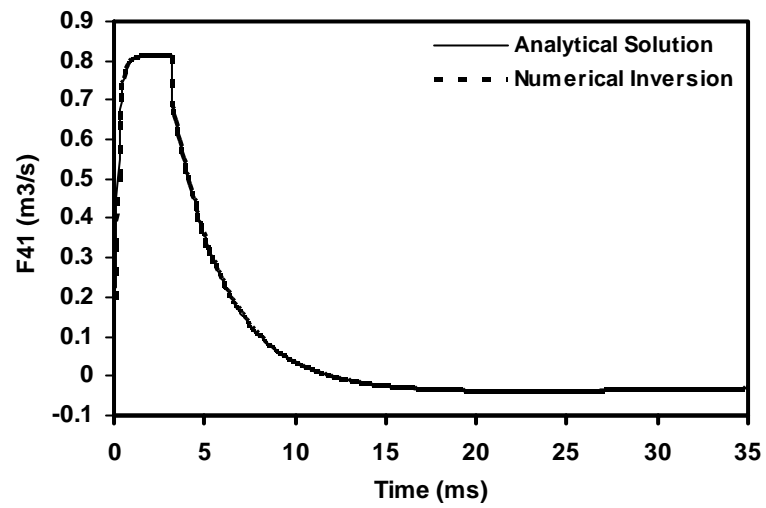


Figure 3. Fluid flow time history on a plane normal to (r) at $(0.1,0.2,0.3)$ due to point force

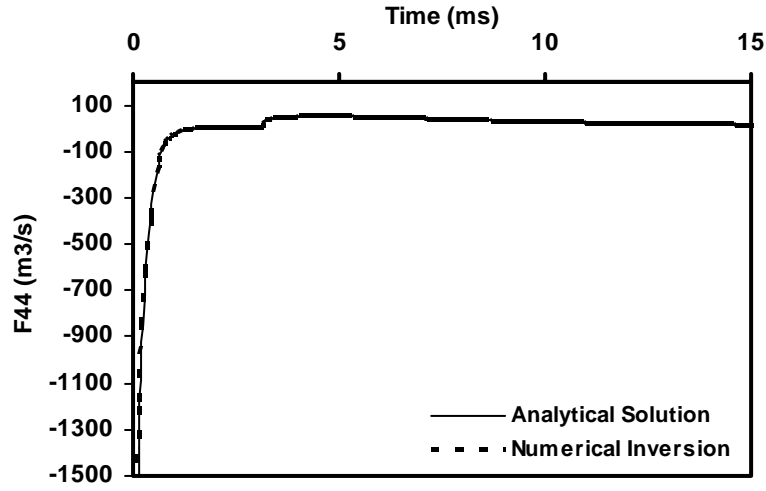


Figure 4. Fluid flow time history on plane normal to (r) at (0.1,0.2,0.3) due to fluid injection

CONCLUSION

Laplace transform domain traction fundamental solutions and simple analytical closed form expressions for time-domain traction fundamental solutions are presented for the well known u-p formulation of 3D saturated porous media with incompressible constituents, in terms of the practical variables of solid skeleton displacement and fluid pressure. A set of numerical results are presented that demonstrate the accuracies as well as some salient features of the derived analytical transient fundamental solutions. The derived traction fundamental solutions could be directly implemented in time domain BEM for modeling the transient behaviour of saturated porous media and enables one to develop more effective numerical procedures for solving 3D nonlinear wave propagation problems in the near future.

APPENDIX: FUNCTIONS $e_i(r, t)$

$$e_1(r, t) = \frac{1}{2\rho\beta} \int_{r/v_d}^t \left[t - \tau - \frac{1}{2\beta} + \frac{\exp(-2\beta(t - \tau))}{2\beta} \right] g_1(r, \tau) d\tau$$

$$e_2(r, t) = \frac{1}{\rho v_d} \int_{r/v_d}^t g_2(r, \tau) d\tau$$

$$e_3(r, t) = \frac{1}{\rho v_d^2} \int_{r/v_d}^t g_1(r, \tau) d\tau$$

$$e_4(r, t) = \frac{t}{\rho v_d^2} g_1(r, t) + \frac{\beta}{\rho v_d^3} g_2(r, t)$$

$$e_5(r, t) = \frac{1}{2\rho} \left[t^2 - \frac{t}{\beta} + \frac{1}{2\beta^2} (1 - \exp(-2\beta t)) \right]$$

$$e_6(r, t) = \int_{r/v_d}^t [1 - \exp(-2\beta(t - \tau))] g_1(r, \tau) d\tau$$

$$e_7(r, t) = \int_{r/v_d}^t \exp(-2\beta(t - \tau)) g_1(r, \tau) d\tau$$

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