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# *A SIMPLE APPROACH TO INVESTIGATE LATERALLY LOADED PILE-SOIL INTERACTION IN CLAY*

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## ABSTRACT

The "p-y" curves method is an acceptable and simple method to model pile-soil interaction by considering nonlinearity in soil behavior. Since soil pressure and frictional components contribute to its total resistance, some procedures have been proposed to reduce pressuremeter data and to obtain “p-y” curves. Soil resistance components are not accessible by back calculation on total resistance. Hence, these curves do not give any information on 3-D pile-soil interaction. In this study, a theoretical solution based on nonlinear Winkler model is exploited to derive “p-y” curve by its components analogous in the procedure used for pressuremeter data reduction. Pile-soil interaction especially the shape factors as the indicators of stress distribution, which are used to reduce pressuremeter test data to obtain “p-y” curves, are studied. Theoretical suggested “p-y” curves show satisfying results in comparison with full scale laterally loaded pile results and in many cases have better fitting rather than common “p-y” curves. Shape factors that are calculated to be 0.75 are valid at low displacements and approach unity as the soil around the pile approaches its ultimate stress. The theoretical study shows high contribution of friction component of pile-soil resistance at service loads like those given in smith’s studies (smith & Slyh, 1985; Smith & Brian, 1986). Smith studied instrumented the pile by pressure cell and concluded that the pile-soil friction contributes mainly to the equilibrium. However, the pressure component has a high contribution to equilibrium as the pile is loaded to its ultimate load.

Keywords: Laterally Loaded Pile, Pile-Soil Interaction, “p-y” curves, Resistance Components, Pressure shape factor, Friction shape factor

## INTRODUCTION

Methods to analyze laterally loaded pile either numerically or analytically can be done in two ways; Continuum based methods and load transfer methods (Kucukarsalan & Bildik, 2003). The “p-y” curves method as a load transfer procedure is the most reliable and simple method that is widely used. Hsiung (2003) and Hsiung & Chen (1997) have proposed a rather simpler and a closed from procedure which assumes bilinear elastic-plastic soil behaviour.

The “p-y” curves may be produced directly by measuring the soil resistance or indirectly by measuring the strain along the pile length like those primarily proposed by McClelland & Focht (1958). The”p-y” curves obtained by pressuremeter test (PMT) can be categorized as a direct method.

Indirect generation of “p-y” curves suffers some shortcomings like the error in the differentiation procedure. P-values are sensitive to the pile curvature while common “p-y” curves (e.g. API,

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1991) are acquired without measurements of the curvature (Janoyan et al., 2001). Janoyan (2001) conducted a research on large diameter shaft which resulted in a stiffer response than that of common “p-y“ curves (e.g. API, 1991). Hence increasing understanding of the contribution of the components of soil resistance may lead to better estimation of pile-soil interaction response. It may be accurate to derive "p-y" curves directly from their components namely; friction resistance (F) and pressure resistance (Q).

Laterally loaded piles are analogous in their soil mechanism to axially loaded piles. They generate resistance from side shear and front pressure only now in a horizontal and not vertical direction (Smith & Brian, 1986).

The front pressure component may be measured by pressuremeter test. The side friction component either can be measured directly by frictionmeter proposed by Kishida et al. (1986) or indirectly by the pressuremeter test results. Briaud et al. (1983) suggested a procedure to obtain the friction component from pressuremeter test. Smith & Slyh (1985) applied the mentioned methodology to predict the deflection at the head of laboratory pile models subjected to lateral load that yielded reasonable results.

It is important to convert the measured peak stresses whether normal or side shear component to soil resistance in kN/m and finally add up “Q-y” and “F-y” curves to get “p-y” curve in the direct method. To do this conversion the shape factors introduced by Briaud et al. (1983) are frequently used as presented in equations 1 and 2.

$$P_p = \sigma_{n(\max)} \times SQ \times B \quad (kN/m) \tag{1}$$

$$P_f = \tau_{\max} \times SF \times B \quad (kN/m) \tag{2}$$

$\sigma_{n(max)}$: Front pressure ahead of the pile
$\tau_{max}$: maximum adhesion at the side of pile
B: Pile diameter
SQ: Shape factor for pressure component
SF: Shape factor for pressure component
$P_p$: Pressure component of soil resistance in (kN/m)
$P_f$: Friction component of soil resistance in (kN/m)

Baguelin et al. (1977) presented a stress analysis on laterally loaded pile in an elastic soil domain. Using elasticity theory, he performed his study on a 2-D model, which consists of a horizontal section of the pile and the soil domain. Briaud *et al*. (1983) derived SQ=0.75 from Baguelin’s solution for a pile with circular section. In the elastic solution, a uniform total stress distribution was assumed. The mentioned value of the shape factor is commonly used for example; Zhang *et al*. (2005) used this value to estimate the ultimate soil pressure. Since soil has a nonlinear behavior and the soil in front of the pile, yields by the pile loading, a plastic zone forms in front of the pile near to the ground. The plastic zone around the pile propagates through the depth as the load increases. While the soil stresses around the pile reach their ultimate value, stress distribution approaches a uniform trend. In this case equation 1 yields SQ=1. So suggested values of the shape factors either for pressure component or for friction component seem to be not valid anymore because the proposed values are based on soil elastic behavior.

In this research, a closed form solution based on Winkler model employing nonlinear characteristic of the springs as the interaction elements is used to investigate pile-soil interaction and to derive “p-y” curve from its components in clay. Shape factors are shown to vary with soil yielding while pile is loaded. Four case studies are adopted to validate the proposed solution, which show good results.

## ANALYTICAL SOLUTION

Winkler model concept is employed to derive soil resistance components separately to achieve “p-y” curves. Winkler model is modified to consider nonlinearity in soil behavior in this solution.

The 2-D simulation in this study consists of a horizontal section of pile, which is surrounded by interface elements as shown in Fig. 1. Two types of spring elements are used at each point of the pile-soil contact. Shear and pressure springs are used to simulate soil friction resistance and soil pressure resistance respectively. A hyperbolic stress-strain relationship is assigned to the behavior of elements. Several modifications are introduced to the Winkler model to overcome the independency between the springs (Hetenyi 1946, Pasternak 1954, Kerr 1965, Horvath 1993). Employing the modifications has been passed up in this procedure because the elements dependency is considered implicitly in the solution to compensate the continuum effect that will be described later.

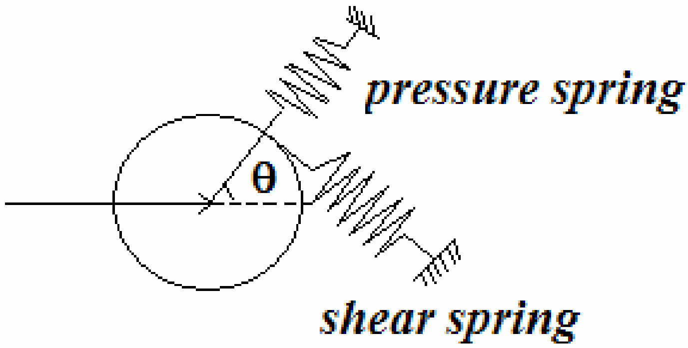



**Figure 1- The Studied model**

A consistency equation is required so that displacement analysis can be implemented instead of strain analysis. “Strain-displacement” relationship can be roughly interpreted by equation 3-a. Equation 3-b is the same expression that is frequently used to obtain $y_{50}$ from $\varepsilon_{50}$ in the “p-y” analysis. In equation 3-b, coefficient A is introduced to be 1 for stiff clay (Reese *et al*. 1975a, b) and 2.5 for soft clay (Matlock 1970). By a mathematical manipulation on analytical solution proposed by Baguelin *et al*. (1977), it is concluded that coefficient A varies between 1.2 and 1.5. Considering the presented values, it can be concluded that A ranges between 1 and 2.5. Thus A=1 is assumed in this study as an average value.

$$\delta = AB\varepsilon \tag{3-a}$$

$$y_{50} = AB\varepsilon_{50} \tag{3-b}$$

δ, B, ε denote the pile displacement, the pile diameter, and the soil strain, respectively. $Y_{50}$, $\varepsilon_{50}$ denote the pile displacement and the soil stain at a load of half of the ultimate load.

## NORMAL COMPONENT OF SOIL RESISTANCE

Rewriting the strain as a function of the displacement acquired from Eq. 3-a into Duncan & Chang hyperbolic “stress-strain” relationship leads us to equation 4. In this equation, δ is a function θ, which was introduced in figure 1.

$$\sigma_n = \frac{\dfrac{\delta_n(\theta)}{B}}{\dfrac{1}{E_i} + \dfrac{1}{\sigma_{ult}}\dfrac{\delta_n(\theta)}{B}} \tag{4}$$

$E_i$: Soil elastic modulus
$\sigma_n(\theta)$: Pile-soil normal stress at a point of azimuth of $\theta$
$\delta_n(\theta)$: Displacement in normal direction at a point of azimuth of $\theta$
$\sigma_{ult}$: the ultimate stress that an element can bear

Expressing normal displacement as a function of total displacement at each point (eq. 5), equation 6 can be derived.

$$\delta_n = \delta \cos(\theta) \tag{5}$$

$\delta$: Rigid body displacement

$$\sigma_n = \frac{\frac{\delta Cos\theta}{B}}{\frac{1}{E_i} + \frac{1}{\sigma_{ult}}\frac{\delta \cos\theta}{B}} \tag{6}$$

In order to achieve "p-y" normal (pressure) component normal stress around the pile section is integrated according to Eq. 7. Substituting $\sigma_n$ in equation 7 equation 8 is obtained.

$$p_p = \oint \sigma_n \cos(\theta) ds \tag{7}$$

$p_p$: Normal component of soil resistance
s: Integration variable on pile section perimeter

$$p_p = \int_0^{\frac{\pi}{2}} \frac{\frac{\delta}{B}\cos^2(\theta)}{\frac{1}{E_i} + \frac{1}{\sigma_{ult}}\frac{\delta}{B}\cos(\theta)} B d\theta \tag{8}$$

As the pile is loaded, a gap occurs between the pile and back soil; thus the integration upper bound is limited to $\pi/2$. The Integration can be solved either numerically or analytically. It has a rigorous analytical solution; hence numerical Simpson method is used to approximate the integration. Equation 9 presents the numerical integration outcome.

$$p_p = (0.21\frac{yB}{\frac{B}{E} + \frac{y}{\sigma_{ult}}} + .70\frac{yB}{\frac{B}{E} + 0.92\frac{y}{\sigma_{ult}}} + .12\frac{yB}{\frac{B}{E} + 0.38\frac{y}{\sigma_{ult}}} + 0.21\frac{yB}{\frac{B}{E} + 0.71\frac{y}{\sigma_{ult}}}) \tag{9}$$

Mair & Wood (1987) proposed an equation to estimate $C_u$ (Undrained shear strength) from limit pressure of pressuremeter test. This equation is suggested in this study to acquire $\sigma_{ult}$ (soil ultimate stress) from soil undrained shear strength ($C_u$). $\sigma_{ult}$ obtained from this equation is more than the ultimate stress that a soil element may bear in compression uniaxial test. Soil continuum effect is taken into account in the formulations accordingly. So the solution presented in this study benefits the advantage that the dependency between the springs of Winkler model is accounted for.

## SHEAR COMPONENT OF SOIL RESISTANCE

The same methodology has been exploited to shear component. A difference between the expression employed for normal and shear component arises because equation 3 can not be used for shear

component and a more complicated relationship is required rather than the simple expression presented in equation 3. Herein, “Force-displacement” relationship is adopted as the original soil shear behavior in charge for Duncan-Chang “stress-strain” relationship for normal component.
Inspired by the so-called “t-z” curves that are proposed for load transfer analysis of axially loaded piles, a non-dimensional hyperbolic equation (equation 10) is introduced to represent laterally loaded pile-soil shear behavior. The ultimate friction stress shown by ”$\tau_{max}$” can be determined by using $\alpha$ method, which is applicable in clay and undrained condition. Smith’s laboratory tests (1985) show that pile-soil slippage occur at displacements in the order of 1 mm.

$$\frac{\tau}{\tau_{\max}} = \frac{\dfrac{\delta_t}{\delta_{slip}}}{1+\dfrac{\delta_t}{\delta_{slip}}} \tag{10}$$

$\tau$: Shear stress between pile and soil

$\tau_{max}$: Maximum shear stress between pile and soil

$\delta_t$: Shear spring displacement

$\delta_{slip}$: Displacement at which slippage occur

Finally, combination of equations 10, 11 and interpretation of displacement component as a function of azimuth angle (Eq. 12) results in equation 13. Equation 14 presents numerical solution of the integration on equation 13. Sympson numerical method is used to calculate the integration.

$$p_f = \oint \tau \sin\theta\, )ds \tag{11}$$

$P_f$: shear component of soil resistance
s: Integration variable on pile section perimeter

$$\delta_t = \delta \sin(\theta) \tag{12}$$

$$p_f = \int_0^{\frac{\pi}{2}} \frac{\dfrac{\delta}{\delta_{slip}}\sin^2(\theta)}{1+\dfrac{\delta}{\delta_{slip}}\sin(\theta)}\tau_{\max} B d\theta \tag{13}$$

$$p_f = \tau_{\max} B\left(0.13\frac{\dfrac{\delta}{\delta_S}}{1+\dfrac{\delta}{\delta_S}} + 0.077\frac{\dfrac{\delta}{\delta_S}}{1+0.39\dfrac{\delta}{\delta_S}} + 0.44\frac{\dfrac{\delta}{\delta_S}}{1+0.9\dfrac{\delta}{\delta_S}} + 0.13\frac{\dfrac{\delta}{\delta_S}}{1+0.7\dfrac{\delta}{\delta_S}}\right) \tag{14}$$

## VALIDATION

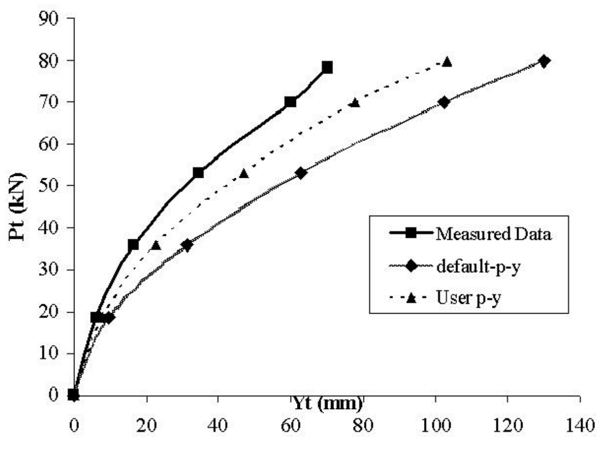
To validate the proposed solution the soil resistances components have been added and accordingly resulted "p-y" curves are employed to analyze four full-scale case studies. A "p-y" curve method based program, Lpile, is used to calculate pile head load-deflection. Two analyses are performed for each case. The first analysis utilizes the program default "p-y" curves, which involves the common "p-y" curves proposed by Welch & Reese (1972), Matlock (1970). Another analysis is performed by user defined "p-y" curves obtained in this study. Soil and pile properties of the case studies are shown in table 1 where $C_u$ denotes soil undrained shear strength, $\varepsilon_{50}$ strain at half of failure stress, D the pile embedment depth, B the pile diameter and EI the pile bending stiffness.

**Table 1- Case Studies Specification**

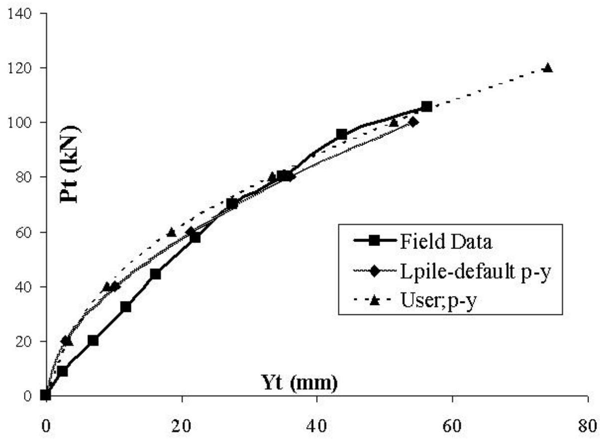
| | Case 1* | Case 2* | Case 3** | Case 4*** |
|---|---|---|---|---|
| $C_u$ (kPa) | 14.4 | 32 | 105 | 268 |
| $\varepsilon_{50}$ | .02 | .01 | .005 | .005 |
| D (m) | 12.8 | 12.8 | 12.8 | 4.876 |
| B(m) | .319 | .319 | .76 | 1.22 |
| EI (kN-m2) | 31280 | 31280 | 400,000 | 2399685.5 |
| Pile head dsitance to ground (m) | .305 | .06 | .076 | .2 |

* Matlock (1970) ** Reese & Welch (1975)
*** Bhushan et al. (1979)

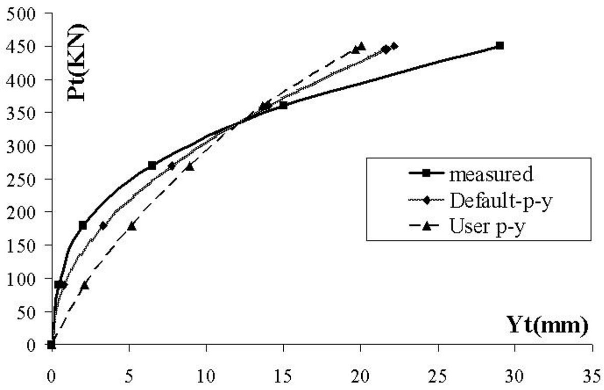
Comparison between test results and the suggested method in this study shows good correspondence in all cases as illustrated in Fig. 2. Analysis results based on user defined "p-y" curves better fit to field results in case 1 & 4. In other cases, there is no significant difference between the two methods.



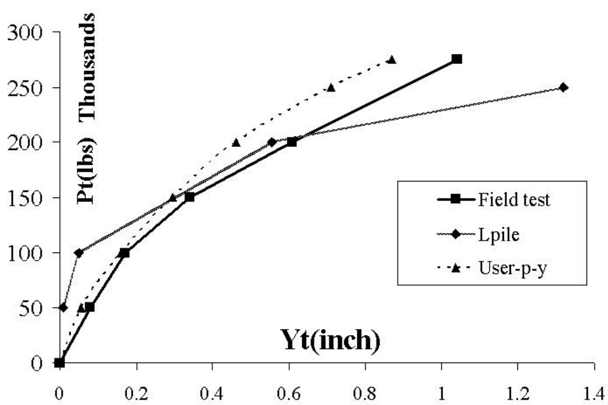

a) Case 1




b) Case 2




c) Case 3



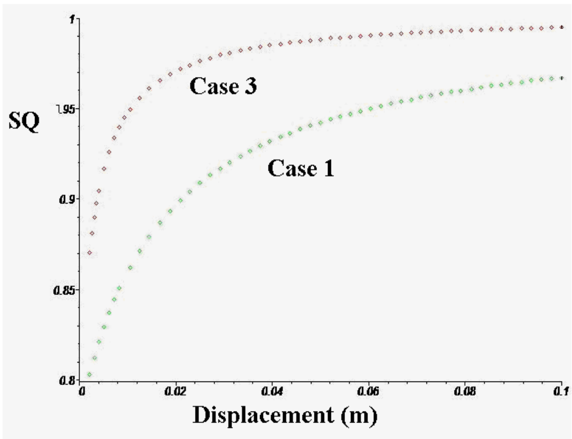

d) Case 4

Figure 2. Comparison of field test results and Lpile output based on suggested “p-y” curves and traditional “p-y” curves

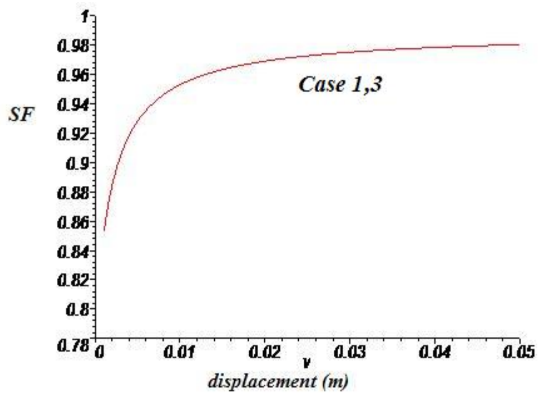
## PILE-SOIL INTERACTION INVESTIGATING

A pile-soil interaction study including stress distribution, stress components, their contribution to equilibrium and shape factors is implemented using the methodology suggested in this study. Cases 1 & 3 are adopted to be investigated for pile-soil interaction. Fig. 3 presents shape factors variation with the pile displacement. SQ of case 1 approaches unity moderately while SQ approaches unity at a smaller displacement in case 3 and the rate of SQ variation in case 3 is much more than that of case 1. SQ=1 hints at uniform distribution of normal stress that denotes all soil elements (in Winkler model) around the pile reach their ultimate stress. Regarding SQ as an indicator of the stress distribution at pile-soil contact it can be inferred that case 3 has more brittle behavior than case 1 because in case 3 the soil around the pile reaches its ultimate stress more rapidly than in case 1. In spite of high soil resistance in case 3, the stresses reach their ultimate value at smaller displacements. SQ=0.75 implies elastic soil behavior around the pile. Moving from SQ=0.75 up to unity shows soil domain bears normal stress until it reaches at its limit state. The upper boundary of soil domain state is not within reach, because the load applied on the pile head is gradually transferred to depth. Fig. 3(b) shows the friction shape factor is not sensitive to the soil and the pile properties and is a function of the displacement.

Contribution of the components of soil resistance is illustrated clearly in Fig. 4. Fig. 4(a) shows gradual mobilization of friction component in soft clay while Fig. 4(b) illustrates that in a stiff clay components contribution approximately remains constant with displacement. The intensity of soil stiffness can be inferred from its elastic modulus. To leave out pile dimension on pile-soil system stiffness the ration of E/B (or Cu/B) is introduced to quantify the pile-soil system stiffness. As a general conclusion, it can be stated that the higher ratio of E/B results in the higher rate of mobilization of the friction component. It is noticeable that E/B is proportional to the subgrade reaction modulus, so it can be stated that high value of subgrade reaction modulus causes brittle behavior of pile-soil interaction and higher rate of friction mobilization.

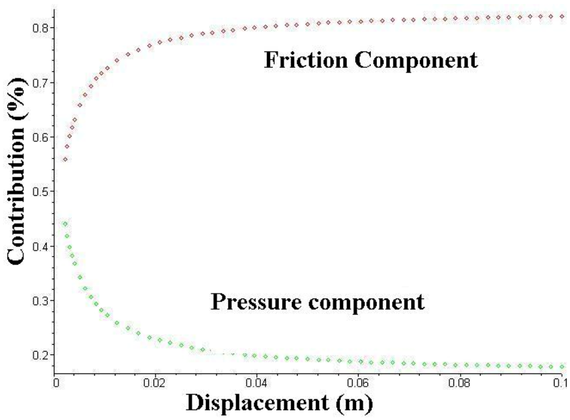



**a)SQ values**

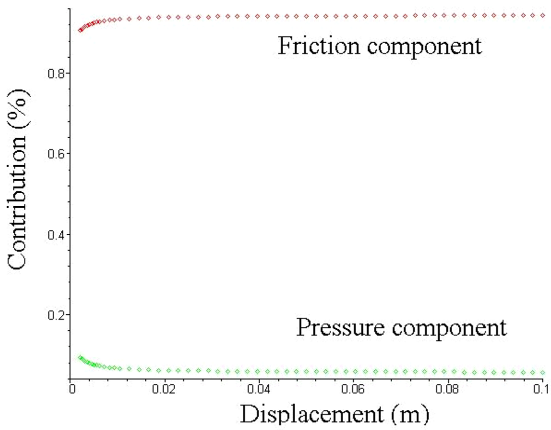



**B) SF values**

**Figure. 3. Pressure and friction shape factors versus displacement**




**a)case 1**




**b)case 3**

**Figure. 4. Contribution of soil resistance components versus displacement**

## CONCLUSION

Foundation on beam theory (Winkler Model) is exploited in this study in which Duncan & Chang hyperbolic relation is assigned to soil behavior. Friction and pressure components of soil resistance are calculated by integrating the pertinent stresses around the pile-soil interface, then “p-y” curve is acquired. Shape factors are calculated as a function of displacement, which is not considered anywhere so far.

Calculated “p-y” curves are used as input “p-y” curves in Lpile program to compare with four field case studies. Results show good agreement with the test results and in some cases, the suggested “p-y” curves have better results than the traditional curves used in Lpile. Shape factor either pressure shape factor or friction approaches unit value as soil displaces which indicates a uniform distribution of the stresses around the pile. Soil and pile properties have no effect on friction shape factor. The two shape factors are frequently used to convert PMT (pressuremeter test) results to “p-y” curves. Therefore, it is noticeable to use more accurate shape factors at various displacements.

Comparison of the case studies shows higher mobilization rate of friction in stiff clay rather than in soft clay. E/B is an indicator of mobilization rate of stress components. High value of E/B, which is proportional to subgrade reaction modulus, shows brittle behavior of pile-soil interaction.

## REFERENCES


Baguelin F., Frank R. and Said Y. H., “Theoretical Study of Lateral Reaction Mechanism of Piles”, Geotechnique,1977

Briaud J. L., Smith T. D., and Meyer B. J., “Laterally Loaded Piles and the Pressuremeter: Comparison of Existing Methods,” Laterally Loaded Deep Foundation, ASTM STP 835, 1983

Bhushan, K. Haley, S. & Fong, P. 1979. Lateral Load Tests on Drilled Piers in Stiff Clays. J. Geotech. Eng. Div. ASCE 105 (GT8): 969-985.

Hetenyi M. Beams on elastic foundations. University of Michigan Press; 1946.

Horvath JS. Beam-column-analogy model for soil–structure interaction analysis. JGeotech Eng 1993;119(2):358–64

Hsiung Yun-mei “Theoretical Elastic-Plastic Solution for Laterally Loaded Piles”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 129, No. 6, June 2003

Hsiung Y.M.,, & Chen, Y.L. (1997). “Simplified method for analyzing laterally loaded single piles in clay.” J. Geotech. Geoenviron. Eng., 123(11), 1018-1029

Janoyan K. Daniel, Stewart J.P. & Wallace J.W. “Analysis of p-y curves from lateral load test of large diameter drilled shaft in stiff clay”, Caltrans 6th seismic design workshop, Sacramento, (2001), paper 5-105

Janoyan K. Daniel, “Interaction between Soil and Full Scale Drilled Shaft Under Cyclic Lateral Load”, PhD. Thesis, University of California Los Angeles, 2001

Kerr AD. A study of a new foundation model, Acta Mech. 1965;I/2:135–47.

Kishida T. Fukaya T. & Hanzawa H., Development of the Friction meter for Evaluating Skin Friction of a Pile, Soils & Foundations, 26 (1), March 1986, 122-128

Mair, R. J. and Wood, D. M. 1987, Pressuremeter Testing, pp 51-52. Available from Construction Industry Research and Information Association, London SW1P3AU, England

Matlock, H. (1970): “Correlations for design of laterally loaded piles in soft clay,” Offshore Technology Conf., OTC Paper 1204, Houston, Texas, Vol. 1, pp. 577-595.

McClelland B. and Focht J A. “Soil Modulus for Laterally loaded Piles”, Journal of Soil Mechanics and Foundation Engineering, ASCE, Vol. 82, No. SM5, 1956

Pasternak PL. On a new method of analysis of an elastic foundation by means of two foundation constants [in Russian]. Gosudarstvennoe Izdatelstvo Literaturi po Stroitelstvu i Arkhitekture, Moscow, USSR. 1954.

Reese, L.C. and Welch, R.C. (1975a): “Lateral Loading of Deep Foundation in Stiff Clay” J. Geotechnic. Engrg Div., ASCE, Vol. 101, No. GT7, pp. 633-649.

Reese, L.C., Cox W.P. and Koop, F.D. (1975b): “Field testing and analysis of laterally loaded piles in stiff clay”, Offshore Technology, Conf. Houston, Texas, Paper OTC 2312, pp. 473-483 Vol. 2, pp. 671-690

Smith T.D., Ray Brian, “Shear Mobilization On Laterally Loaded Shafts”, Geotechnical Aspects of Stiff and Hard Clay, New York, April 1986, pp. 60-68

Smith T.D., Slyh R., “Lateral friction Mobilization rates For Laterally Loaded Piles”, 38th Canadian Geotechnical Conference, Edmonton, (1985), pp. 393-398

S. Kucukarsalan, P.K. Banerjee, N. Bildik, “Inelastic Analysis o Pile Soil Structure Interaction”, Engineering Structure, 2003, pp 1231-1239

Welch, R. C. & Reese, L. C. 1972. Laterally loaded behavior of drilled shafts. Research Report 3-5-65-89. Center for Highway Research. University of Texas, Austin

Zhang Lianyang, Silva Francisco and Grismala Ralph, “Ultimate Lateral Resistance in Cohesionless Soils”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 131, No.1, Jan 2005