### **Example 3.7a**

> restart:

> with(linalg):with(plots):

> N:=4;



> L:=1;



> eq:=diff(y(x),x$2)-Pe\*diff(y(x),x);



> bc1:=y(x)-1;



> bc2:=y(x);



Central difference expressions for the second and first derivatives are

> d2ydx2:=(y[m+1]-2\*y[m]+y[m-1])/h^2;



> dydx:=(y[m+1]-y[m-1])/2/h;



The governing equation in finite difference form is:

> Eq[m]:=subs(diff(y(x),x$2)=d2ydx2,diff(y(x),x)=dydx,y(x)=y[m],x=m\*h,eq);



A 'for loop' can be written for the interior node points as

> for i to N do Eq[i]:=subs(m=i,Eq[m]);od;



> Eq[0]:=y[0]=1;



> Eq[N+1]:=y[N+1]=0;



> y[0]:=solve(Eq[0],y[0]);



> y[N+1]:=solve(Eq[N+1],y[N+1]);



> h:=L/(N+1);



> for i to N do Eq[i]:=eval(Eq[i]);od;



> eqs:=[seq(Eq[i],i=1..N)];



> vars:=[seq(y[i],i=1..N)];



> A:=genmatrix(eqs,vars,'B1');



> evalm(B1);



Maple generates a row vector, which can be converted to a column vector as:

> B:=matrix(N,1):for i to N do B[i,1]:=B1[i]:od:evalm(B);



The solution is obtained as:

> X:=evalm(inverse(A)&\*B);



> for i to N do y[i]:=X[i,1];od;



> y[0]:=eval(y[0]);y[N+1]:=eval(y[N+1]);



Next, the result obtained is compared with the exact analytical solution:

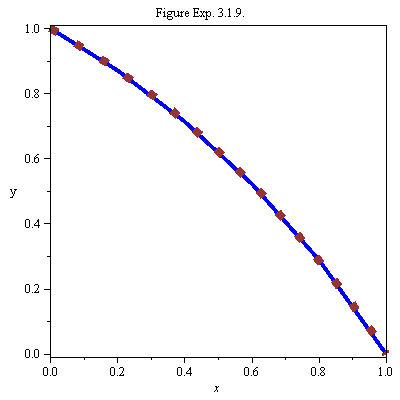
> ya:=(exp(Pe)-exp(Pe\*x))/(exp(Pe)-1);



> p1:=plot([seq([i\*h,subs(Pe=1,y[i])],i=0..N+1)],thickness=4,color=blue,axes=boxed):

> p2:=plot(subs(Pe=1,ya),x=0..1,thickness=8,color=brown,axes=boxed,linestyle=2):

> display({p1,p2},title="Figure Exp. 3.1.9.",labels=[x,"y"]);

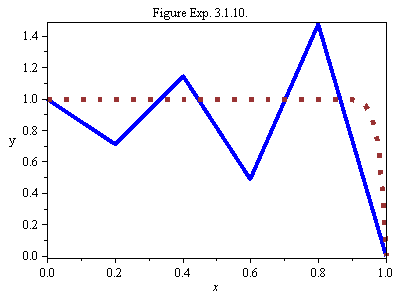


We observe that both the finite difference solution and the analytical solution match exactly when the Peclet number is 1. New plots can be obtained for different values of the Peclet number as follows:

> p1:=plot([seq([i\*h,subs(Pe=50,y[i])],i=0..N+1)],color=blue,thickness=4,axes=boxed):

> p2:=plot(subs(Pe=50,ya),x=0..1,thickness=5,color=brown,axes=boxed,linestyle=2):

> display({p1,p2},title="Figure Exp. 3.1.10.",labels=[x,"y"]);



This shows that for Pe = 50, four interior node points are not enough and we observe oscillations.[11][12] This happens usually when central difference approximations are used for the convective term . Use a forward approximation for the first derivative to solve this problem. Only dydx in the Maple program needs to be changed:

