Searching for Good Blank Regions in the Sky for Flatfielding

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Abstract T
he most important advantage of widefield cameras is, precisely, the "widefield", since this offers the observers the possibility of obtaining vast amounts of data in a much shorter observing time. However, for a reliable data interpretation it is necessary a proper data calibration. Concerning the flatfielding of images, many times it is required to obtain several integrations in blank regions (sky patches without bright sources) nearby to the science target areas. In this work we present a systematic approach to obtain a catalogue of useful blank regions, based on the application of the Delaunay triangulation of the sky.

1 The Delaunay Triangulation

The Delaunay triangulation (Delaunay 1934) consists in a subdivision of a geometric object (e.g. a surface or a volume) into a set of simplices. A simplex, or n-simplex, is the n-dimensional analogue of a triangle. More precisely, a simplex is the convex hull (convex envelope) of a set of $(n+1)$ points.

In particular, for the Euclidean planar case, given a set of points (also called nodes) the Delaunay triangulation becomes a subdivision of the plane into triangles, which vertices are nodes. For each of these triangles, it is possible to determine its associated circumcircle, the circle passing exactly through the three vertices of the triangle, and which center, the circumcen-

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tre, can easily be computed as the intersection of the three perpendicular bisectors.

Interestingly, in a Delaunay triangulation all the triangles satisfy the **empty circumcircle interior property**, which stays that all the circumcircles are empty, i.e., there are no nodes inside any of the computed circumcircles. The Delaunay triangulation is closely related to the Voronoi tessellation (Voronoi 1908), also called Dirichlet tessellation, which in the simple planar case is a decomposition of the plane into individual regions surrounding each node, such as all the points within a given region are closer to its corresponding node than to any other node. The Voronoi tessellation can be immediately built from the Delaunay triangulation by joining the circumcentres of all the circumcircles.

The Delaunay triangulation is not restricted to the Euclidean 2-dimensional (2D) case. It can be applied to other 2D surfaces (e.g. the surface of a 3D sphere) and to objects of higher dimension. For example, when considering the Euclidean three-dimensional space, the Delaunay triangulation becomes a partition of that space into tetrahedrons, and the empty circumcircle interior property implies that there are no nodes within the corresponding circumspheres.

2 Applying the Delaunay triangulation to the celestial sphere

The empty circumcircle interior property of the Delaunay triangulation provides a straightforward method for a systematic search of regions in the celestial sphere free from bright objects. If one computes the Delaunay triangulation in the 2D surface of a sphere, using as nodes the location of the stars down to a given threshold magnitude, the above property guarantees that all the circumcircles are void of stars brighter than that magnitude. Thus, the circumdiameter of every circumcircle determines the maximum field of view that can be observed in that region of the sky without including bright stars.

In order to proceed with the triangulation, we have made use of STRIPACK (Renka 1997), a Fortran 77 software package that employs an incremental algorithm to built a Delaunay triangulation of a set of points on the surface of the unit sphere. For N nodes, the storage requirement for the triangulation is 13N integer storage locations in addition to 3N nodal coordinates. The triangulation can be constructed with time complexity O(N log N). The original software was written using single-precision floating arithmetic. For the work presented here, we have modified the software to work using double-precision floating arithmetic, which is required in order to properly compute the triangulation when dealing with star separations approaching a few arcseconds.

3 Preparing the nodes for the triangulation

In this work we have applied the Delaunay triangulation to the celestial sphere using as source for the star coordinates the Bright Star Catalogue, 5th Revised Edition (Hoffleit & Warren 1991; available at CDS). This catalogue contains 9110 objects, although we have imposed a threshold visual magnitude of 6.5, which leads to an initial sample of 8404 stars. This initial collection of stars is still not suitable to compute the Delaunay triangulation due to the presence of stars with a too small separation to other stars (or even identical coordinates). For that reason, we decided to "merge" into single objects all the stars closer than a predefined angular resolution value. The resulting visual magnitude for the combined objects was computed as the sum of the fluxes of the merged stars. The coordinates of the new objects were placed in the line connecting the merged stars, closer to the brightest star (using a weighting scheme dependent on the individual brightnesses of the combined stars). Finally we have adopted an angular resolution of 5 arcseconds, which transform the initial magnitude-filtered catalogue of 8404 stars into a new catalogue containing 8373 objects.

4 Results

The resulting Delaunay triangulation leads to 16742 spherical triangles and, thus, the same number of circumcircles. For every circumcircle we determined its associated circumradius, measured as half the angle subtended by the circumcircle as viewed from the centre of the celestial sphere. Twice this angle correspond to the maximum field of view void of bright stars in that region of the sky. The main results, displayed in Figures 1 and 2, show that, as it is expected, the larger empty regions are found at high galactic latitudes.

5 Future work

We plan to extend this work in the future by using star catalogues including fainter magnitudes. That extension must take into account some critical aspects associated with these kind of catalogues that must be considered with caution. For example: (1) the completeness of the catalogues to a given limiting magnitude; (2) the accuracy and availability of photometric measurements in different bands; (3) the accuracy of the star positions (this is not such a critical issue since in our treatment we are applying a spatial resolution threshold of the order of a few arcseconds); and (4) avoid duplicity in the catalogues.

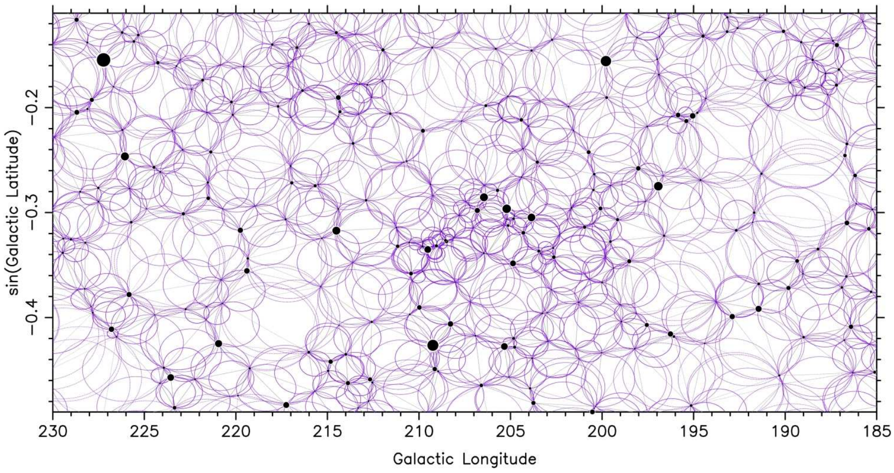

Fig. 1 Sky region around the Orion constellation. The Delaunay triangulation is shown with gray lines, while the associated circumcircles are displayed in magenta.

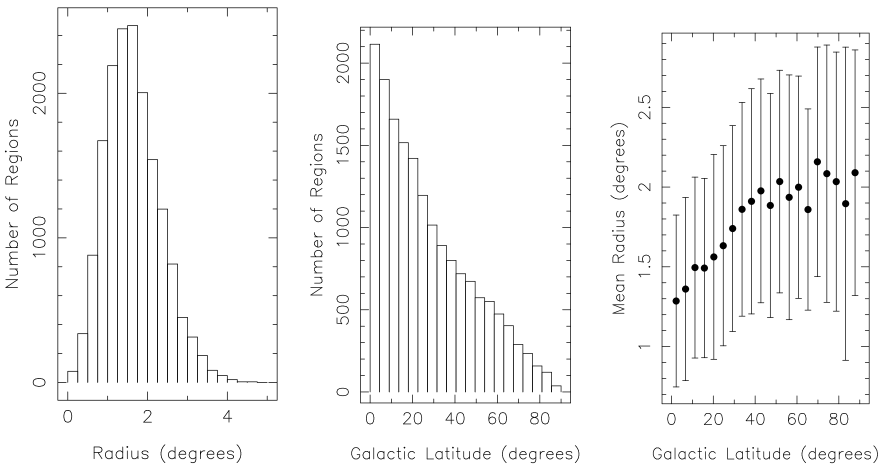

Fig. 2 *Left:* histogram of the different angular sizes (radii of the computed circumcircles) of the regions void of stars. *Center:* number of regions as a function of the galactic latitude. *Right:* variation of the mean circumcircle radius as a function of the galactic latitude. The error bars indicate the root mean square exhibited by the radii within each galactic latitude bin.

The plan is to generate a catalogue of blank regions that can be easily accessed through the WEB. Obviously, in this work we have ignored Solar System objects. They must be taken into account in order to make use of regions close to the Ecliptic at a given date.

References

1. B. Delaunay: *Sur la sphère vide*, Bull. Acad. Sci. USSR, **7**, 793–800 (1934)
2. E. D. Hoffleit, Jr. Warren: `http://cdsarc.ustrasbg.fr/vizbin/Cat?V/50` (1991)
3. R. J. Renka: *Algorithm 772: STRIPACK: Delaunay Triangulation and Voronoi Diagram on the Surface of a Sphere*, ACM Transactions on Mathematical Software, **23**, 416–434 (1997)
4. G. Voronoi: *Nouvelles applications des parametres continuis à la theorie des formes quadratiques: Deuxième mémoire: Reserches sur les parallèloèdres primitifs*, J. Reine Angew Math, 134, pp 198–287 (1908)