The Temporal Evolution of Linear Fast and Alfvén MHD Waves in Solar Coronal Arcades

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Abstract The excitation and temporal evolution of fast and Alfvén magnetohydrodynamic oscillations in a two-dimensional coronal arcade are investigated. The approach is to consider an equilibrium magnetic and plasma structure and then to introduce a perturbation trying to mimic a nearby disturbance, such as a flare or filament eruption. By numerically solving the time-dependent linearised MHD wave equations the properties of the solutions have been studied. First, the properties of uncoupled fast and Alfvén waves are described. Then, longitudinal propagation of perturbations is included, and the properties of coupled waves are determined.

1 Introduction

Observations obtained with imaging telescopes onboard SoHO, TRACE and, more recently, Hinode spacecrafts have clearly demonstrated the existence of waves and oscillations in solar coronal structures (see [2], for a recent review). The theoretical understanding on the propagation of magnetohydrodynamic (MHD) waves in such structures is important to increase our knowledge as well as to infer unknown physical properties of the corona, by means of the novel technique known as coronal seismology. The wave propagation properties of MHD waves are only now starting to be explored and understood in realistic magnetic configurations in a comprehensive way ([4]). In this paper, we study the excitation and subsequent temporal evolution of oscillations in a two-dimensional coronal arcade model. The approach is to consider a given equilibrium configuration and then to study the properties of waves by numerically solving the time-dependent, linearised ideal MHD wave equations. As we neglect gas pressure, we can only study the properties of fast and

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Alfvén waves in our magnetic and plasma configuration. In this work, we allow the three-dimensional propagation of waves.

2 Equilibrium configuration and excitation of linear waves

We model our coronal arcade by means of a static two-dimensional equilibrium configuration in which the plasma and field variables do not vary along the y-axis of a Cartesian frame (x, y, z). If we neglect gravity and gas pressure (plasma-$\beta = 0$), the magnetic field components of a two-dimensional potential arcade can be written as

$$B_x(x,z) = B_0 \cos\left(\frac{x}{\Lambda_B}\right) e^{-\frac{z}{\Lambda_B}}, \qquad B_z(x,z) = -B_0 \sin\left(\frac{x}{\Lambda_B}\right) e^{-\frac{z}{\Lambda_B}}$$

with $B_0 = B(x=0, z=0)$ and $\Lambda_B = 2\pi/L$ the magnetic scale height, L being the half-length of the arcade. In the $\beta = 0$ approximation the density or Alfvén speed profiles are arbitrary. Considering a vertically stratified atmosphere they can be written as

$$\rho(z) = \rho_0\, e^{-z/\Lambda}, \qquad V_A^2(z) = \frac{B^2}{\mu\rho} = v_{A0}^2\, e^{-(2-\delta)\frac{z}{\Lambda_B}},$$

where v_{A0} is the Alfvén speed at the photosphere ($z = 0$) and we have defined $\delta = \Lambda_B/\Lambda$ as the ratio of magnetic scale height to the density scale height. The case $\delta = 2$ corresponds to constant Alfvén speed, i.e., exponentially decreasing density profile, while $\delta = 0$ corresponds to a constant density configuration. Note that in both models the Alfvén speed is constant in the x-direction.

We now consider small amplitude perturbations of the form $\exp(\imath k_y y)$, the linear and adiabatic MHD wave equations can then be cast as 6 coupled equations for the three components of the perturbed velocity, v_x, v_y, and v_z, and the three components of the perturbed magnetic field, b_x, b_y, and b_z. Numerical solutions to these equations have been obtained for the temporal evolution of perturbations after an initial ($t = 0$) impulsive disturbance of the form

$$v_p = v_0\, e^{\left[-\frac{(x-x_s)^2+(z-z_s)^2}{a^2}\right]},$$

with v_0 the amplitude of the velocity perturbation, x_s and z_s the coordinates of the centre of the perturbation, and a the width of the two-dimensional Gaussian at half height. We impose line-tying boundary conditions at the photosphere, $\mathbf{v} = \mathbf{0}$ at $z = 0$ and flow-through conditions at the rest of the boundaries.

3 The coupling of fast and Alfvén waves

Uncoupled and coupled fast and Alfvén normal modes in our potential coronal arcade were studied by [3] and [1], respectively. We now describe the propagation of waves after an initial disturbance, such as a flare or filament eruption, which are often seen to induce oscillations in the solar corona. Consider first that $k_y = 0$. When perturbing in the normal direction ($v_p = v_n$, $v_y = 0$), a fast wave front is excited which propagates in all directions (see Fig. 1 left). Initially the front has an almost circular shape, but, as in our constant density model the Alfvén speed decreases with height, the wave propagates faster towards the photosphere where it is reflected. For large times all the energy associated to the wave leaves the system. Note that $v_y = 0$ for all times, hence a pure fast wave is excited. When perturbing in the longitudinal direction ($v_p = v_y$, $v_n = 0$), two wavefronts representing pure Alfvén waves are created (see Fig.1 right). They propagate along field lines and deform as they travel towards the photosphere, due to Alfvén speed variations and field line convergence. There, they undergo reflection and, for large times, a pure Alfvénic oscillation, confined to a given magnetic surface, remains. When including longitudinal propagation of perturbations, $k_y \neq 0$ an identical disturbance ($v_p = v_n$, $v_y = 0$) produces a wave with mixed fast and Alfvén wave properties (see Fig. 2). The excited wave has both $v_n \neq 0$ (left panels) and $v_y \neq 0$ (right panels). The behaviour of the normal velocity component is akin to the previous pure fast wave solution. In spite of only having perturbed in v_n, a longitudinal velocity perturbation arises, which, as the fast part of the wave crosses the full arcade, deposits some part of the energy in different magnetic surfaces all over the structure. By performing a Fourier analysis of the signal at different points in the arcade (not shown here), it is seen that the oscillatory frequency of each magnetic surface is in very good agreement with the analytical predictions obtained by [3], so we are confident that the eigenmodes of the different surfaces have been excited.

4 Conclusion

When MHD waves in solar coronal structures are allowed to propagate in the three-directions no pure fast and Alfvén waves exist. A fast-like wave front propagating across the structure excites Alfvénic oscillations at different magnetic surfaces in the arcade. These oscillations correspond to the natural Alfvén modes of the arcade at different heights. These standing Alfvén waves remain for long times, while the energy associated to the fast part leaves the system. Future studies will extend this to a fully three-dimensional magnetic structure.

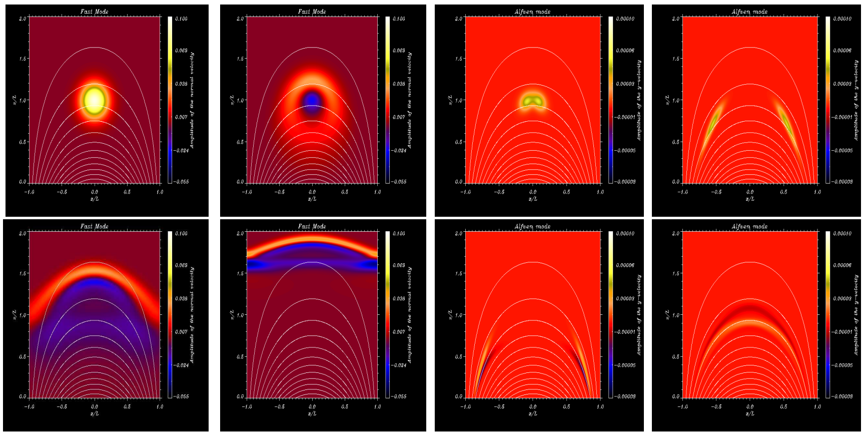

Fig. 1 *Left:* Fast wave: time evolution of the $v_n = B_z/B\ \hat{\mathbf{e}}_x - B_x/B\ \hat{\mathbf{e}}_z$ velocity field for the constant density model ($\delta = 0$). The initial perturbation of v_n is given by Eq. (5) with $x_s = 0$, $z_s = 1$, $a = 0.1L$, and $v_0 = 10^{-4} v_{A0}$. Magnetic field lines are represented with white lines. The time, shown at the top of each plot, is normalised to the Alfvén transit time, $\tau_A = L/v_{A0}$. Snapshots at $t/\tau_A = 0, 1.16, 3.63$, and 9.72 are shown. *Right:* Alfvén wave: time evolution of the v_y velocity field for $\delta = 0$. Snapshots at $t/\tau_A = 0.43, 2.61, 4.21$, and 29.04 are shown. The numerical solutions have been computed in a grid with $N_x = 400$ and $N_z = 400$ points.

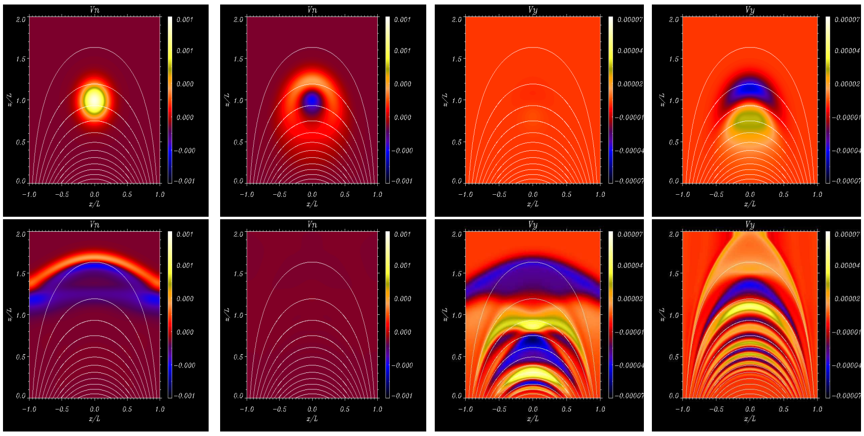

Fig. 2 Time evolution of the v_n and v_y velocity fields for the same model as before, but with $k_y L = 0.8$. The initial perturbation is given in v_n with $x_s = 0$, $z_s = 1$, $a = 0.1L$, and $v_0 = 10^{-4} v_{A0}$. Snapshots at $t/\tau_A = 0, 2.32, 5.8$, and 29.09 are shown. A wave with mixed properties is excited. The fast component (*left*) propagates isotropically in the arcade, while Alfvénic perturbations (*right*) are generated in almost all field lines. The numerical solution has been computed in a grid with $N_x = 400$ and $N_z = 400$ points.

References

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