The Importance of the Nucleus Rotation on the Size of the Dust Particle Ejected from Comets

A. Molina, F. Moreno, and F.J. Jiménez-Fernández

Abstract Dust particles attached on the nucleus surface can be dragged by cometary gas and, sometimes, these particles may remain around the comet in pseudostable orbits. In a previous work, Molina et al. (2008), we show the equation of motion, including rotational terms, and we estimate the maximum diameter of a cometary dust particle that could be lifted from the nucleus surface. The purpose of this work is to analyze the importance of those rotational terms in order to obtain the values of the size of the largest grain ejected from the nucleus. We consider a strong sunward anisotropy emission as reported by Fulle (1997). We discuss the obtained values and we make a comparison with those obtained using radar measurements.

1. Introduction

Particles attached to the nucleus surface can be ejected due to the following forces:

a) Drag force, $\vec{F}_D$. This force is due to the gas drag, and it can be derived from Navier-Stokes equations after suitable assumptions.

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We consider radially-symmetric outgassing. If the gas velocity is assumed to be constant, drag force is $\vec{F}_D = (1/32)\, C_D d^2 \dot{m}_g v_g r_d^{-3} \vec{r}_d$,
We consider radially-symmetric outgassing. If the gas velocity is assumed to be constant, drag force is $\vec{F}_D = (1/32)\, C_D d^2 \dot{m}_g v_g r_d^{-3} \vec{r}_d$,
which is a particular case of that one given by Wallis (1982), where C_D is the drag coefficient, d is the diameter of the grain, $\dot{m}_g$ is the gas loss rate, v_g is the gas velocity, and r_d is distance from the center of the nucleus to the dust grain position.

b) Gravitational force, $\vec{F}_G$. This force can be written as $\vec{F}_G = -m_d \left[M_c G r_d^{-3} \vec{r}_d + M_S G r_S^{-3} \vec{r}_S \right]$, being m_d the dust grain mass, M_c the comet mass, M_S the Solar mass, $\vec{r}_s$ the vector joining the centre of Sun to the dust grain and G the universal gravitational constant.

c) Solar radiation pressure force, $\vec{F}_{rad}$. Introducing the dimensionless parameter β as the ratio of the radiation pressure to the Sun gravitational force, the radiation pressure becomes $\vec{F}_{rad} = m_d M_S G \beta r_S^{-3} \vec{r}_S$.

d) Inertial forces, $\vec{F}_I$. In this work, we used a nucleus-attached reference system with origin at the centre of the comet. Obviously, this frame is non inertial and then we must consider two inertial forces. One of them is due to the gravitational comet attraction by the Sun and the other is due to the rotation of the comet (spin). The final expression of this force is $\vec{F}_I = G M_S m_d r_c^{-3} \vec{r}_c + m_d \left\{ \vec{\Omega} \times (\vec{r}_d \times \vec{\Omega}) + 2 \vec{v}_d \times \vec{\Omega} \right\}$, where $\vec{\Omega}$ is the nucleus angular velocity and $\vec{v}_d$ is the dust particle velocity. Here, $\vec{\Omega}$ is considered constant with time and therefore no angular acceleration terms are included.

The motion equation can be written as follows:

$$\frac{d^2\vec{r}_d}{dt^2}=\left(\frac{3C_D\dot{m}_g v_g}{16\pi C_p Q_p}\beta-GM_c\right)\frac{\vec{r}_d}{r_d^3}+\beta M_S G\frac{\vec{r}_c}{r_c^3}+\frac{\mu M_S G}{r_c^3}\left(\frac{3(\vec{r}_d\cdot\vec{r}_c)}{r_c^2}r_c-\vec{r}_d\right)+$$
$$+\Omega^2\vec{r}_d-(\vec{\Omega}\cdot\vec{r}_d)\cdot\vec{\Omega}+2(\vec{v}_d\times\vec{\Omega}) \quad (1)$$

Here, C_p= 1.19 10^{-3} kg m^{-2}, Q_p is the scattering efficiency of the grain, $\mu = 1\text{-}\beta$, being t the time. This expression is similar to that used by Fulle (1997), but now terms due to the nucleus rotation are included.. From equation (1), we obtain an expression for the particle with largest diameter than can be lifted from the surface (see, for example, Molina et al. 2008):

$$d_{\max}=\frac{1}{\rho_d g_{eff}}\frac{3C_D\dot{m}_g v_g}{16\pi^2 R^2} \quad (2)$$

being R the radius of the cometary nucleus and $g_{eff} = g - \Omega^2 R\cos^2\varphi$ (g is the gravity of the comet and φ is the latitude on the surface place where the dust particle is located). In this paper, we are going to show the importance of including the rotational terms in the equation motion, and, particularly, in the values obtained for d_{max} for several comets.

2. Grain ejection from Comet Halley

The state of rotation of Halley´s comet has been the subject of many works. An explanation to the two observed periods is to consider the Halley´s nucleus to be in a complex rotation state (see Molina et al. 2003, and references therein). In any case, the observed rotation is slow enough to consider $g_{eff} \approx g$ because $\Omega^2 R\cos^2\varphi \ll g$. Thus, the obtained value of d_{max} is independent of the nucleus rotation for the Halley´s comet, and, in general, for those comets with rotation periods τ such as $\tau^2 \gg \frac{3\pi}{\rho_n G}$, being ρ_n the density of the nucleus. Then, the value of d_{max} will be independent of the rotation of the comet when $\tau^2 \gg 1.4 \times 10^{11}\,\rho_n^{-1}$ if the international system of units is used. As g_{eff} must be positive, only rotation periods larger than a critical period $\tau_{crit} = \sqrt{\frac{3\pi}{G\rho_n}}$ are possible. Figure 1 shows the curve of τ_{crit} for different values of nucleus densities. Only the values of rotational period belonging to the region above the curve are possible.

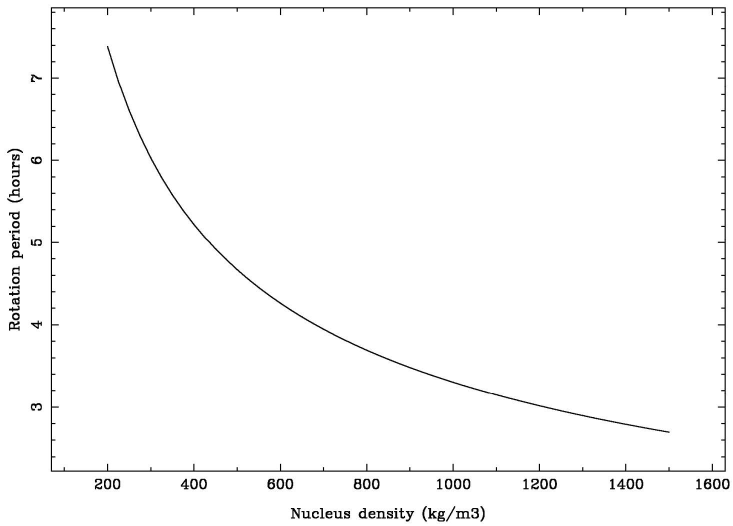

Figure 1. Critical rotation period versus density of the cometary nucleus. The region under the curve is forbidden.

We define the parameter α as the percentual relative increase in d_{max} due to the nucleus rotation. From equation (2), we obtain $\alpha = 100 \frac{1}{\frac{\rho_n G \tau^2}{3\pi} - 1}$, which is shown versus cometary rotation in figure 2 for a density $\rho_n = 1000$ kg m^{-3}.

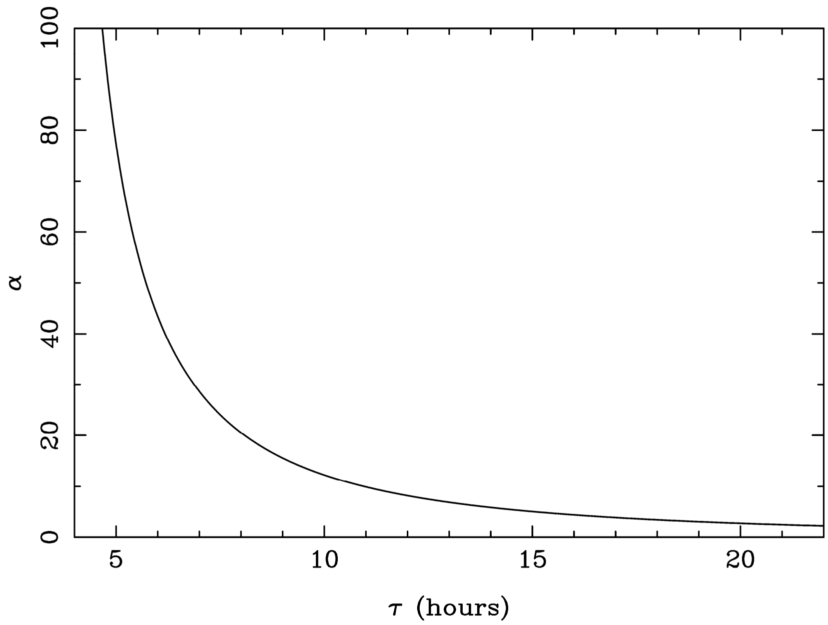

Figure 2. Percentual relative increase in $d_{\max}$ due to cometary rotation for 1000 kg m^{-3} nucleus density.

From that curve we can see, for example, as the size of the largest ejected particle from the nucleus for comets rotating with 8 hours period are a 20% bigger than for non-spining comets.
As shown in Equation 2, $d_{\max}$ is proportional to the quantity $\dot{m}_g v_g$ and the grains would not be lifted from the surface for low values of it. Fulle (1997) assumed the value of 1.5 10^7 kg m s^{-2} obtained by Krankowsky et al. (1986) for 1P/Halley. However, if that value is considered in the integration of the equation of motion, any grain is lifted from the surface of the comet. Fulle considered that the emission grains of the Halley nucleus is not an isotropic emission, suggesting a strong anisotropy towards the sun as the third

exponent of cosine of the zenital angle. Then, the value of the gas mass flux must be multiplied by an integration constant.

Due to the uncertainties of the water molecules density (around fifty percent) derived from the neutral mass spectrometer experiment aboard of the Giotto spacecraft (Krankowsky, 1986), considering volume mixing ratios for H_2O, CO_2, NH_3 and CH_4, and the uncertities of the gas expansion velocity measurements, we find a range of possible values for $\dot{m}_g v_g$ from 5.0 10^7 to 3.1 10^8 kg m s^{-2}. Then, we obtain values of $d_{\max}$ between 0.17 and 1.06 m if dust and nucleus densities are equal to 1000 kg m^{-3}.

3. Comparison with d_{max} values obtained from radar measurements.

Radar observations have indicated the presence of large grains in several comets long time ago (see Harmon et al. 2004, and references therein). However, a solid detection of large boulders from radar measurements was published by first time in Nolan et al. (2006), who analyzed radar measurements of Comet C/2001 A2 (LINEAR). These authors used a gas-drag model and obtained values for a velocity factor C_v fitting a model to radar Doppler spectra. This velocity factor is $C_v = \left(3C_D v_g ZR / 4\rho_d\right)^{1/2}$ where Z is the mass gas flux at the surface. To obtain the radius of the largest particle that can be lift from the nucleus surface, a_m, Nolan et al. (2006) used the expression (their eq. 4): $a_m = \dfrac{9C_D v_g Z}{32\pi G R \rho_n \rho_d} = \dfrac{3C_v^2}{8\pi G R^2 \rho_n}$. They got a good fit to the spectra with C_v = 36 $cm^{1/2}$ m s^{-1}. They assumed ρ_n = 1000 kg m^{-3}, ρ_d = 500 kg m^{-3}, R = 1 km, and temperature at the surface T = 250 K. Thus, they indicated a value for Z of 7 10^{-4} g cm^2 s^{-1} and a value for the radius of the largest grain that can be lifted a_m = 10 m. However, if C_v = 36 $cm^{1/2}$ m s^{-1} is inserted in equation (4) by Nolan et al. (2006), other values, in fact, are obtained: Z=1.6 10^{-3} g cm^2 s^{-1}, a_m = 23.2 m. For the moment we do not know the reason of such discrepancy.

Comet C/2001 A2 (LINEAR) can be considered as a fast rotator with period 3 or 6 hours (Woodney et al. 2001). Then, the inclusion of the rotational effects should enlarge the estimated maximum

radius of the lifted particles by a factor 1.43 as the rotation period is 6 h. Therefore, the estimated a_m by Nolan et al. (2006) should be more than 14 m. The period of 3 h is less than the critical period as can be shown in figure 1.
In order to compare d_{max} values obtained from radar measurements with other techniques we consider the molecular abundances obtained by Magee-Sauer et al. (2008) on 10 July 2001 using NIRSPEC instrument on the Keck-2 telescope at Mauna Kea. After considering that the composition is not only water, although water molecules are near 90% of the total molecular abundance, we obtain $\dot{m}_g$ = 1.4 10^3 kg s^{-1}. This value is in agreement with that reported by Nolan et al. (2006) assuming isotropic grain ejection: 1-3 10^3 kg s^{-1}. It is an excellent result because both values were obtained using completely different techniques.
Assuming a grain temperature of 250 K, we have a value of 270 m s^{-1} for v_g. Then, inserting $\dot{m}_g$ = 1.4 10^3 kg s^{-1} and v_g = 270 m s^{-1} in equation (2), we obtain a value of d_m = 32 cm assuming isotropic ejection and omitting the rotation term. If we consider an anisotropic ejection as the third exponent of cosine of the solar zenital angle and the fast rotation of the Comet C/2001 A2 (six hours period) we obtain a value of d_m = 3.6 m, which is a large value but much smaller than that one reported by Nolan et al. (2006) using C_v = 36 $cm^{1/2}$ m s^{-1}.
Other comet studied by radar measurements is the Comet IRAS-Araki-Alcock (IAA). It is a very slow rotator (nucleus rotation period of 2-3 d), therefore the value of d_m is unaffected by the rotation terms. Harmon et al. (1989) obtained an a_m of only 0.6 mm assuming $\rho_n = \rho_d = 10$ kg m^{-3}, v_g = 280 m s^{-1}, and R = 5 km and using the gas mass-loss rate measured by Feldman et al. (1984). Nevertheles, Harmon et al. reported a value of a_m = 0.6 mm assuming uniform outgassing over the sunlit hemisphere and a value of a_m = 3 cm if only 1 per cent of the total surface area of the comet was assumed active. Following our assumption of an anisotropic ejection as the third exponent of cosine of the solar zenital angle and using a mass-loss rate of 6.0 10^5 g s^{-1} as measured by Feldman et al. (1984) and the same parameter values used by Harmon et al. (shown

above) we obtain a value of d_m = 0.5 cm. If we insert the value of C_v = 8 $cm^{1/2}$ m s^{-1} as shown by Nolan et al. (2006) in their equation (4) we obtain a value of a_m = 6 cm, which is inconsistent with the value a_m = 0.6 mm reported by Harmon et al. (1989).

4. Comet 67P/ Churyumov-Gerasimenko

The Rosetta lander will land upon the surface of comet Churyumov-Gerasimenko in late 2014. For that reason, is important to illustrate d_m obtained with the present model. Although first estimates favored a near 3 km radius, more precise studied indicate a radius less than 2 km. Lamy et al. (2007), from the analysis of several light curves, concluded that the comet is an irregular body with an effective radius of 1.72 km and they found that the comet is rotating around a principal axis with a period of 12.4-12.7 hours. They estimated a nuclear density of 370 kg m^{-3} although densities between 100 and 500 kg m-3 were reported by Davidsson and Gutierrez (2005). Considering gas production rate values shown in Lamy et al. (2007), and that ρ_d = 300 kg m^{-3} and v_g = 270 m s^{-1}, we obtain values of d_m = 0.17-0.56 m.

5. Conclusions

We have used a classical outgassing model to estimate the diameter of the largest boulder that can be lifted from the cometary nucleus surface. Our results are compared with those one obtained from radar measurements being smaller our results. In the case of the Comet 67P/ Churyumov-Gerasimenko, an upper limit of a half of meter of diameter.

The role of the rotation of the cometary nuclei on the size of the ejected particles is discussed. A nucleus spinning in 6 hours can lift a 40 per cent larger grains that a comet with very low rotation.

The introduction in the equations (2) of a strong anisotropy toward the sun as the third exponent of cosine of the zenital angle as proposed by Fulle (1997) allow us to obtain reasonable values of d_m.

The dependence on the heliocentric distance, excentricity of the orbit of the comet, true anomaly, obliquity and longitude of the place where the grain is lifted, will be studied soon.

Acknowledgements

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