Observing supermassive black hole binary systems with LISA

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Abstract We study parameter estimation of supermassive black holes by LISA using the inspiral full post-Newtonian gravitational waveforms up to 2PN. The analysis shows that LISA will observe the coalescence of a system with total mass $(10^4 - 10^8)\ M_\odot$ with signal-to-noise ratios of hundreds and errors in masses and luminosity distance smaller than 10%. For sources at $z = 1$ we find that at least 20% could be localized within a $(1^o \times 1^o)$ patch.

1 Introduction

The future Laser Interferometer Space Antenna (LISA) [7] will observe gravitational waves (GWs) in the frequency band of $(10^{-4} - 1)$ Hz, which are generated by supermassive black hole binary systems or by rapidly rotating compact objects in our Galaxy. Among the wide diversity of different LISA sources, the observation of supermassive black holes (SMBHs) will address many of LISA's science objectives in understanding the mechanism of their formation [8], performing fundamental tests of gravitational theory [4] and putting constraints on the equation of state of dark energy [10].

The real impact of these observations will depend on how accurately the source parameters can be estimated. Here, we summarize the main results of the work that has been done in studying the LISA parameter estimation of SMBH binary systems [11, 12] , which are in agreement with other recent works on the same topic [2, 3, 9].

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2 Gravitational waveform, method and assumptions

SMBH binaries are long lived sources in the LISA band. The whole coalescence of the compact binary system is usually divided into three phases: the adiabatic inspiral, the merger and the ringdown. Most of the SNR accumulates during the last days of the coalescence (merger), but one critically relies on long integration times to disentangle the source parameters. Here, we are considering the inspiral phase of the coalescence and in particular, the last year before merger.

The inspiral post-Newtonian (PN) waveforms in the two polarizations h_+ and $h_\times$, take the general form

$$h_{+,\times} = \frac{2M\eta}{D_L}(M\omega)^{2/3}\left\{H^{(0)}_{+,\times} + v^{1/2}H^{(1/2)}_{+,\times} + vH^{(1)}_{+,\times} + v^{3/2}H^{(3/2)}_{+,\times}\dots\right\} \quad (1)$$

where we have set $G = c = 1$, $v \equiv (M\omega)^{2/3}$ is the PN expansion factor, ω is the orbital frequency, D_L is the luminosity distance to the source, and M and η are the observed total mass and the symmetric mass ratio respectively. The explicit expressions for $H^{(m/2)}_{+,\times}$ [1, 5] include contributions from several harmonics of the orbital frequency. In this paper, we study LISA parameter estimation of SMBH binary systems modeled by Eq. (1) (so-called *full* waveform, FWF) up to 2PN order and we compare the results with the commonly used *restricted* approximation (RWF) where all the amplitude terms except the leading Newtonian quadrupole one, $H^{(0)}_{+,\times}$, are neglected.

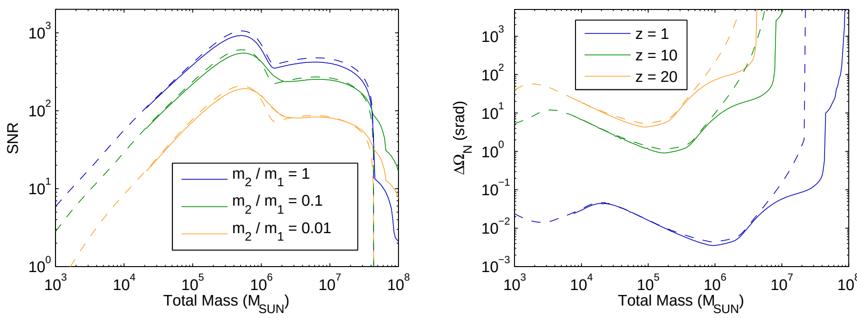
For the SMBH binary inspirals, most of the SNR accumulates at frequencies below 10 mHz, so it is adequate to use the low frequency approximation for the LISA response [6] in which LISA can be regarded as two independent Michelson interferometers. The parameter estimation is based on the Fisher information matrix formalism, which assumes signals with high SNR. Moreover, we are assuming circular orbits with negligible spin precession effects (the gravitational waveform is characterized by 11 parameters) and the model of Universe that we use to relate redshift and D_L is described by the cosmological parameters $H_0 = 71$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$.

3 Results

In Fig. 1a we are plotting the evolution of the SNR as a function of the total mass of the binary for different mass ratios. The high SNR values we obtain (several hundreds) and the wide range of masses $M \sim (10^4 - 10^8)\ M_\odot$, makes LISA a powerful instrument to observe SMBH binaries and study their dynamics up to very high redshifts. The main impact of working with FWF rather than RWF is that we increase LISA's reach to higher masses.

In Fig. 1b we can see how the errors in sky location change with the total mass of the system for different values of redshift. When the redshift is increased, the errors grow up proportionally to D_L but also there is a shift to lower physical masses[1]. The sky resolution of any GW detector is always poorer than the electromagnetic telescopes, and the reason is because the GW detectors are omnidirectional (all sources and sky locations are recorded in the same single measured strain), so the distinction between different sky locations is done as part of the posterior data analysis.

The amplitude of the gravitational wave signal increases as the two black holes approach to the plunge and in fact, most of the SNR is accumulated during the last day before merger. For this reason, the total SNR and the errors in different parameters depend importantly in the sky location of the source as it can be seen in Figs. 7-10 of [11]. So, in order to extract general conclusions about the science that can be done from LISA observations, we performed Monte Carlo (MC) simulations over sky location and orientation angles for different systems at $z = 1$, the SNR and error distributions can be found in Fig. 11 of [11]. In Tab. 1 we are presenting the percentage of sources from these MCs with an error smaller than a certain value. What we see from these numbers is that LISA will be able to estimate masses and luminosity distances better than a 10% error in almost all the cases and the sky resolution will be better than $(1^{\circ} \times 1^{\circ})$ in almost 20% of the cases. These percentages are much worse when one just considers the RWF, so the improvement added by modeling the signal with the complete waveform is very sensitive in terms of the science that can be done.



(a) (b)

Fig. 1 LISA observations of SMBHs during the last year before merger, all located at a fixed sky position and orientation given by the spherical coordinates: $\cos\theta_N = -0.6$, $\phi_N = 1$, $\cos\theta_L = 0.2$ and $\phi_L = 3$. (solid: FWF ; dashed: RWF) **(a)** SNR versus total mass of the system in the source rest frame for different mass ratios having all sources at $z = 1$. **(b)** Angular resolution also as a function of total mass for an equal mass case considering different redshifts

[1] The observed masses are redshifted respect the physical ones as: $M_{obs} = (1 + z)M_{phys}$. Masses plotted in the x-axis of Fig. 1 are physical, i.e. measured in the source rest frame.

Table 1 Percentage of sources from the MC simulations with a measurement error less than the given values. It corresponds to 1000 MCs over sky locations and orientations uniformly distributed in the sky, assuming that all sources were located at $z = 1$

$m_1, m_2 (M_\odot)$:	$10^7 - 10^7$		$10^7 - 10^6$		$10^6 - 10^6$		$10^6 - 10^5$	
	RWF	FWF	RWF	FWF	RWF	FWF	RWF	FWF
$\Delta\Omega_N < (2.5^o \times 2.5^o)$	2.4	34.2	10	43	35	50	35	57
$\Delta\Omega_N < (1^o \times 1^o)$	0.8	19.3	3.1	24.1	11	28	10	35
$\Delta\Omega_N < (0.5^o \times 0.5^o)$	0.4	10.7	1.0	7.4	3.9	14.0	3.5	18.5
$\Delta D_L/D_L < 10\%$	0	42	2	92	86	100	85	100
$\Delta\mathcal{M}/\mathcal{M} < 10\%$	0	99	10	100	100	100	100	100
$\Delta\mu/\mu < 10\%$	0	99	0	99	0	100	91	100

4 Conclusions

LISA will be able to observe the coalescence of two SMBHs with total mass $M \sim (10^4 - 10^8)\ M_\odot$ up to high redshifts, with SNRs of several hundreds and errors in masses and luminosity distance smaller than 10% in all the cases (in some of them even less than 1%). For sources at $z = 1$ we have found that at least 20% could be localized in the sky with an error smaller than $(1^o \times 1^o)$, allowing us to try to observe the potential electromagnetic counterpart emission during the coalescence. Working with the FWF instead of RWF, increases LISA's reach at higher masses and also provides a sensitive improvement in the parameter estimation of the high mass systems.

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