Cosmological Vector Perturbations and CMB Anomalies.

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Abstract Recently, it has been proved that large scale vector modes could explain most of the CMB anomalies in the first temperature multipoles. Some divergenceless (vortical) velocity fields, –which are superimpositions of vector modes– can explain both the alignment of the second and third multipoles and the planar character of the octopole. In this paper we comment: (i) some papers trying to account for the mentioned anomalies, (ii) our explanation based on vector modes, and (iii) some current ideas about the possible origin of these modes.

1 Introduction

It may be stated that the analysis of the WMAP (Wilkinson Microwave Anisotropy Probe) five years data has confirmed the standard ΛCMB concordance model [1]. In fact, this model explains most of the observations on: (i) the CMB anisotropy, (ii) the far Ia supernovae, (iii) the statistical properties of big and deep galaxy surveys, and so on. Nevertheless, some anomalies have appeared in the CMB angular power spectrum (C_ℓ quantities) extracted from the cleaned WMAP maps of the CMB sky (five maps corresponding to different frequencies). We are here concerned with the anomalies in the first temperature multipoles (quadrupole and octopole); in particular, we focus our attention on: (a) the small value of the quadrupole Q_2, (b) the planar structure of the octople Q_3 and, (c) the alignment between the quadrupole and the octopole [2]. Other anomalies are described in the literature, e.g., (α) the asymmetry between the ecliptic hemispheres [3, 4], (β) the existence of a preferred

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axis [5], (γ) the existence of a preferred plane without any internal rotational symmetry [6], (δ) the observation of strange alignments with characteristic directions of the solar system [7], and so on.

The temperature distribution in the sky is usually expanded as follows: $T(\theta,\phi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{+\ell} a_{\ell m} Y_{\ell m}$, where functions $Y_{\ell m}$ are the well known spherical harmonics and, then, functions $Q_2(\theta,\phi) = \sum_{m=-2}^{+2} a_{2m} Y_{2m}$ and $Q_3(\theta,\phi) = \sum_{m=-3}^{+3} a_{3m} Y_{3m}$ are the quadrupole and the octopole, respectively. The question is: how could be characterized these two multipoles? In order to do that, some parameters have been defined. The quadrupole can be described by the amplitude $C_2 = (\sum_{m=-2}^{+2} |a_{2m}|^2)/5$ and the direction $\mathbf{n}_2$, whereas the octopole can be characterized by the amplitude $C_3 = (\sum_{m=-3}^{+3} |a_{3m}|^2)/7$, the planarity parameter t, and the direction $\mathbf{n}_3$. The nonlinear rigorous definition of t, $\mathbf{n}_2$, and $\mathbf{n}_3$ can be found in [2]; nevertheless, the meaning –from an intuitive point of view– of these three parameters is illustrated in Fig. 1. The centers of the spots appearing in the quadrupole (left panel) and the octopole (right panel) presented in Fig. 1 are all very close to the equatorial circle. In other cases, the spots of the octople are not located close to any maximum circumference. The parameter t measures the planar character of the octopole. If the centers of the octopole spots are very close to some maximum circumference in the celestial sphere, it is said that the octopole is very planar and, then, the t value is very close to unity. For planar octopoles (quadrupoles), vectors $\mathbf{n}_3$, ($\mathbf{n}_2$) are orthogonal to the plane containing the spot centers (equatorial plane in Fig. 1). The octopole obtained from the WMAP data is very planar ($t \simeq 0.94$), and the angle between its direction $\mathbf{n}_3$ and that of the quadrupole $\mathbf{n}_2$ is $\alpha_{23} \simeq 10^{\circ}$; in other words, Q_2 and Q_3 are very aligned. The amplitudes appear to be $C_2 \simeq 3\times 10^{-11}$ and $C_3 \simeq 7\times 10^{-11}$.

In the framework of the concordance model, the whole probability of all the above anomalies is very small; e.g., the probability of the observed quadrupole amplitude is ~ 0.16, the probability of the planar character of the octopole is ~ 0.07 and, finally, the probability of the alignment between the quadrupole and the octopole is ~ 0.015; therefore, if these three anomalies are assumed to be statistically independent (for rough estimates), the whole probability of them would be $\sim 1.7\times 10^{-4}$. If we also consider the anomalies (α), (β), (γ), and (δ), the whole probability would be very small and, consequently, it could be stated that the concordance model does not explain the WMAP anisotropies for small ℓ values, that is

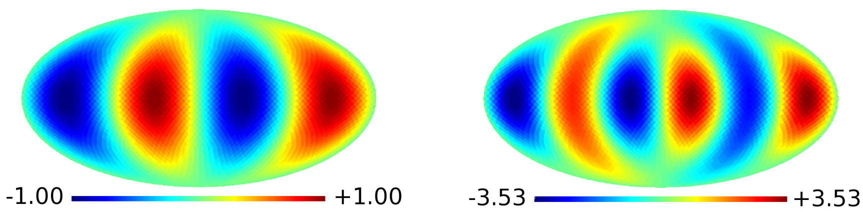

Fig. 1 Mollwide representation of the quadrupole $Q_2(\theta,\phi)$ (left) and the octopole $Q_3(\theta,\phi)$ (right) of a certain sky map. Normalization is arbitrary and irrelevant

to say, for large angular scales $\theta = \pi/\ell$. It seems obvious that, in any cosmological explanations of the mentioned anomalies, the alignments inside the solar system should be casual; nevertheless, even in this case, the whole probability of the remaining anomalies would be too small. It seems that something must be modified in the current cosmological paradigm.

The above probabilities have been calculated in the framework of the concordance model, in which, the distributions of linear scalar perturbations and CMB temperatures are Gaussian and statistically isotropic. Since anomalies imply preferred directions and alignments (see above), which are very unlikely in statistically isotropic models, we should look for nonisotropic scenarios. Furthermore, privileged directions in cosmology strongly suggest the existence of appropriate vector perturbations. Scalar (seeds for structure formation) and tensor (gravitational waves) perturbations have been extensively considered in the literature; however, the so-called vector perturbations (divergenceless vector fields) did not deserve much attention. We think it was due to the possible incompatibility between vector modes and CMB statistical isotropy (absolutely accepted in cosmological grounds before WMAP observations). Nowadays, statistical isotropy seems to be violated and, consequently, appropriate levels of vector perturbations could be the key to account for this violation. This was our point of view in papers [8, 9].

Along this paper, Latin indexes run from 1 to 3. Units are defined in such a way that $c = \kappa = 1$ where c is the speed of light and $\kappa = 8\pi G/c^4$ is the Einstein constant. The unit of length is the Megaparsec. Symbols a, τ, z, H_0, and Ω_m stand for the scale factor, the conformal time, the redshift, the Hubble constant, and the density parameter of matter (baryonic plus CDM), respectively. Whatever quantity A may be, A_0 (A_e) stands for the value of A at present (CMB emission) time. Quantity a_0 is assumed to be unity. This choice is possible in any flat background.

2 Explaining the large angular scale CMB anomalies

Let us now describe and discuss a few proposals designed to explain the low ℓ anomalies (an exhaustive study is not presented by the sake of briefness). The chosen explanations are due to Jaffe et al. [10] and Inoue & Silk [11].

In the first case [10], a Bianchi VII_h model is considered to introduce vorticity and shear in the universe. Although the authors proved that the subtraction of the CMB Bianchi component leaves a statistically isotropic sky at large angular scales (first CMB multipoles), other observations are not explained by the model.

The second explanation [11] requires two large voids at well defined positions. These voids would be located at redshifts $z \sim 1$. They would have a density contrast $\delta \sim -0.3$ and a size $\sim 300h^{-1}\ Mpc$. The separation distance would be $\sim 400h^{-1}\ Mpc$. The center of one of these voids would be in the direction $(l,b) = (-153^\circ, -59^\circ)$ and the direction of the equidistant point (between void centers) would be $(l,b) = (-30^\circ, -30^\circ)$. In our opinion, the main problem with this model is that voids with the proposed size (too large) are very unlikely in the concordance model. We could change the spectrum of the energy density perturbations and other elements to justify the presence of these voids; nevertheless, it would be expectable the failure of the new spectrum in order to explain various current observations (including the peaks of the CMB angular power spectrum).

Recently [8, 9], the authors of the present paper proposed another explanation, in which, superimpositions of vector modes with very large spatial scales (superhorizon vector perturbations) would produce vorticity, shear, and appropriate CMB anisotropy explaining most of the low ℓ anomalies (without conflicts with other observations). Let us discuss this proposal in more detail. If only very large superhorizon vector modes are superimposed, the resulting divergenceless vector fields must be almost constant in big regions, maybe in regions as large as the volume limited by the large scattering surface. In particular, we could have vortical velocity fields having an almost constant angular velocity $\mathbf{w}$ in the region crossed by the CMB photons. The direction of $\mathbf{w}$ would then define a preferred direction in an absolutely natural way. A first realization of this possibility can be found in [9].

3 Simulations and results

Vector perturbations can be expanded in terms of the so-called vector harmonics [12], whose form is $\mathbf{Q}^{\pm} = \varepsilon^{\pm}(\mathbf{k}) exp(i\mathbf{k}\cdot\mathbf{r})$, where $\mathbf{k}$ is the wavenumber vector. A representation of vectors ε^{+} and ε^{-} is: $\varepsilon_1^{\pm} = (\pm k_1 k_3/k - ik_2)/\sigma\sqrt{2}$, $\varepsilon_2^{\pm} = (\pm k_2 k_3/k + ik_1)/\sigma\sqrt{2}$, and $\varepsilon_3^{\pm} = \mp\sigma/k\sqrt{2}$, where $\sigma = (k_1^2 + k_2^2)^{1/2}$.

In standard cosmology there are vector perturbations associated to: the peculiar velocity v_i, the metric components $h_i = g_{0i}$, and the anisotropic stresses Π_{ij}. Condition $\Pi_{ij} = 0$ is assumed. The expansions of vectors $\mathbf{h}$ and $\mathbf{v}$ read as follows $\mathbf{h} = B^{+}\mathbf{Q}^{+} + B^{-}\mathbf{Q}^{-} \equiv B^{\pm}\mathbf{Q}^{\pm} \equiv \int B^{\pm}(\mathbf{k})\varepsilon^{\pm}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r})d^3k$ and $\mathbf{v} = v^{\pm}\mathbf{Q}^{\pm}$. Functions $B^{\pm}(\mathbf{k},\tau)$ and $v^{\pm}(\mathbf{k},\tau)$ describe the perturbation in momentum space. The differences $v_c^{\pm}(\mathbf{k},\tau) = v^{\pm}(\mathbf{k},\tau) - B^{\pm}(\mathbf{k},\tau)$ are gauge invariant.

These expansions can be introduced into Einstein equations to get the evolution equations of $B^{\pm}$ and $v^{\pm}$, which can be easily solved to obtain the following evolution laws in the matter dominated era

$$v_c^{\pm}(\tau,\mathbf{k}) = v_{c0}^{\pm}(\mathbf{k})/a(\tau)\ ,\ \ B^{\pm}(\tau,\mathbf{k}) = 6H_0^2\Omega_m v_{c0}^{\pm}(\mathbf{k})/k^2a^2(\tau)\ , \qquad (1)$$

By using the equations of the null geodesics, the above expansions, and Eq. (1), the relative temperature variation $\Delta T/T$ –along the $\mathbf{n}$ direction– can be easily written as follows:

$$\frac{\Delta T}{T} = \mathbf{v}_{c0}\cdot\mathbf{n} - \mathbf{v}_{ce}\cdot\mathbf{n} + 6H_0^2\Omega_m \int_0^{r_e} \frac{dr}{a^2(r)} F(\mathbf{r}), \qquad (2)$$

where $F(\mathbf{r}) = F_{pq}(\mathbf{r})n^p n^q$, $\mathbf{v}_{c0}\cdot\mathbf{n} - \mathbf{v}_{ce}\cdot\mathbf{n} = -z_e \int v_{c0}^{\pm}(\mathbf{k})\varepsilon^{\pm}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r})d^3k$, and

$$F_{pq}(\mathbf{r}) = -i \int \frac{k_p}{k^2} v^{\pm}_{c0}(\mathbf{k})\, \varepsilon^{\pm}_q(\mathbf{k}) \exp(i\mathbf{k}\cdot\mathbf{r})\, d^3k \; . \tag{3}$$

Finally, the components of the angular velocity in momentum space are [9]:

$$W_1 = iv^{\pm}_c(\varepsilon^{\pm}_2 k_3 - \varepsilon^{\pm}_3 k_2) \; , \; W_2 = iv^{\pm}_c(\varepsilon^{\pm}_3 k_1 - \varepsilon^{\pm}_1 k_3) \; , \; W_3 = iv^{\pm}_c(\varepsilon^{\pm}_1 k_2 - \varepsilon^{\pm}_2 k_1) \; . \tag{4}$$

Appropriate Fourier transforms allow us to get the angular velocity in position space from (W_1, W_2, W_3) (and vice versa)

From the equations in this section, it follows that only the gauge invariant quantity $v^{\pm}_{c0}(\mathbf{k}) = v^{\pm}_c(\mathbf{k}, \tau_0)$ is necessary to calculate both the CMB anisotropy $\Delta T/T$ and the angular velocity $\mathbf{W}$. Both calculations involve Fourier transforms, which can be numerically performed in appropriate boxes (simulations). The observers are located in the central part of the simulation box in position space. They receive the CMB photons from the last scattering surface, which is much smaller than the box. Since the scales of the vector modes are assumed to be of the order of 10^4 Mpc (superhorizon scales), the size of the simulation boxes is assumed to be of the order of 10^5 Mpc and, then, 512 cells per edge suffice to reach the required resolution. Photons are moved inside the box along the directions corresponding to the 3072 pixels of a HEALPIx map. Quantity $\Delta T/T$ is calculated for each direction and the resulting map is analyzed to get parameters t, $\mathbf{n}_2$, and $\mathbf{n}_3$ (see [9] for details).

Since big regions where the direction of the angular velocity is almost constant are very natural (in the presence of vector perturbations with large angular scales, see Sect. 2) we have superimposed appropriate vector modes (W-modes in [9]) to simulate: (i) A big region undergoing a differential rotation (DR) with angular velocity $W^N_z(\rho) = N_1[e^{-(\rho^2/2m^2)} - e^{-2}]$ for $\rho \leq 2m$, and $W^N_z(\rho) = 0$ for $\rho > 2m$, where N_1 is a normalization constant and $m = 5 \times 10^3$ Mpc. For appropriate N_1 values, there are observers (with non preferred positions) which would measure C_2 and C_3 amplitudes similar to those obtained from the WMAP data analysis. Moreover, the octopole would be planar ($t \simeq 0.94$) and the quadrupole and octopole would be fully aligned ($\alpha_{23} \simeq 0^\circ$) and, (ii) Parallel vorticity regions (PVR) where $W_z = W_z(x,y)$. In these region, the angular velocity is everywhere parallel to a certain direction (identified with the z axis), but there is no DRs around any axis. For PVRs (combinations of superhorizon vector modes), it is very likely (probability between 0.05 and 0.1) the existence of observers, with non preferred positions, who would measure $t \simeq 0.94$, $\alpha_{23} = 0^\circ$, and appropriate values of C_2 and C_3.

4 Discussion and conclusions

Either DRs of very big regions or PVRs in large enough zones can explain the anomalies (a), (b) and (c) and, moreover, there are no problems (*a priori*) to explain the anomalies (α) to (γ) of Sect. 1 (see [9]). Anomaly (δ) is not cosmological (alignments with respect to solar system directions are expected to be casual). Since DRs and PVRs lead to a perfect alignment between $\mathbf{n}_2$ and $\mathbf{n}_3$ ($\alpha_{23} = 0^\circ$), either residual large scale scalar perturbations or deviations from perfect parallelism in **W** should account for the ten degrees α_{23} angle obtained from the WMAP observations. This point deserves more attention.

Vector perturbations might appear in models involving brane words [13], topological defects [14], either magnetic fields or the divergenceless part of other vector fields (vector–tensor theories, see [15]), and so on. In spite of this fact, we must recognize that the main problem with the model proposed here and also in [9] is that the origin and evolution of the required vector perturbations is not well understood. The creation and evolution of these structures deserves further attention. After evolution, only large enough superhorizon vector perturbations must be present, whereas smaller scales should disappear soon enough. We are looking for models of this type in vector-tensor theories of gravitation.

Acknowledgements This work has been supported by the Spanish Ministerio de Educación y Ciencia, MEC-FEDER project FIS2006-06062.

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