Figure A4.1 GRAPHIC SOLUTION TO LINEAR PROGRAM

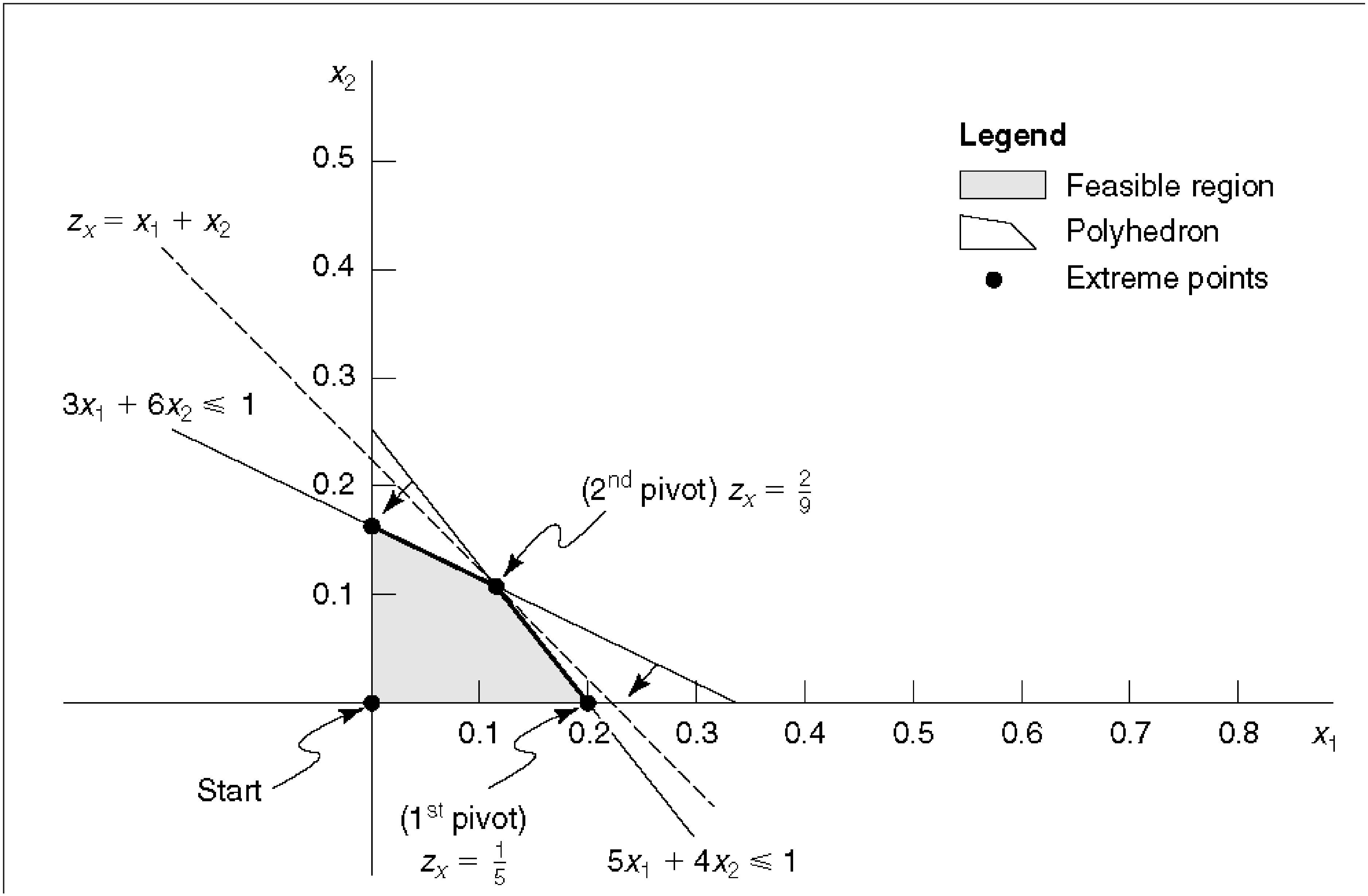


Figure A4.1A Graphical interpretation of LP

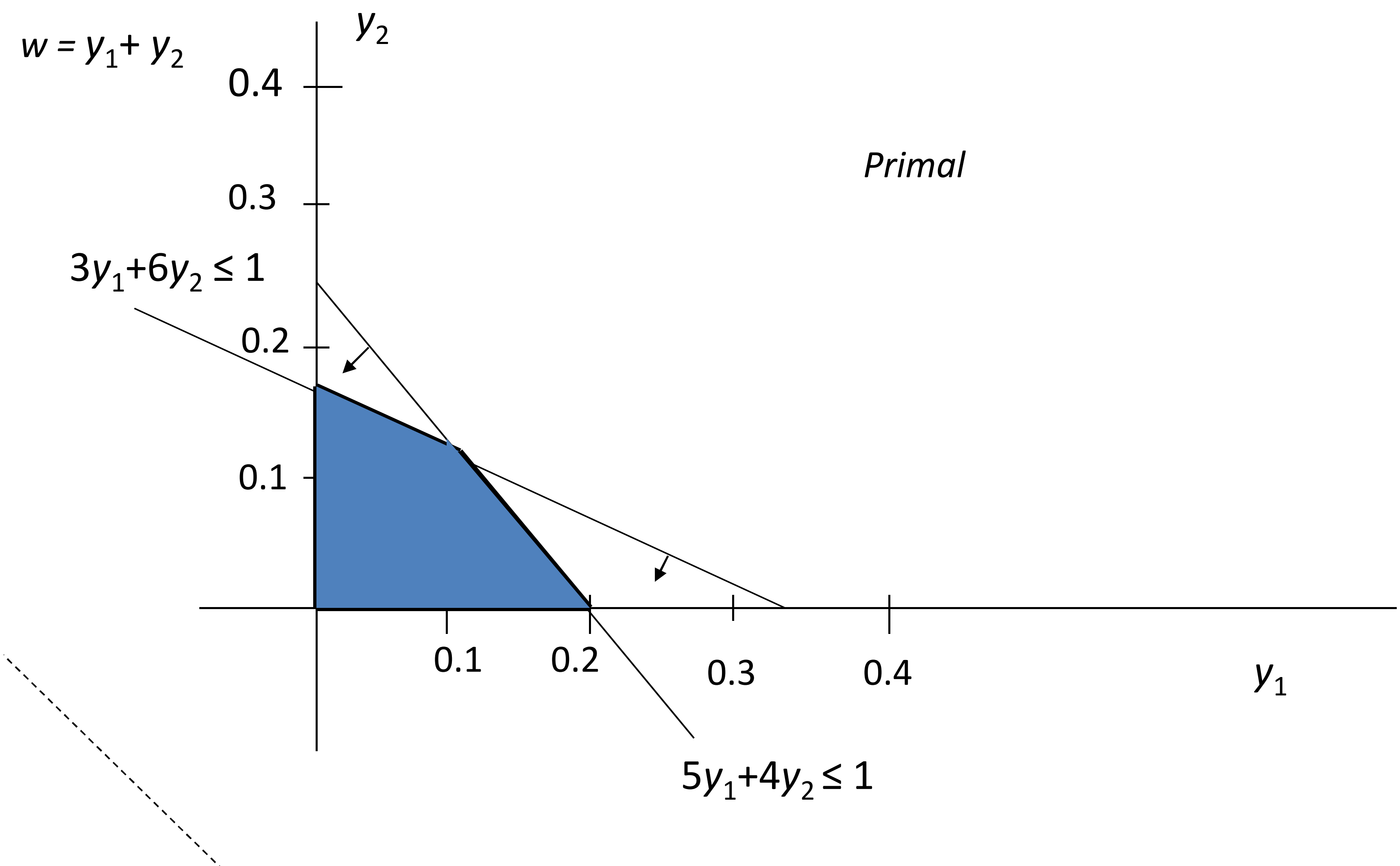


Figure A4.2 LINEAR PROGRAMMING TABLEAUX SHOWING PIVOTING OPERATIONS

Smallest negative entry

z_x	x_1	x_2	x_3	x_4	RHS	
1	-1	-1	0	0	0	
0	3	6	0	1	1	$\rightarrow \frac{1}{3}$
0	5	4	1	0	1	$\rightarrow \frac{1}{5}$

(Smallest ratio: All variables guaranteed positive)

Only pick positive values in column

First tableau

1	0	$-\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	
0	0	$\frac{18}{5}$	$-\frac{3}{5}$	1	$\frac{2}{5}$	$\rightarrow \frac{2}{5} \times \frac{5}{18} = \frac{1}{9}$
0	1	$\frac{4}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\rightarrow \frac{1}{5} \times \frac{5}{4} = \frac{1}{4}$

Search among vertices (only 1st vertex)

Second tableau (optimum)

2nd vertex

1	0	0	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{2}{9}$
0	0	1	$-\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{9}$
0	1	0	$\frac{1}{3}$	$-\frac{2}{9}$	$\frac{1}{9}$

Figure A4.3 DUAL OF LINEAR PROGRAMMING EXAMPLE

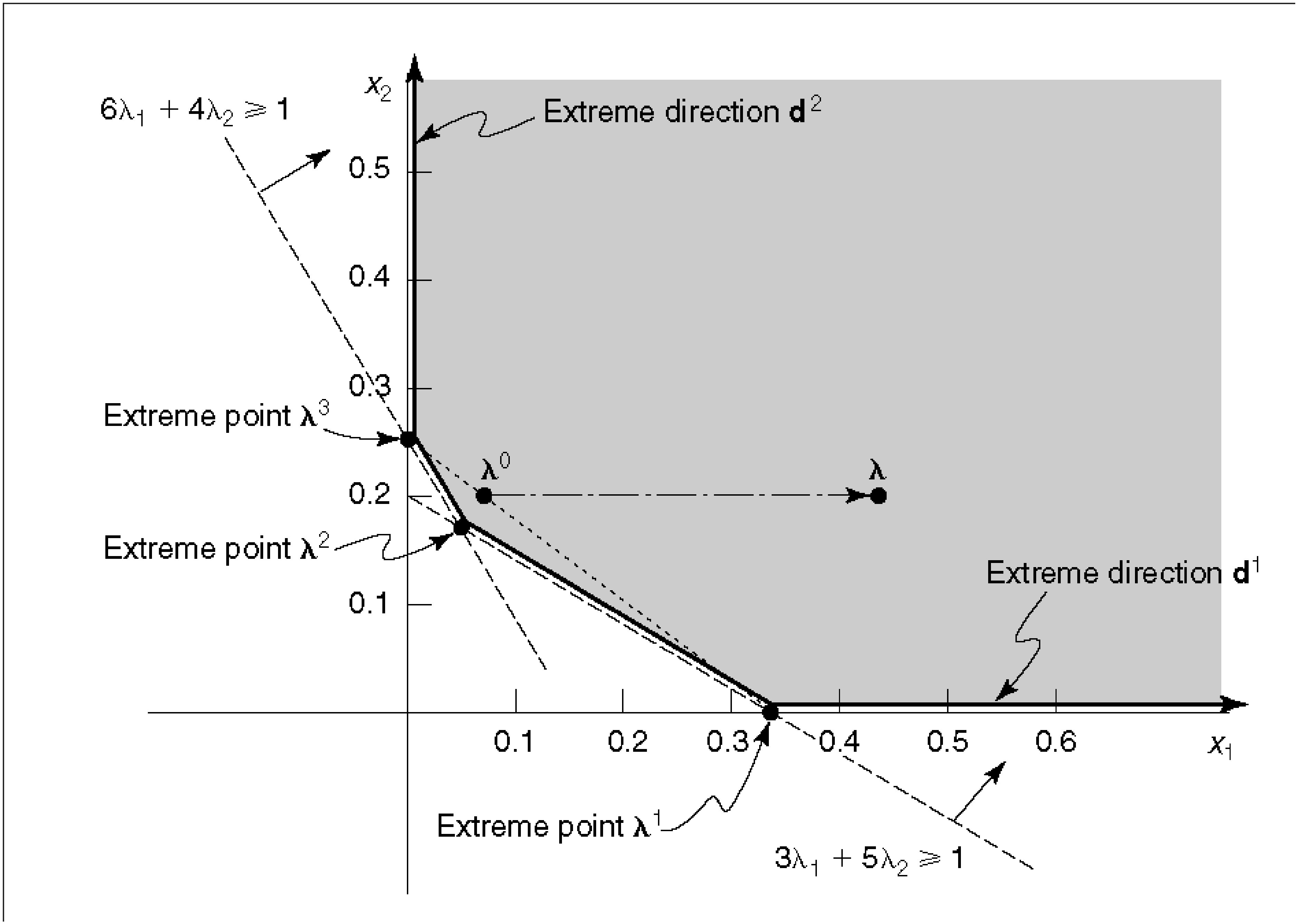


Figure A4.3A Graphical interpretation (Dual LP)

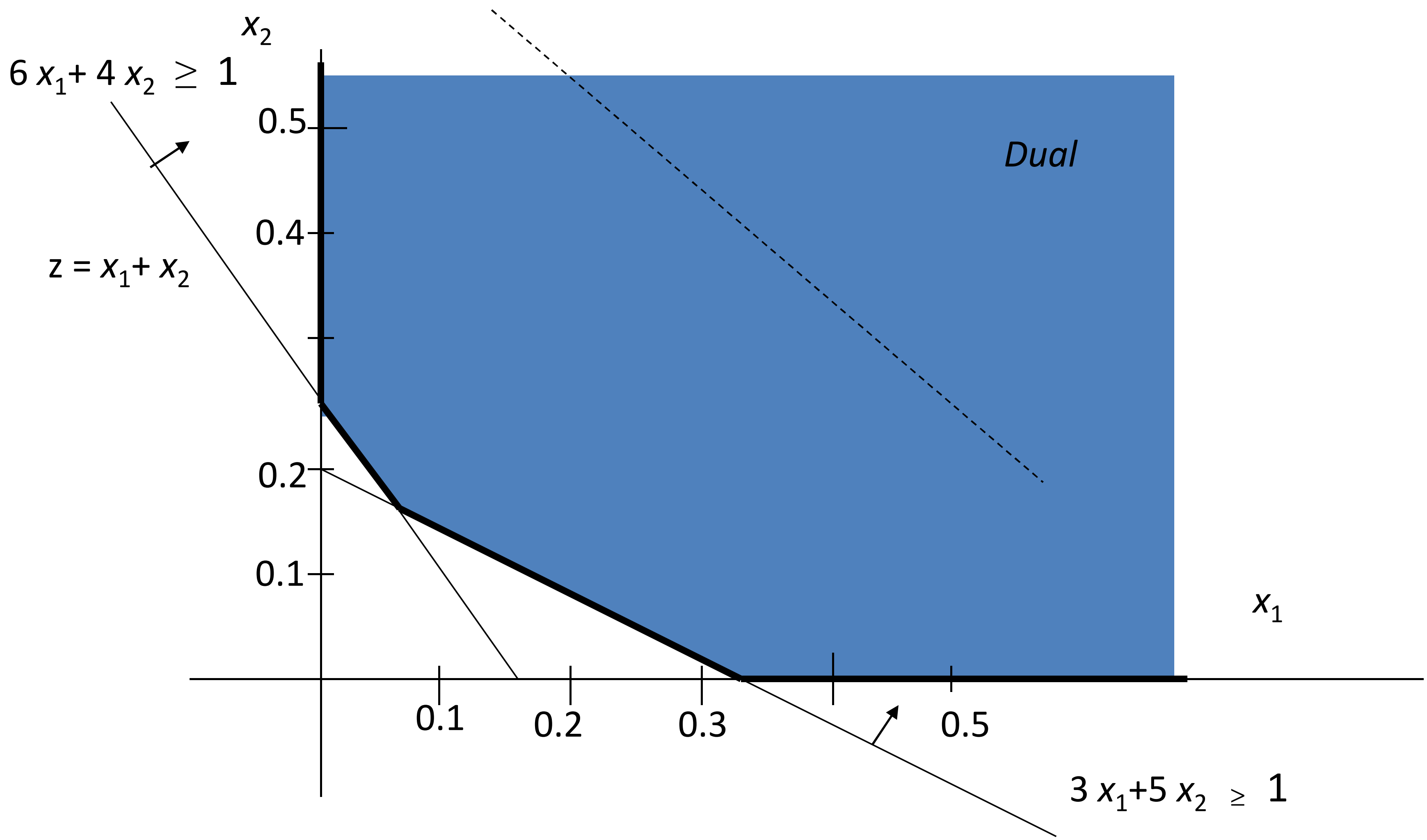
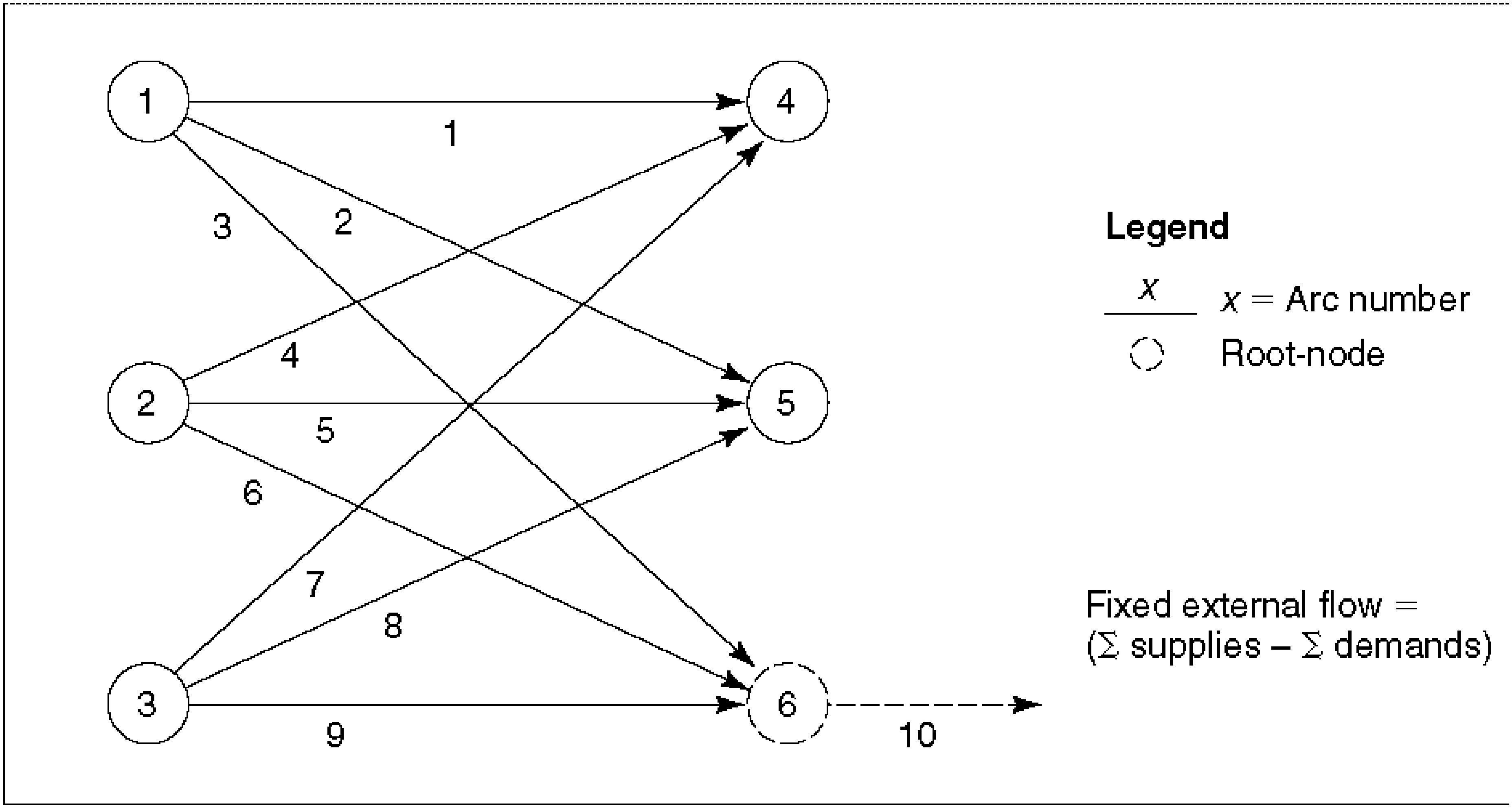
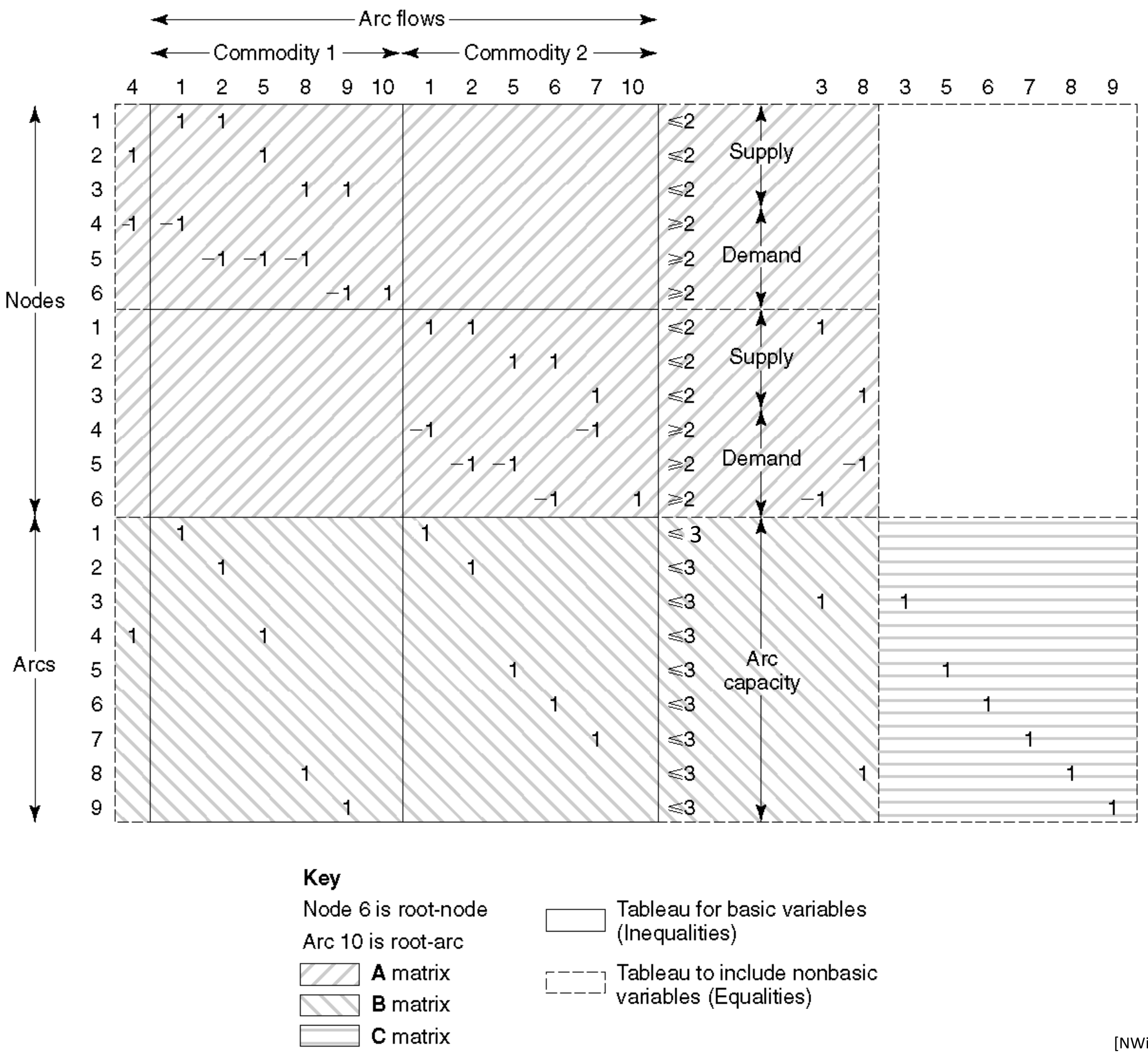


Figure A4.4 SAMPLE MULTICOMMODITY-FLOW NETWORK



SOURCE: Kennington and Helgason (1980). Reprinted with permission.

Figure A4.5 Multicommodity flow Network with Side Constraints



[NWithSideConstraints-1]

Figure A4.6(a) Basic tree 1

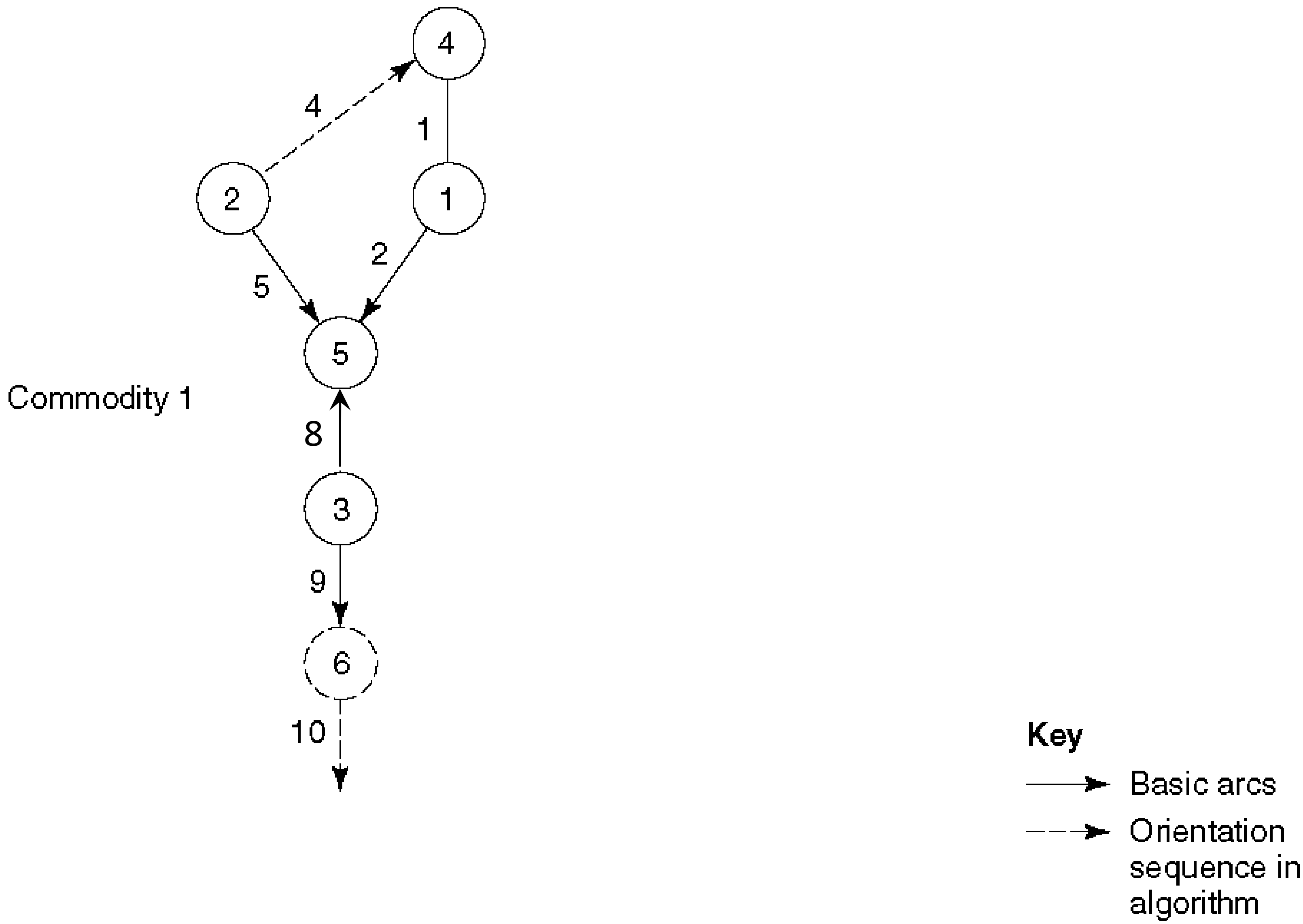
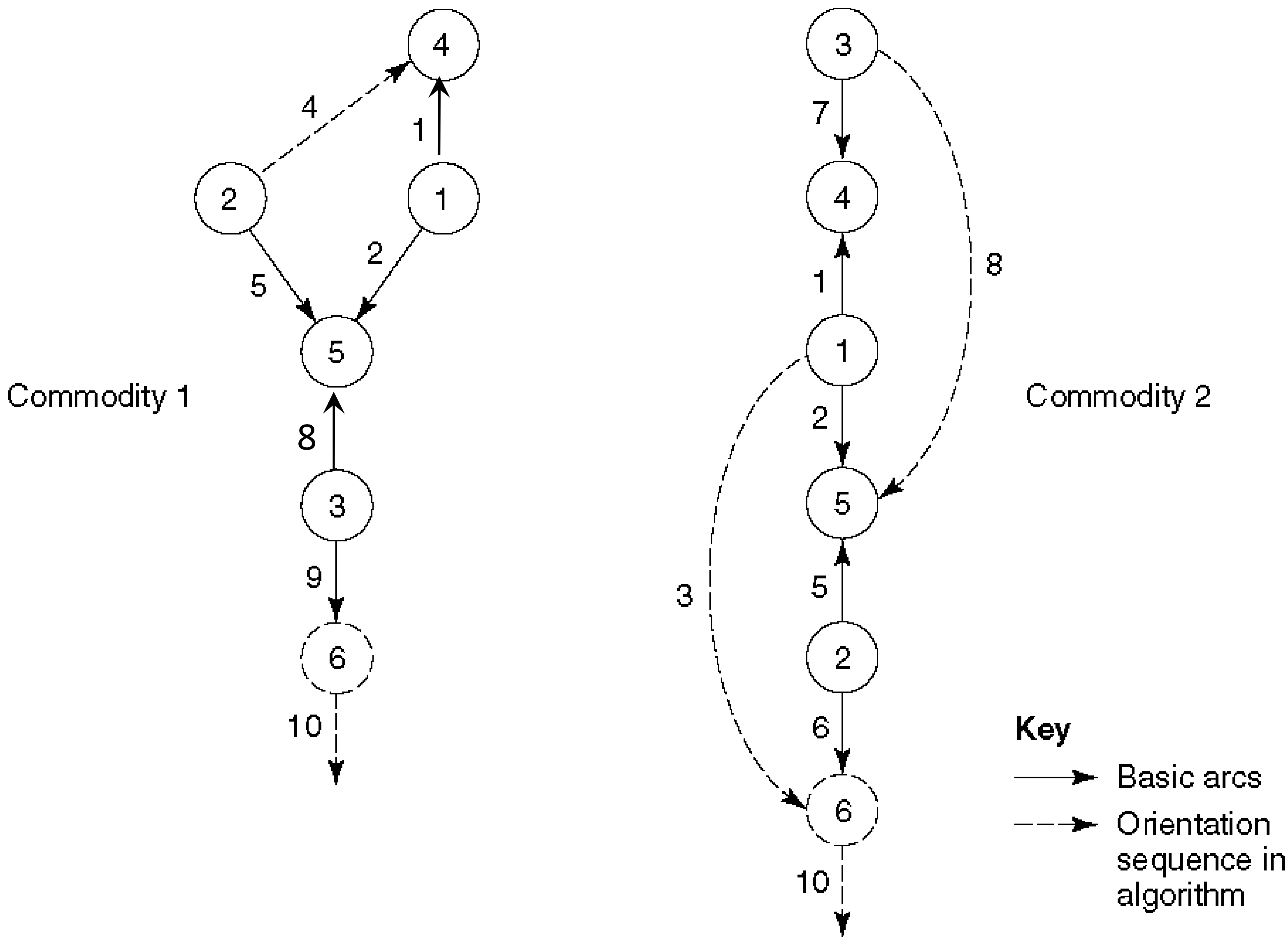
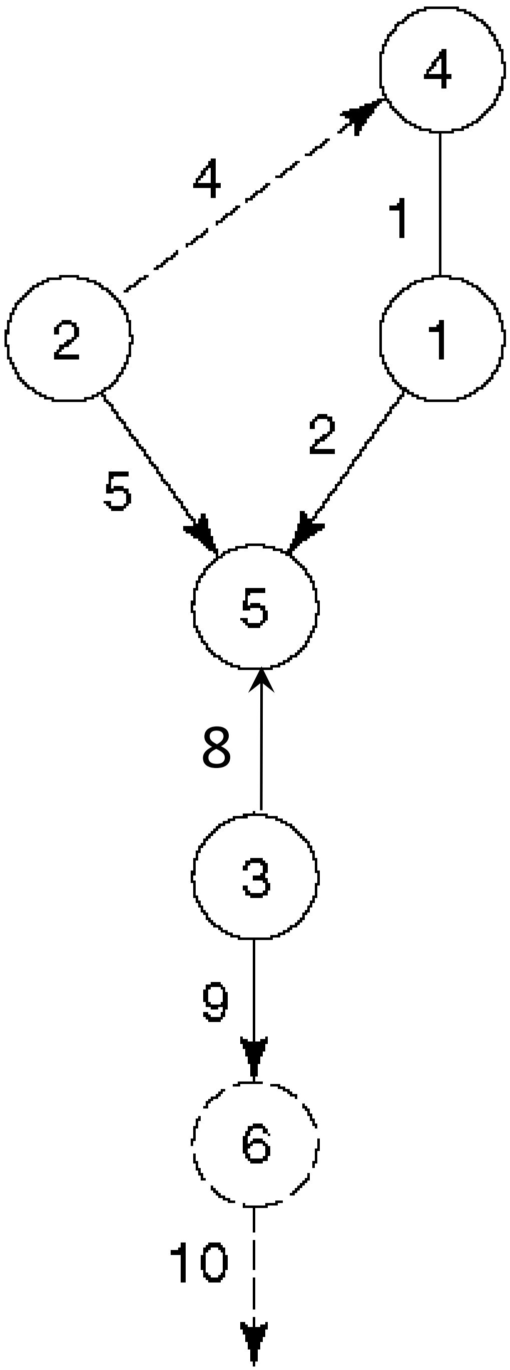


Figure A4.6(b) Basic tree 2 of same orientation sequence



Orientation sequence for basic tree 1



$\longleftarrow$ Commodity 1 $\longrightarrow$

$$
\begin{array}{c|c|cccccc}
 & 4 & 1 & 2 & 5 & 8 & 9 & 10 \\
\hline
1 & & 1 & 1 & & & & \\
2 & 1 & & & 1 & & & \\
3 & & & & & 1 & 1 & \\
4 & -1 & -1 & & & & & \\
5 & & & -1 & -1 & -1 & & \\
6 & & & & & & -1 & 1
\end{array}
$$

Figure A4.7 EXAMPLE TO ILLUSTRATE NETWORK-WITH-SIDE-CONSTRAINT ALGORITHM

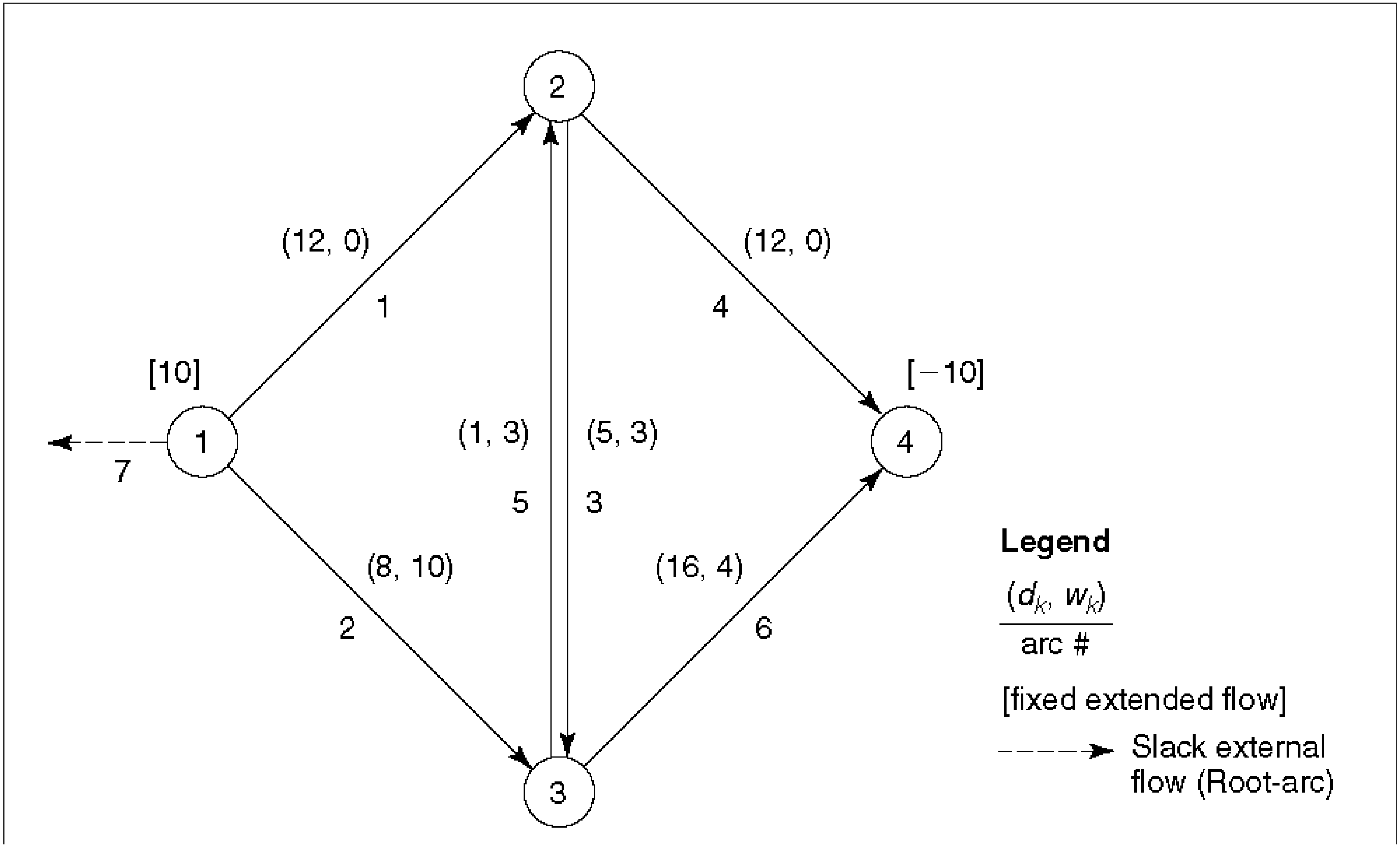
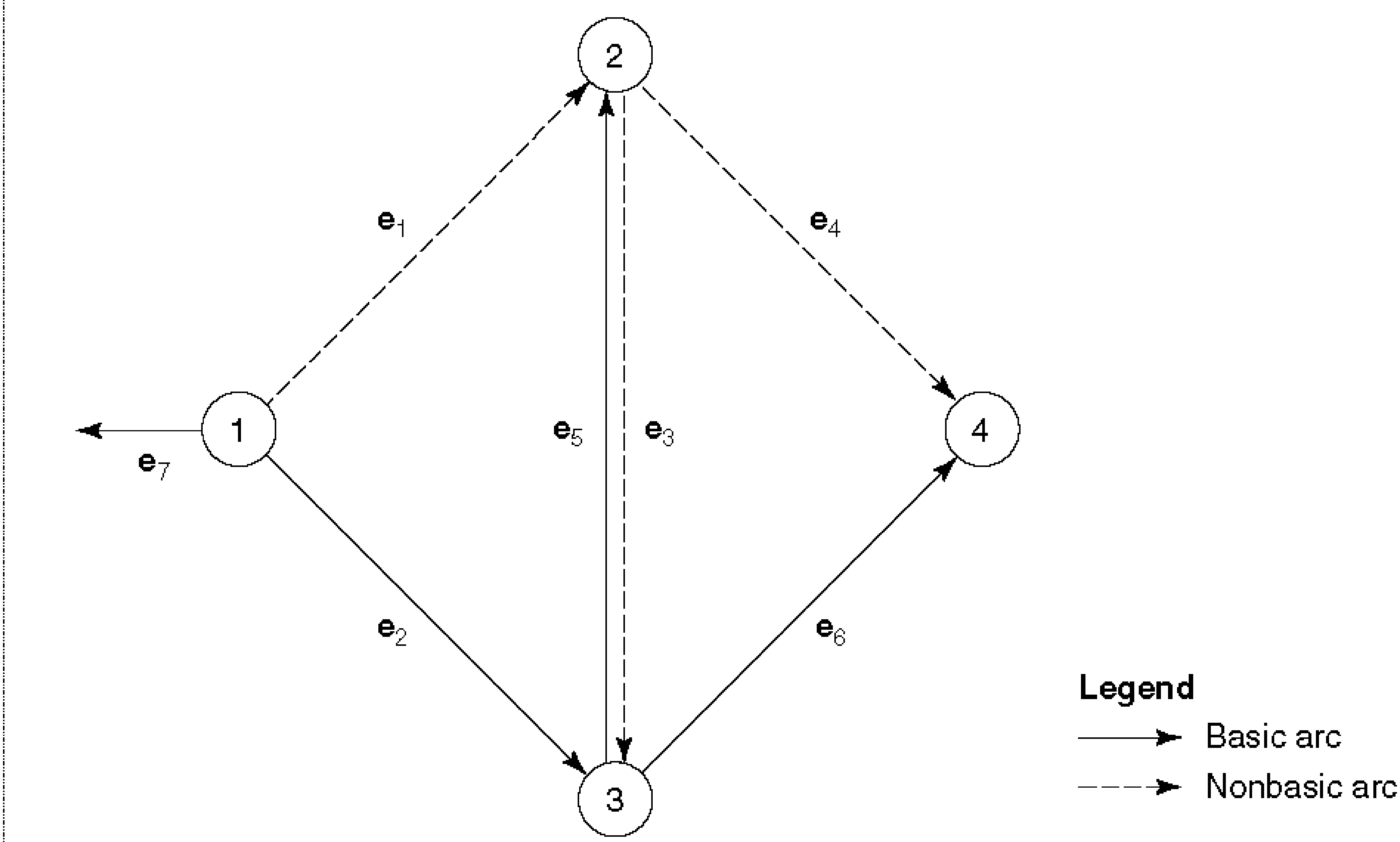


Figure ***A4.8*** TREE REPRESENTING INITIAL BASIS



[NWsideConstr9]

NSC2: Network synthesis example

Network basic variables

$$
\begin{array}{llllllllllll}
\min & & & 10x_2 & +\,2x_3 & & +\,3x_5 & +\,4x_6 & & & & \\
\hline
\text{s.t.} & x_7 & +\,x_1 & +\,x_2 & & & & & | & & & =10 \\
 & & -\,x_1 & & +\,x_3 & +\,x_4 & -\,x_5 & & | & & & =0 \\
 & & & -\,x_2 & -\,x_3 & & +\,x_5 & +\,x_6 & | & & & =0 \\
 & & & & & -\,x_4 & & -\,x_6 & | & & & =-10 \\
\hline
 & & 10x_1 & & -\,2x_3 & +\,3x_4 & -\,2x_5 & & | & +\,y_1 & & =16 \\
 & & x_1 & & +\,4x_3 & & +\,x_5 & & | & & +\,y_2 & =10 \\
\hline
\end{array}
$$

$\mathbf{v}^1$

$\mathbf{v}^2$

Entering variable

Non-network basic variables

Dictionary notation of LP simplex

	z	$\mathbf{x}_B$	$\mathbf{x}_N$	RHS
Row 0	1	$\mathbf{0}$	$\mathbf{c}_B{}^T\mathbf{A}_B{}^{-1}\mathbf{A}_N - \mathbf{c}_N$	$\mathbf{c}_B\,\mathbf{A}_B{}^{-1}\mathbf{b}$
Row $1 \to m$	$\mathbf{0}$	$\mathbf{I}$	$\mathbf{A}_B{}^{-1}\mathbf{A}_N$	$\mathbf{A}_{\mathrm{B}}{}^{-1}\mathbf{b}$

Figure ***A4.9*** CALCULATION OF π^1 FOR PRICING

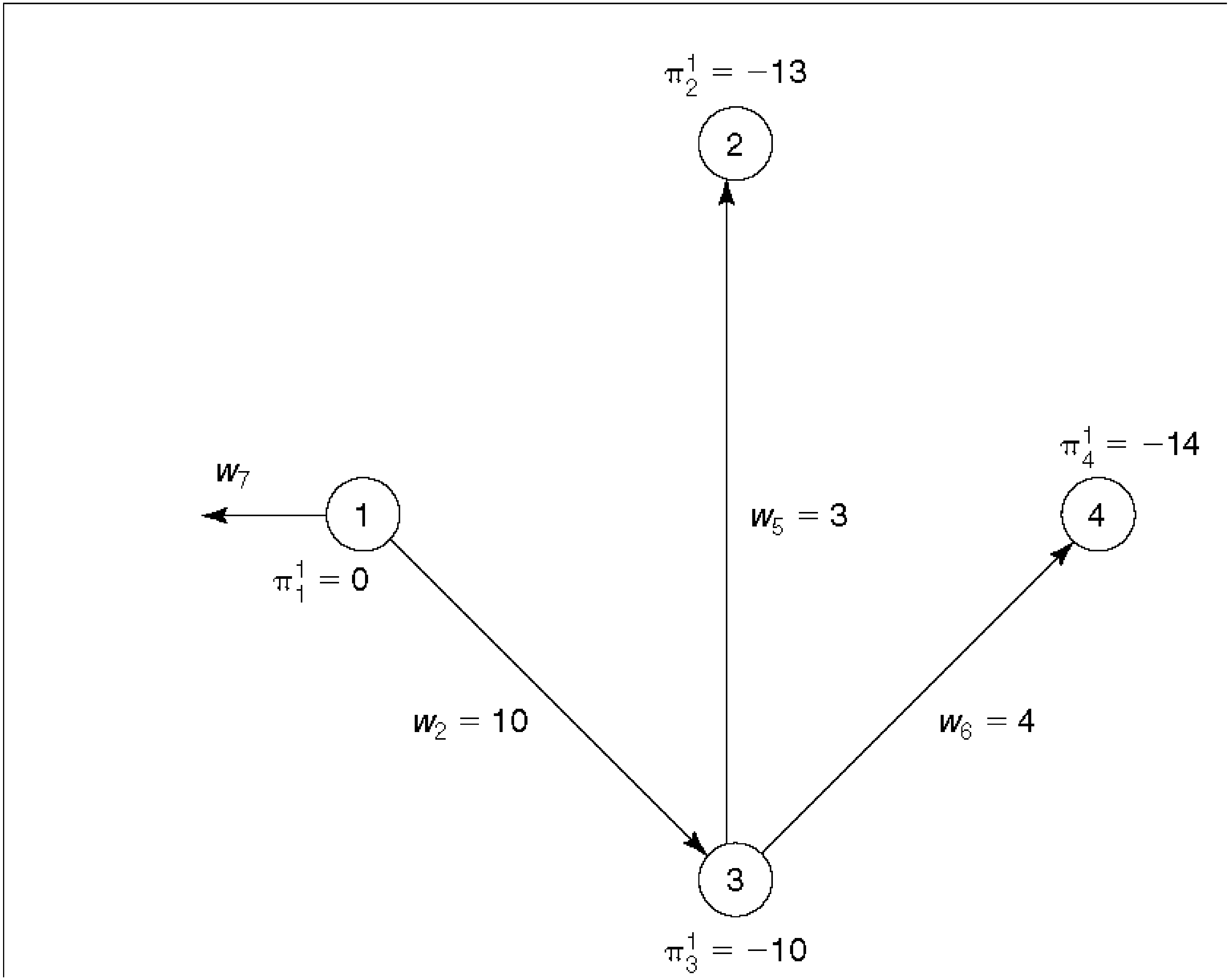
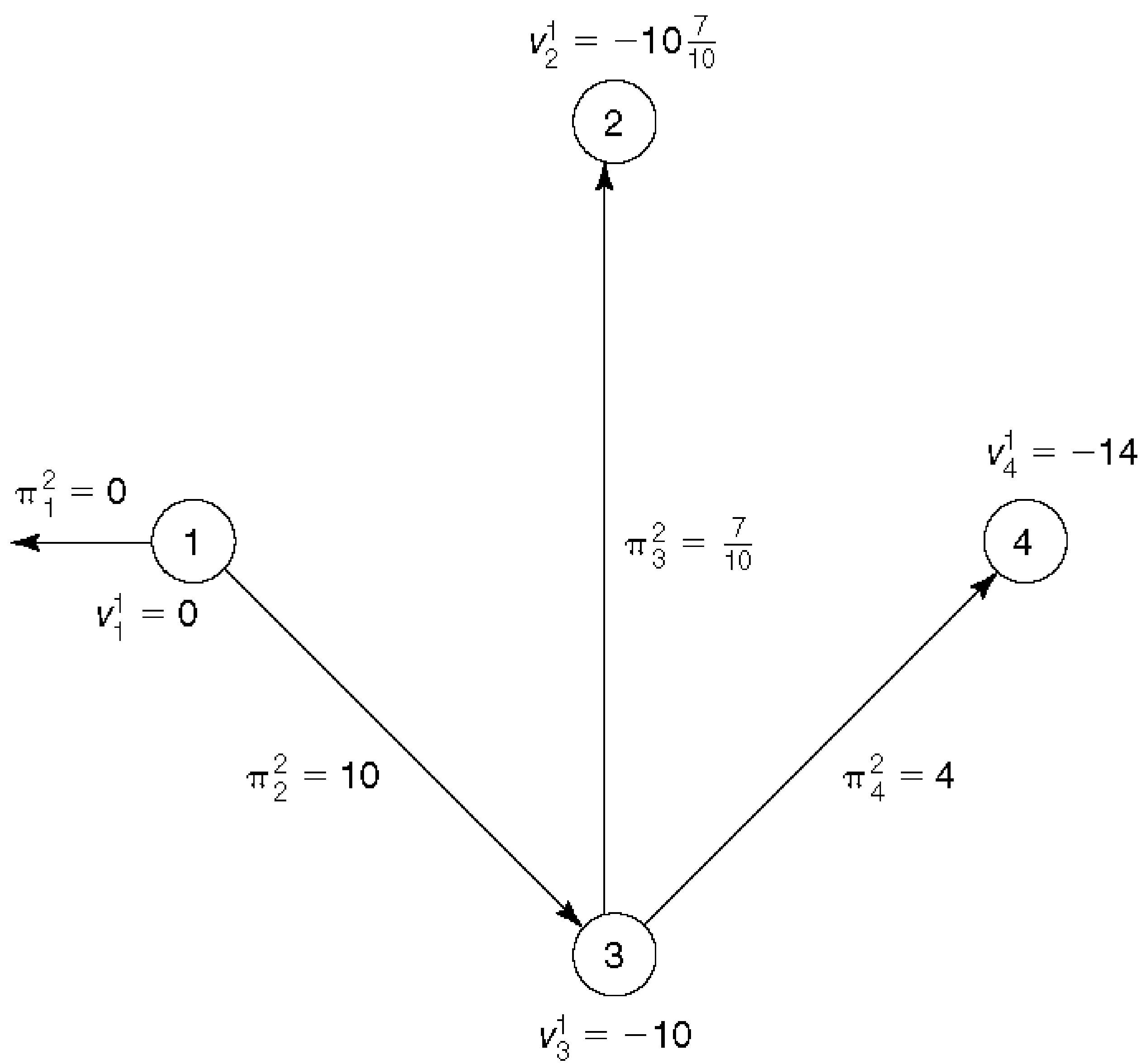
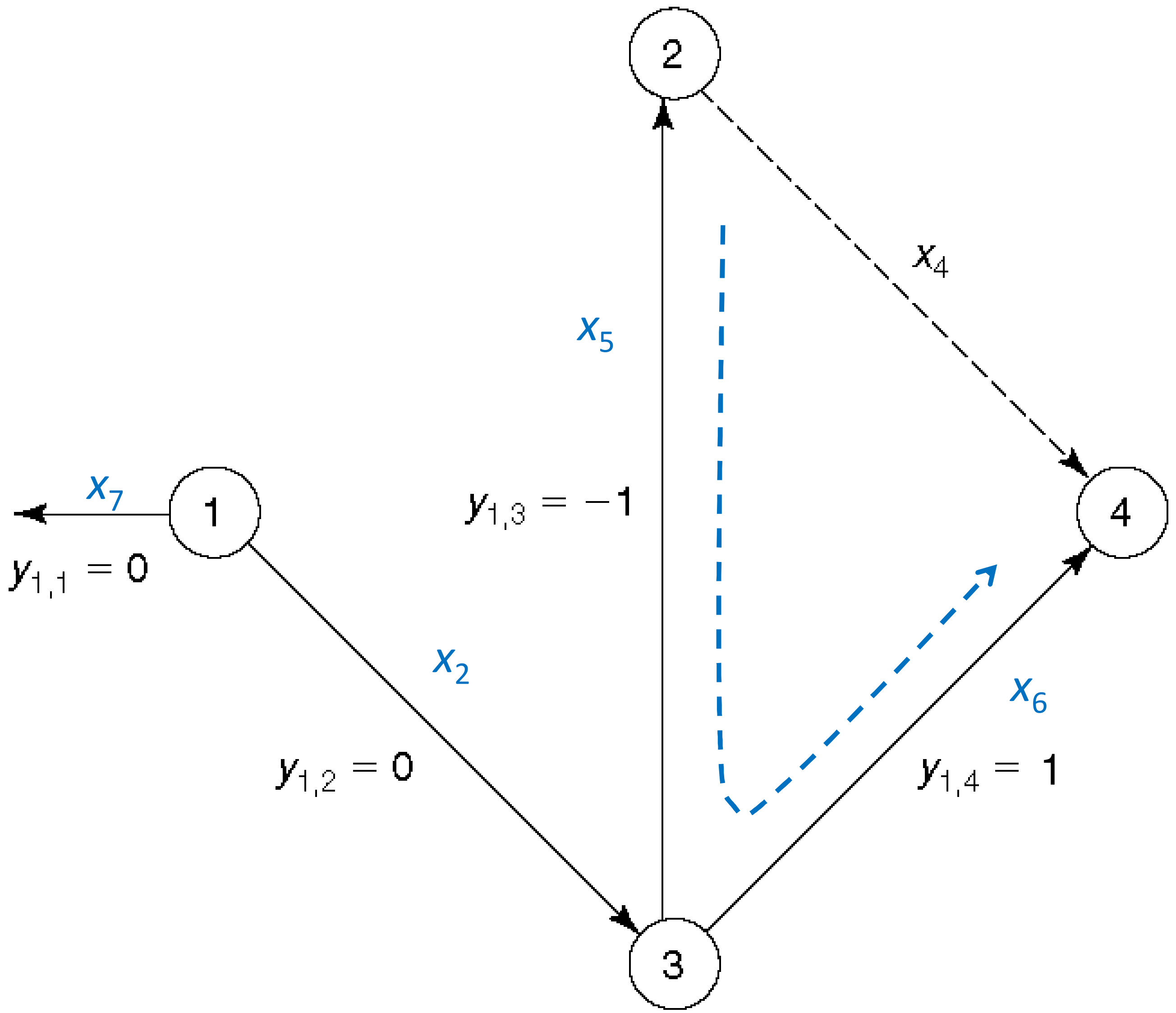


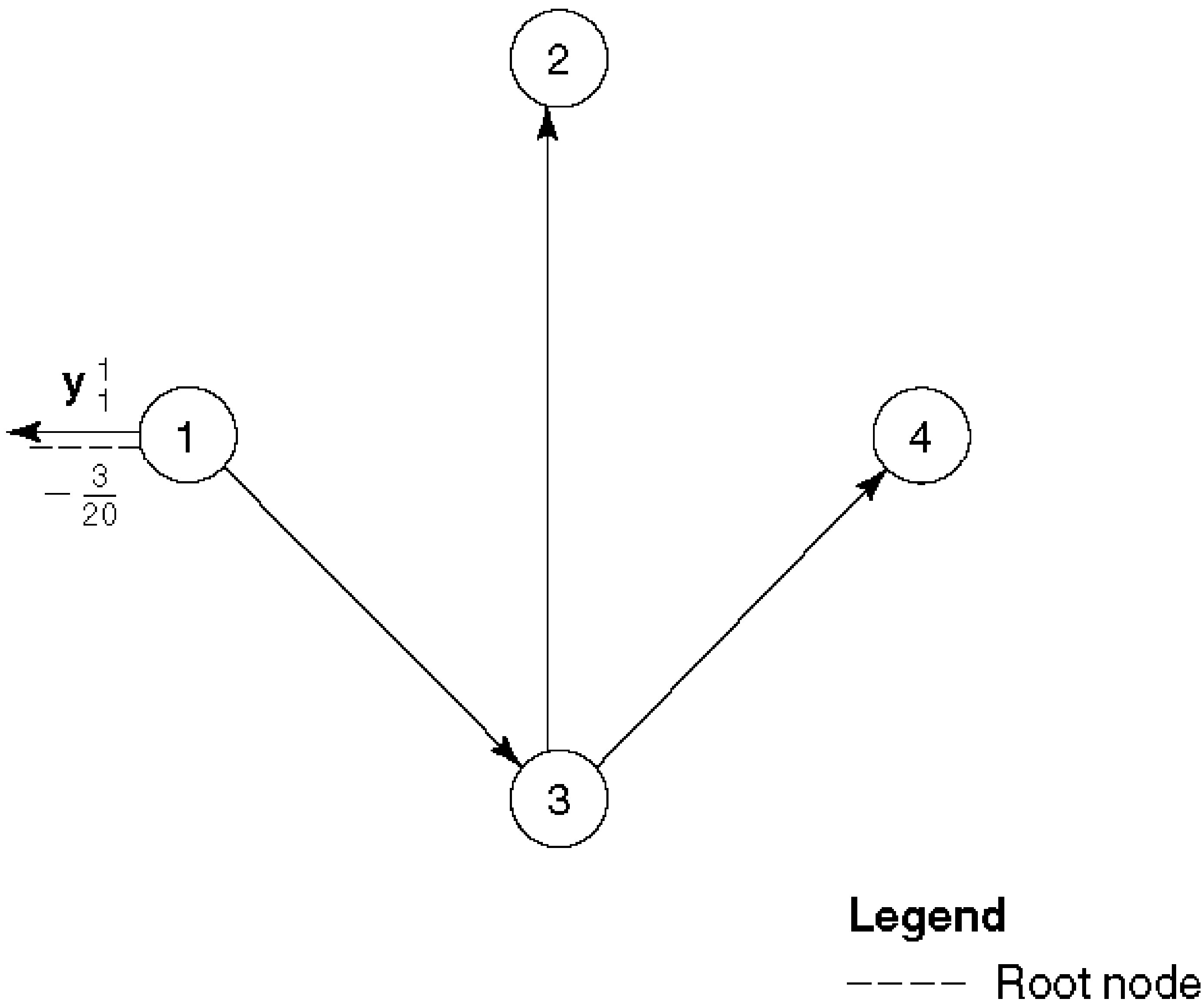
Figure ***A4.10*** CALCULATION OF V^1



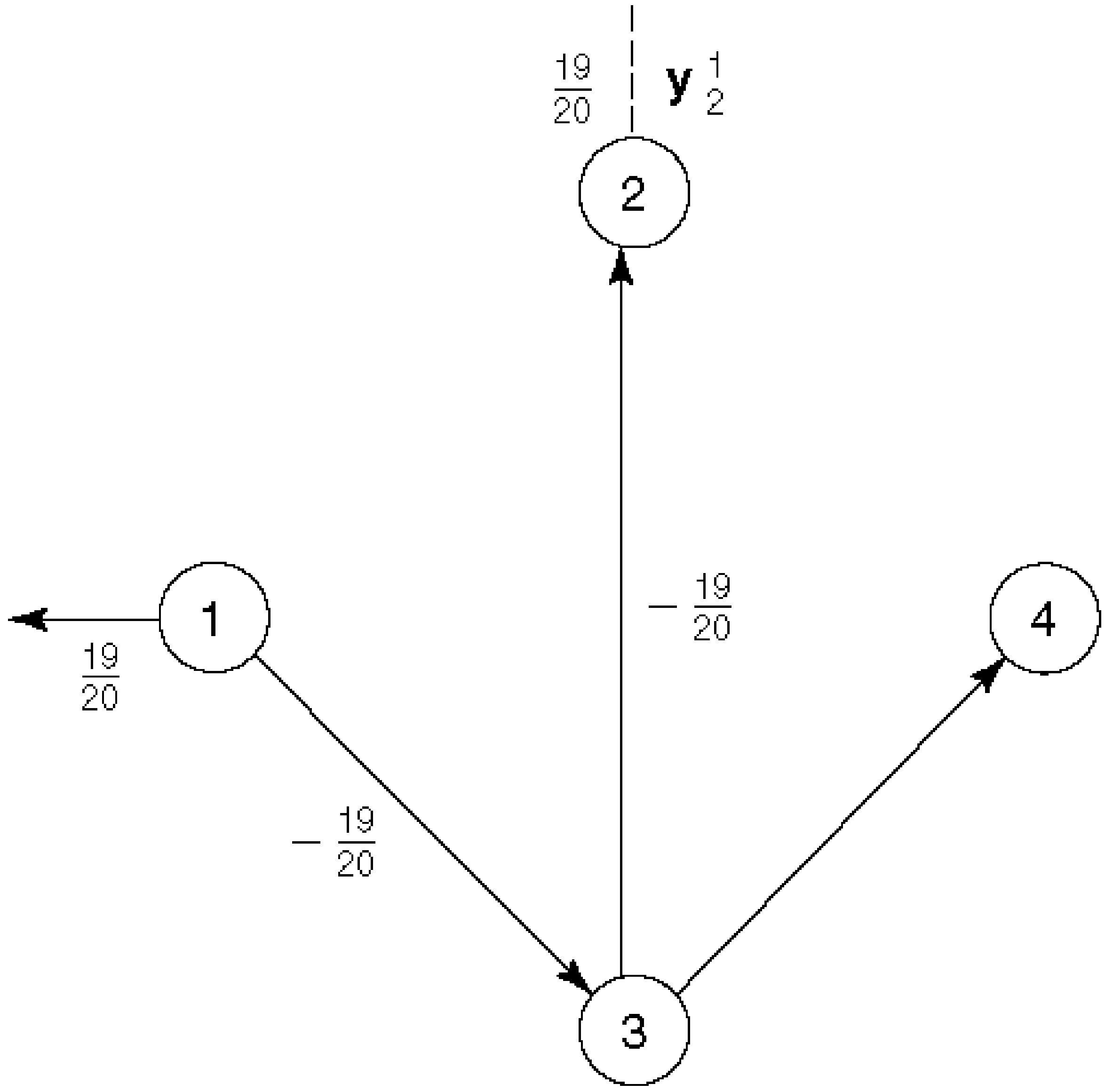
A4.11 $\boldsymbol{y}^1$ calculation in ratio test



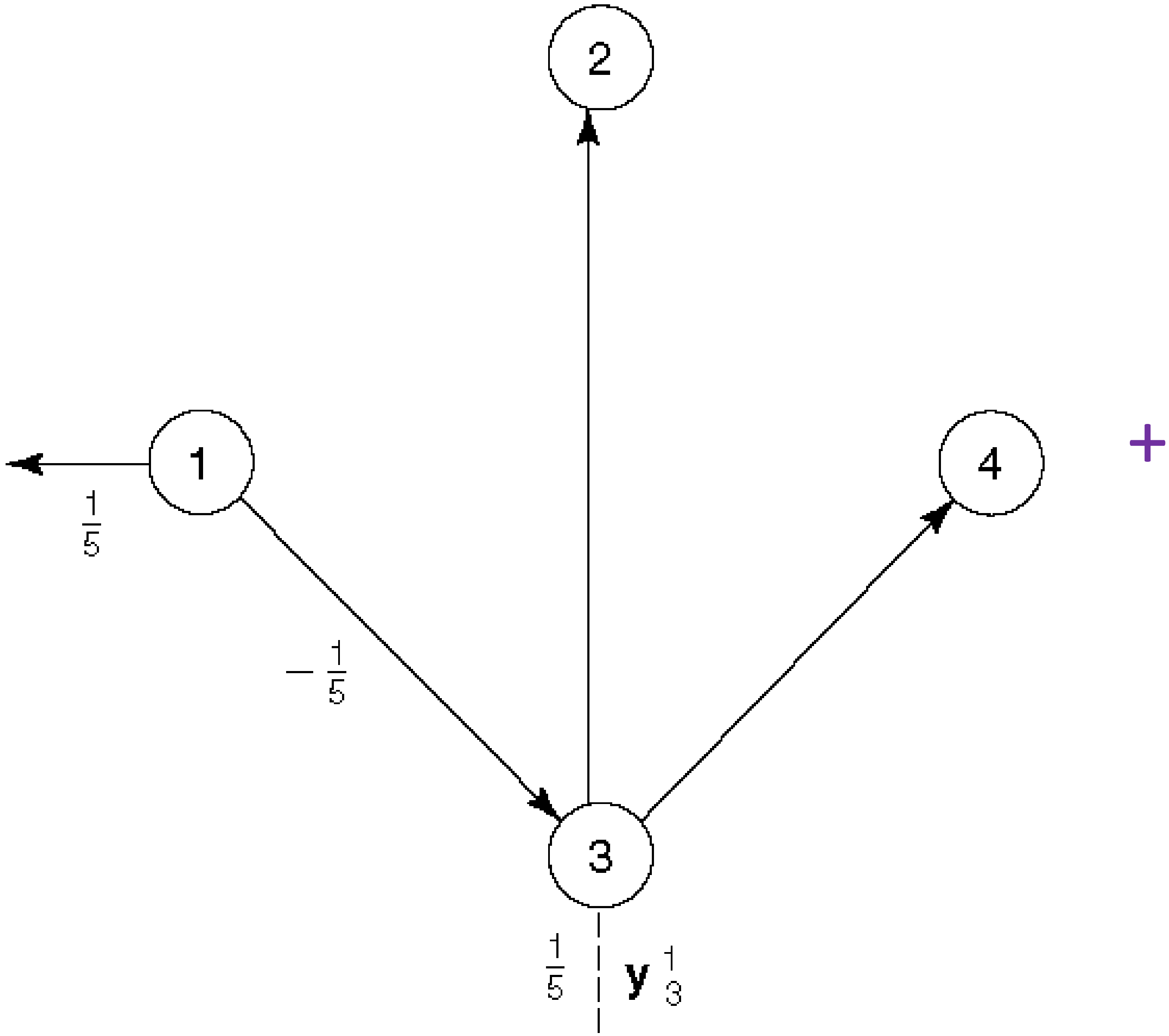
Calculation of $\boldsymbol{y}^1$ (Node 1 to node 1)



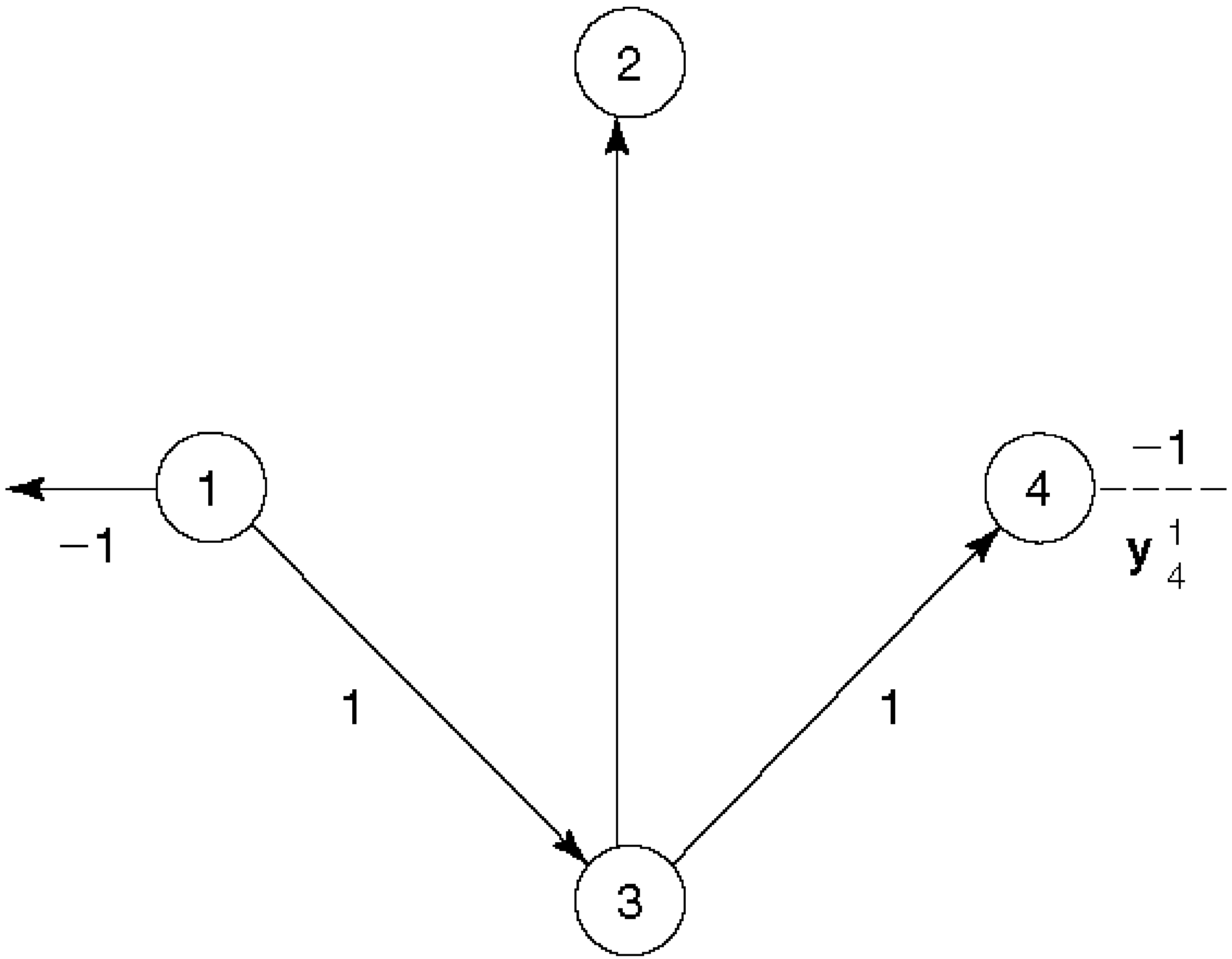
Calculation of $\boldsymbol{y}^1$ (Node2 to node 1)



Calculation of $\boldsymbol{y}^1$ (Node 3 to node 1)

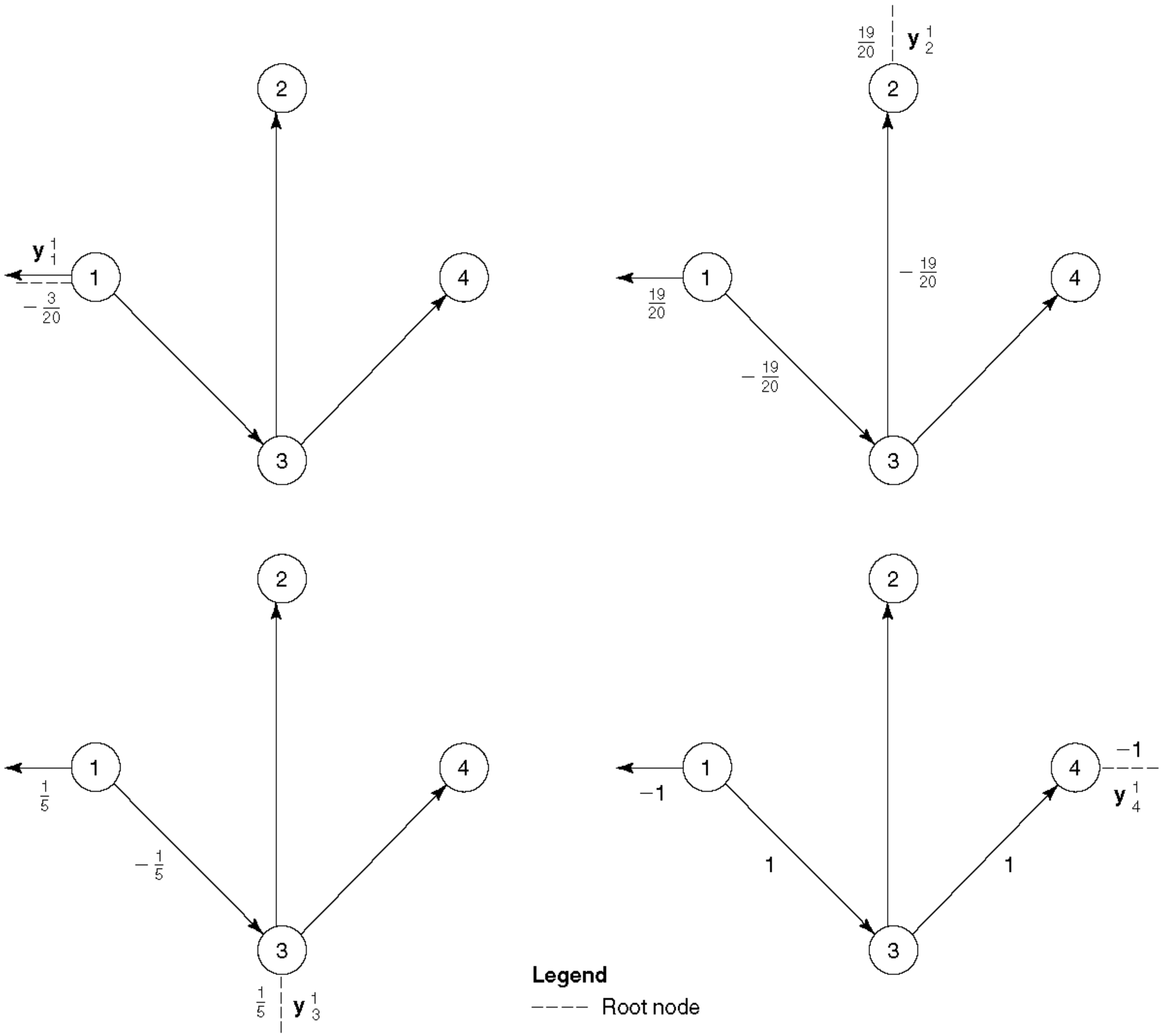


Calculation of $\boldsymbol{y}^1$ (Node 4 to node 1)



=

Figure A4.12 Calculation of y^1 (All other nodes to node 1)



Refreshed column x_4

[NWsideConstr7]

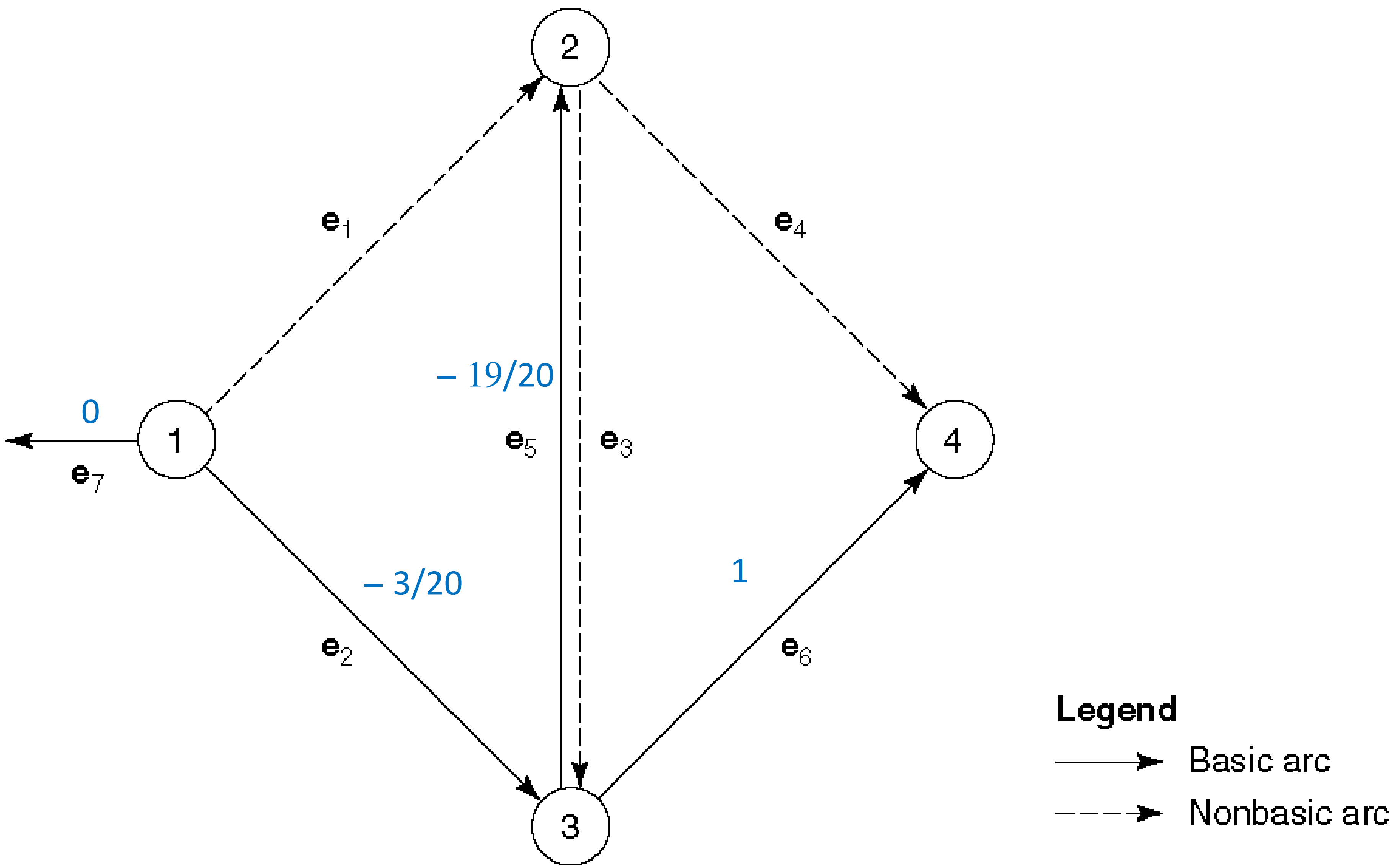


Figure A4.13 MIN IN x AND MAX IN λ FOR A WEAK DUALITY

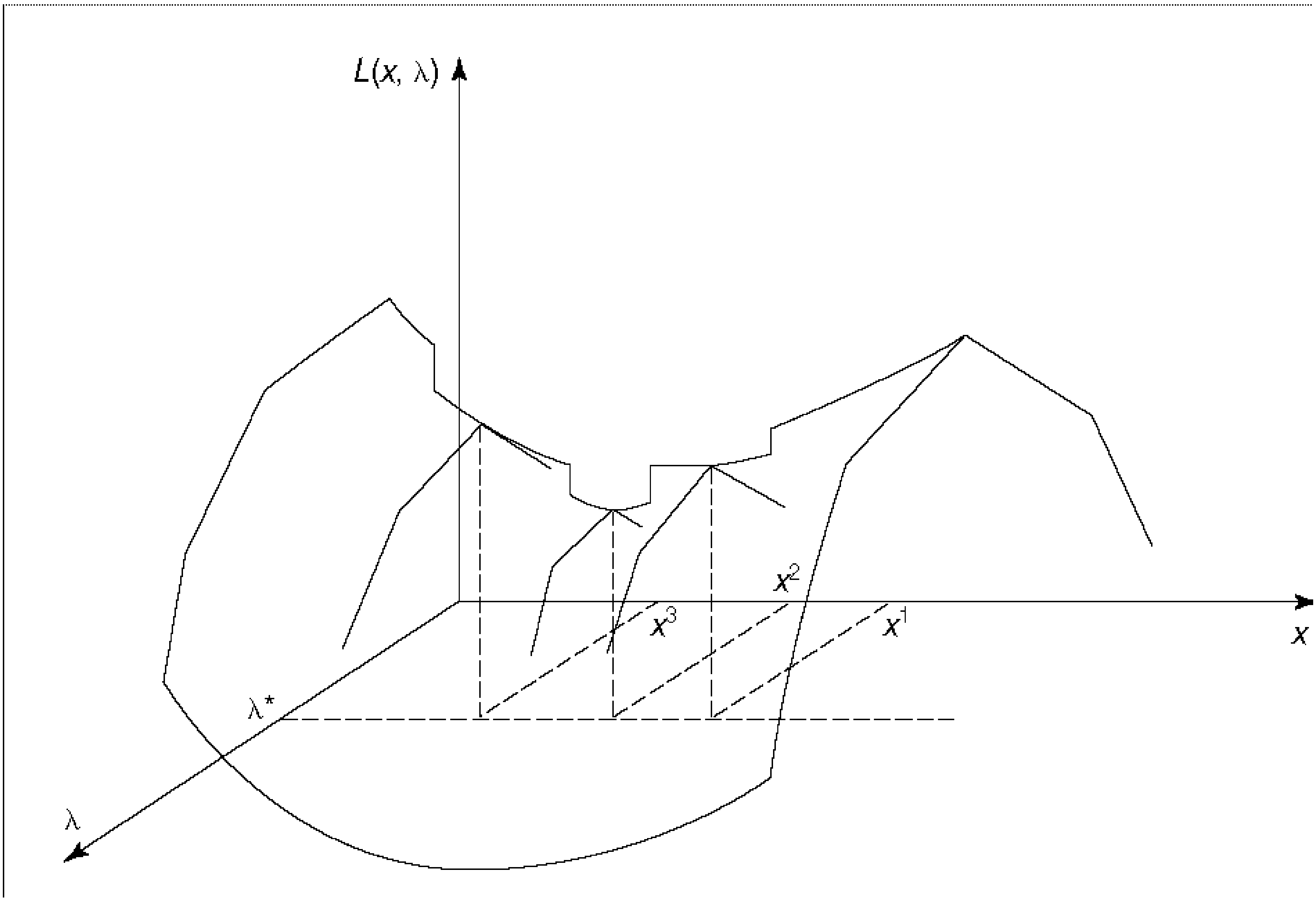
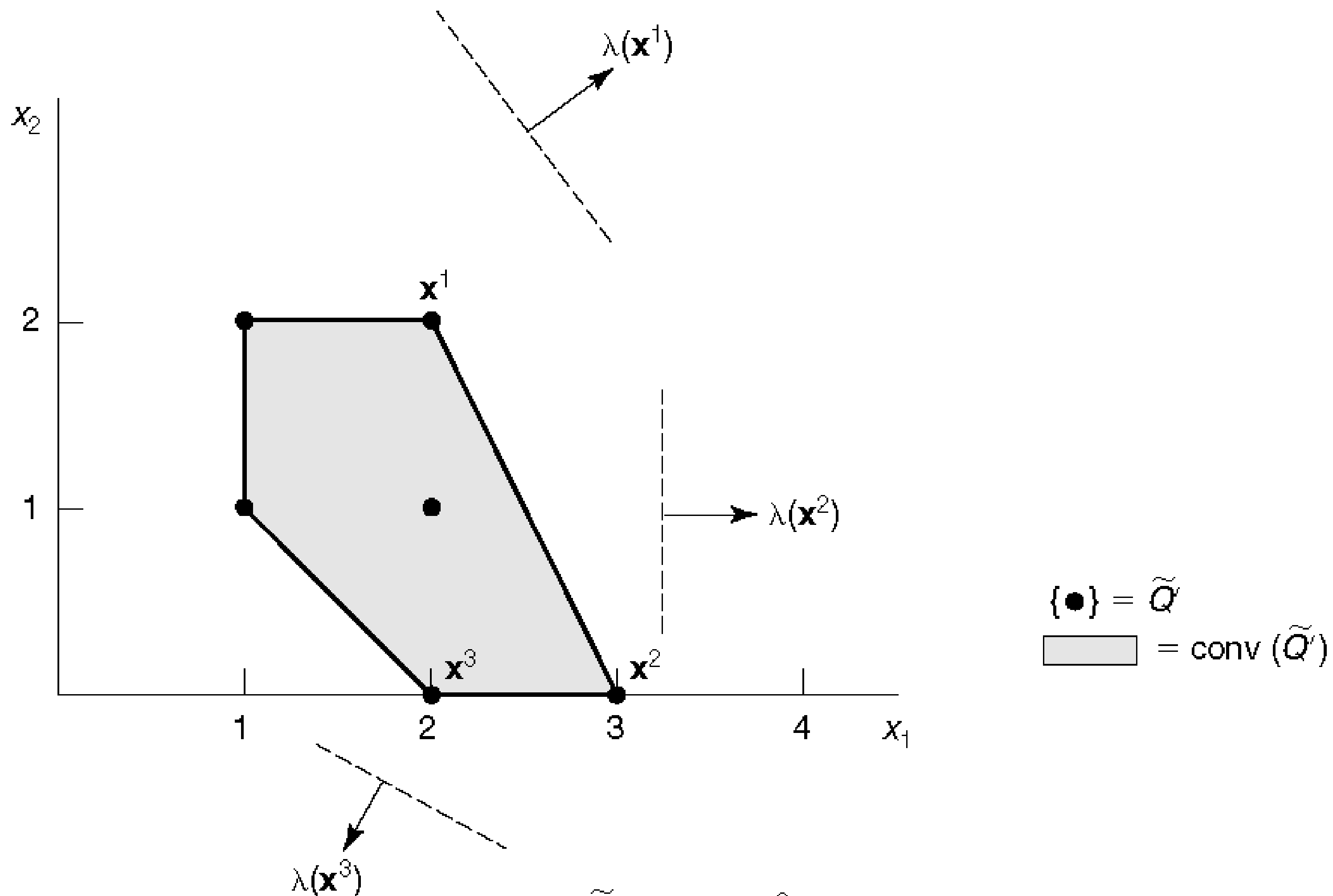


Figure A4.14 OPTIMIZING OVER A CONVEX HULL



$$\text{conv}(\widetilde{Q}') = \{\mathbf{x} \in R^2:\quad \begin{aligned} -x_2 &\leqslant 0 \\ -x_1 \quad &\leqslant 1 \\ x_2 &\leqslant 2 \\ -x_1 - x_2 &\leqslant -2 \\ 2x_1 + x_2 &\leqslant 6 \}\end{aligned}$$

Figure A4.15 Maximization in λ

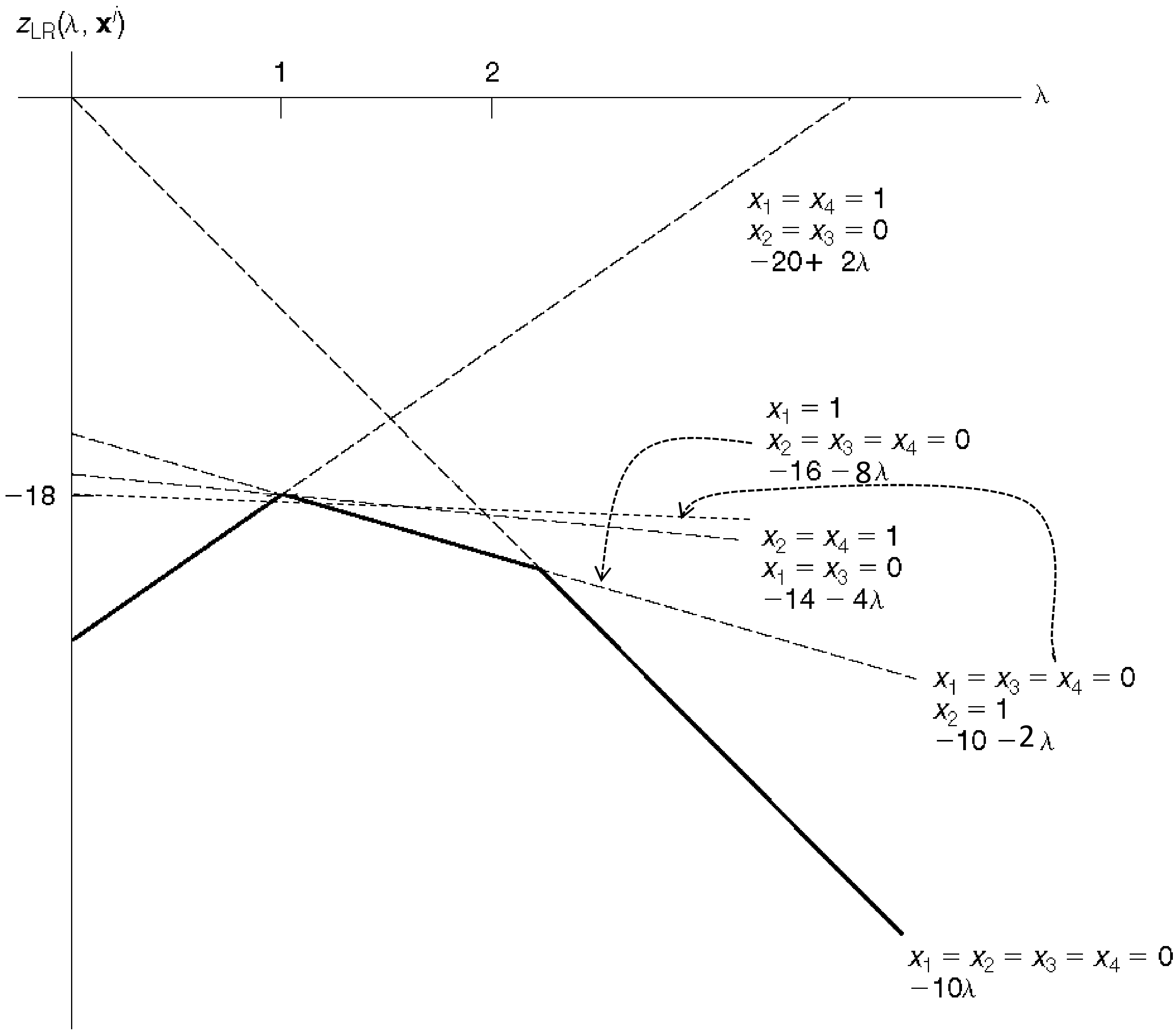


Figure A4.16(a) Generic LR algorithm

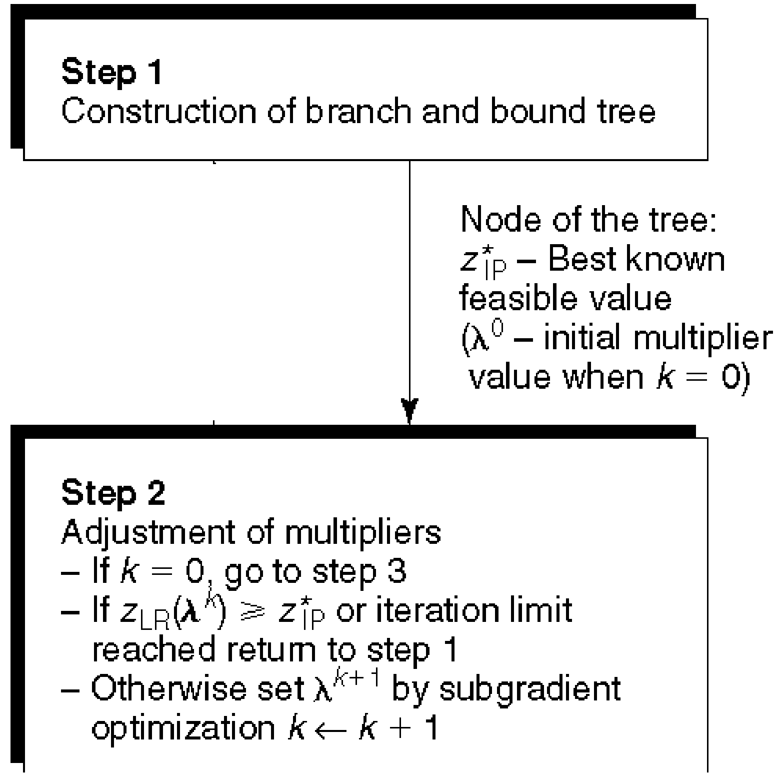


Figure A4.16(b) Generic LR algorithm

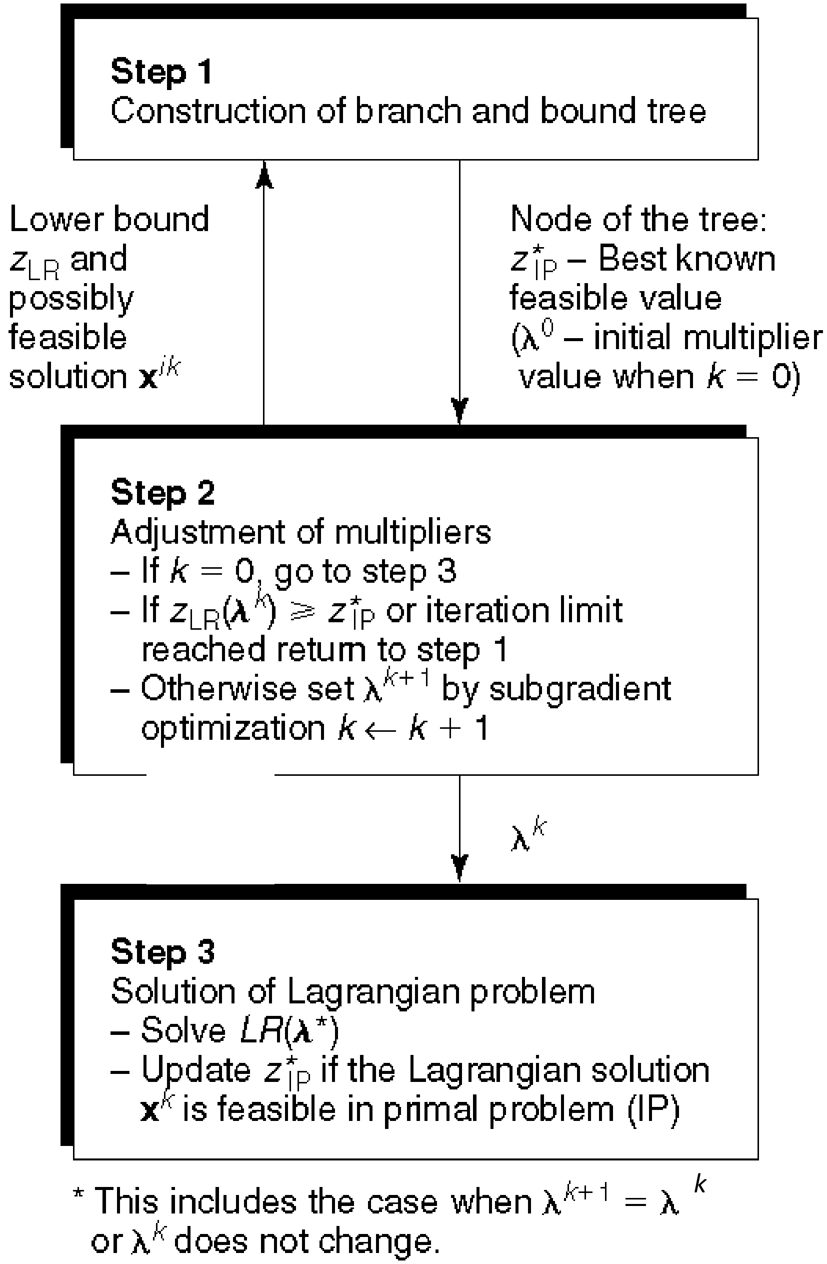
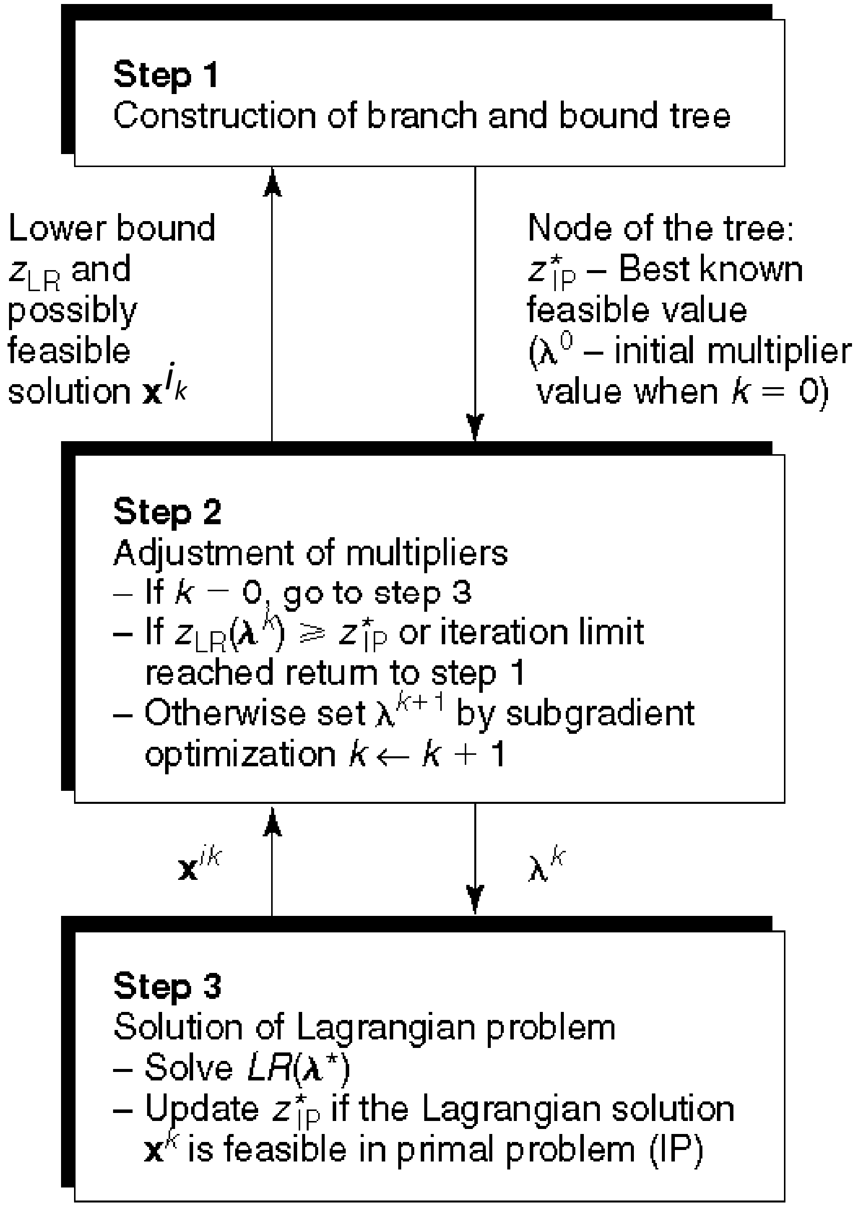


Figure A4.16(c) Generic LR algorithm



* This includes the case when $\lambda^{k+1} = \lambda^k$ or λ^k does not change.

Figure ***A4.17*** DUAL POLYHEDRON FOR BENDERS' EXAMPLE

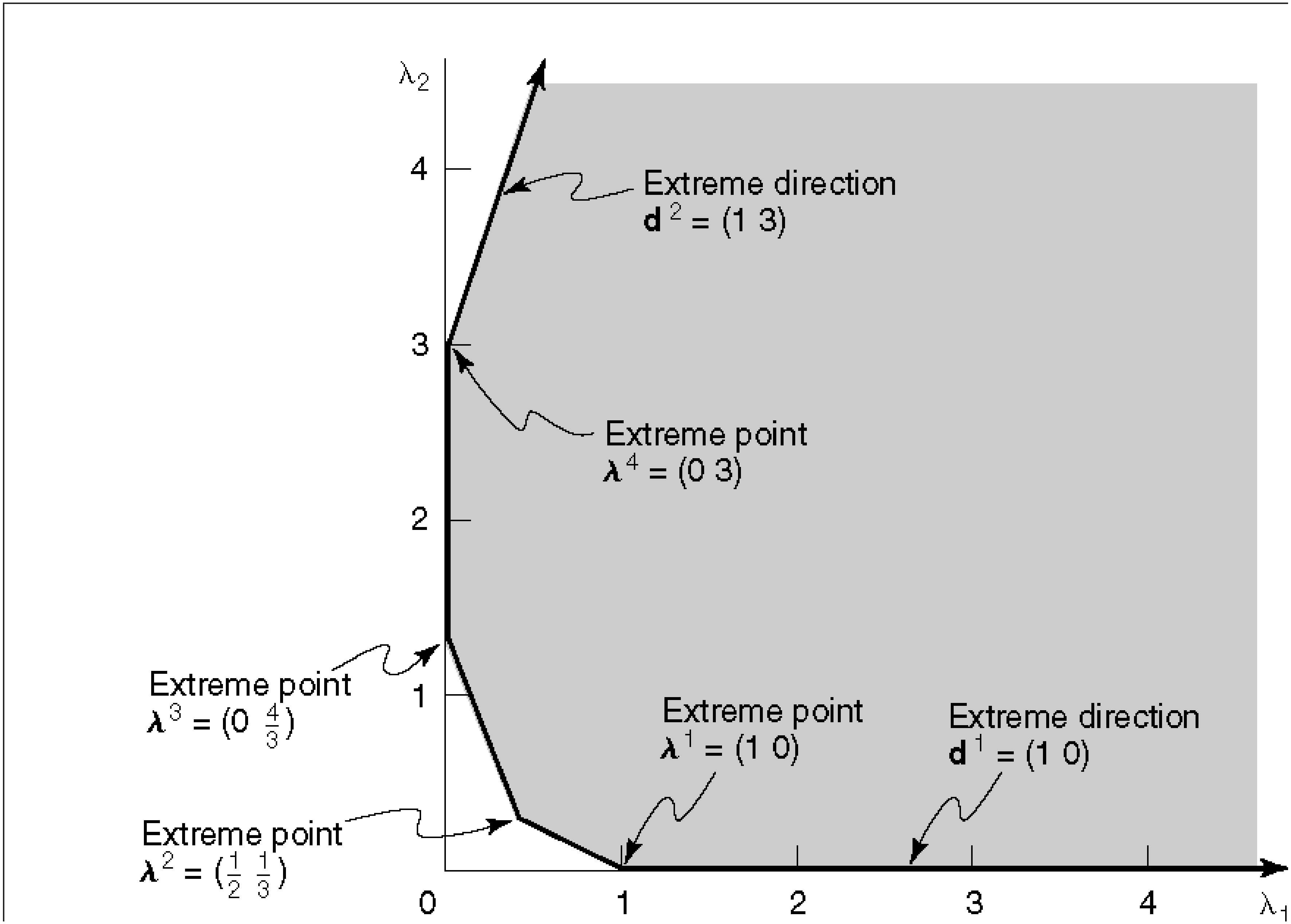
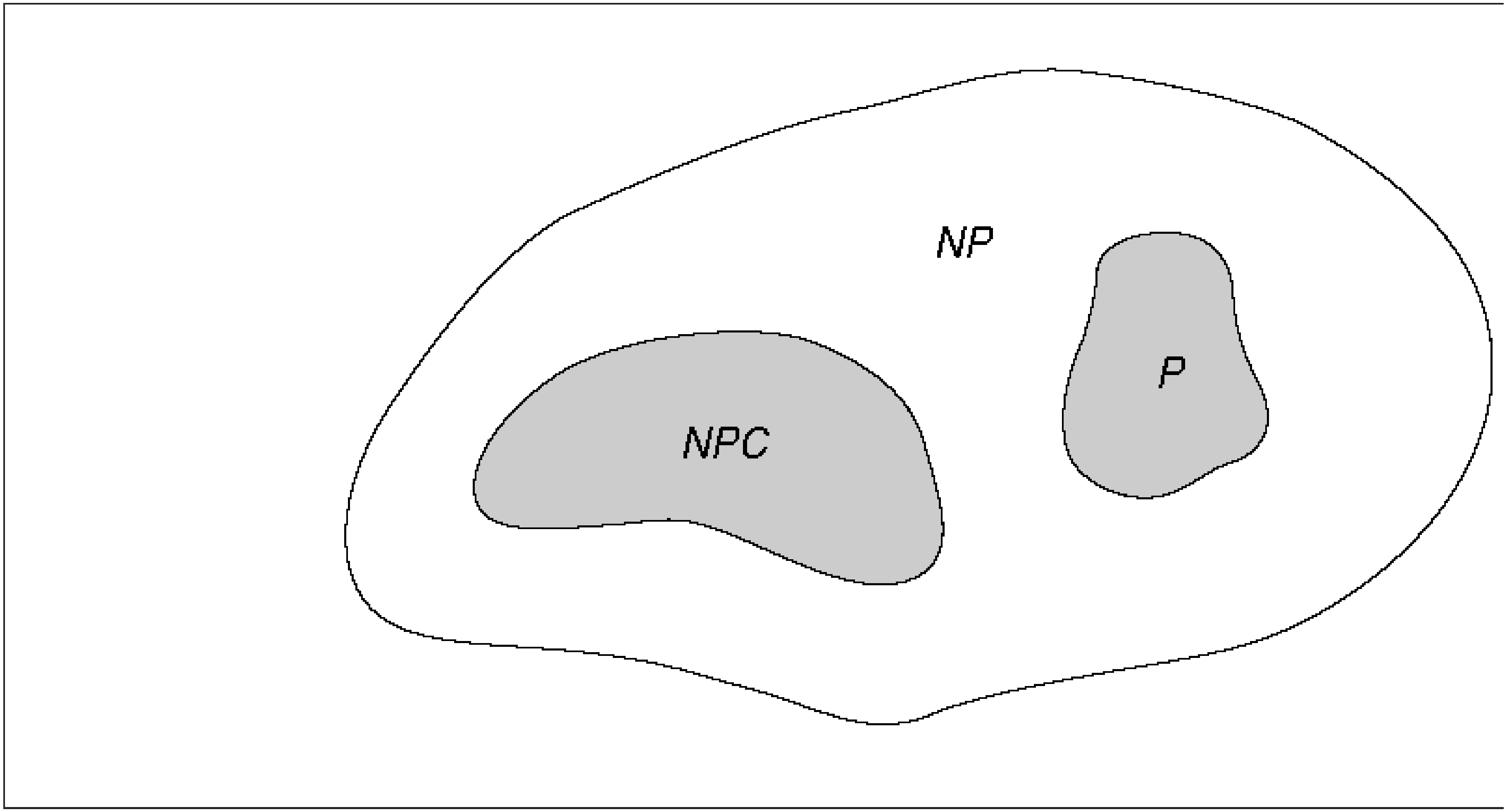


Figure A4.18 TYPES OF COMPLEXITY



SOURCE: Nemhauser & Wolsey (1988). Reprinted with permission.