

Fig. 8.1. Joint probability density $\rho_D(x_1, x_2)$ for a system of two particles. It forms a surface over the x_1, x_2 plane. The marginal distributions $\rho_{D1}(x_1)$ and $\rho_{D2}(x_2)$ are plotted as curves over the margins parallel to the x_1 axis and the x_2 axis, respectively. In each plot the classical position x_{10}, x_{20} is indicated by a black dot in the x_1, x_2 plane as well as by its projections on the margins. Also shown is the covariance ellipse. The three plots apply to the cases of (a) uncorrelated variables and (b) positive and (c) negative correlation between x_1 and x_2 .

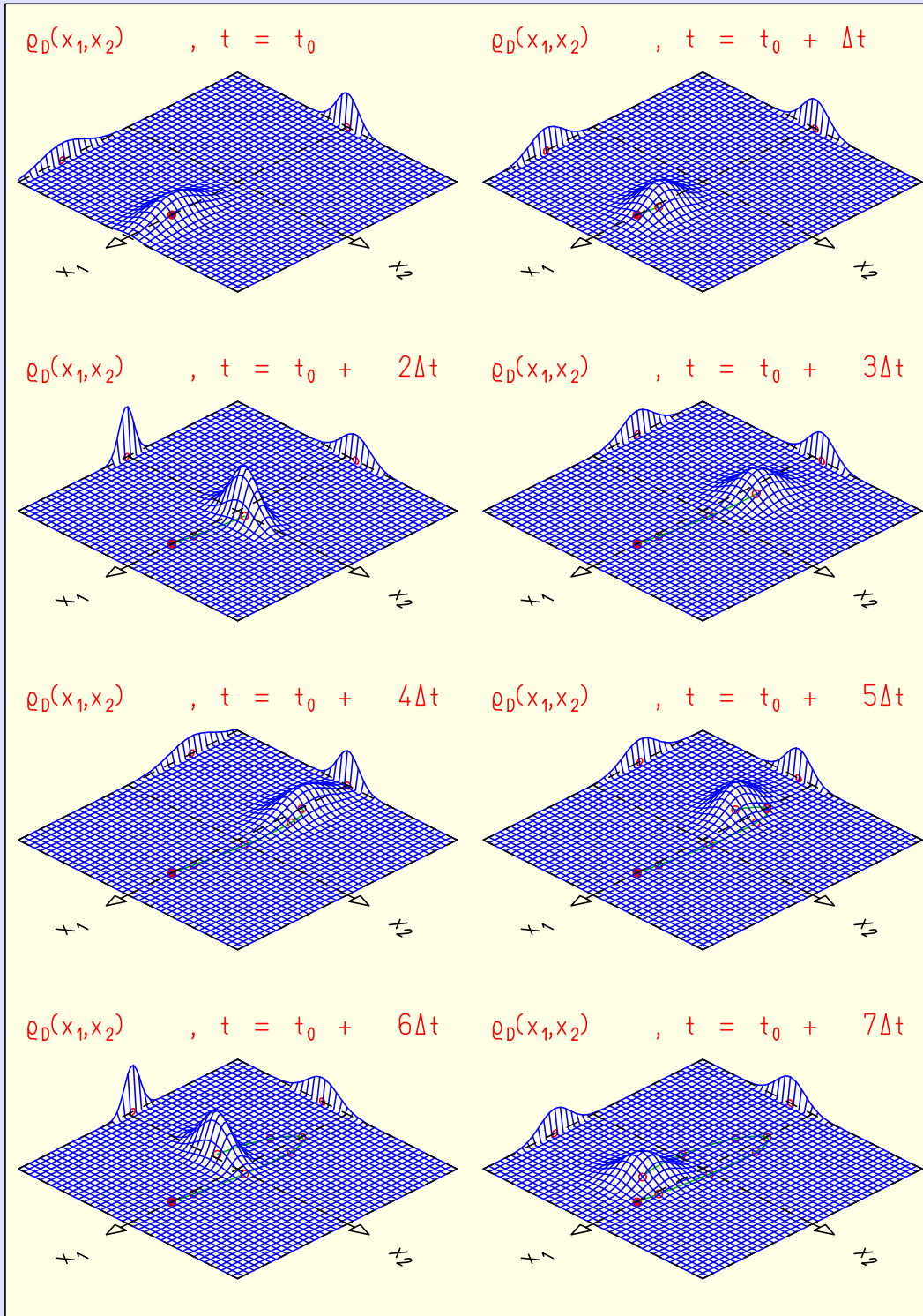


Fig. 8.2. Joint probability density $\rho_D(x_1, x_2, t)$ and marginal distributions $\rho_{D1}(x_1, t)$, $\rho_{D2}(x_2, t)$ for two distinguishable particles forming a system of coupled harmonic oscillators. The different plots apply to various times $t_j = t_0, t_1, \dots, t_N$. The classical position of the two particles at the various moments in time is marked by a dot in the x_1, x_2 plane and by two dots on the margins. The initial dot for $t_j = t_0$ is black. The classical motion between t_0 and t_j is represented by the trajectory drawn in the x_1, x_2 plane.

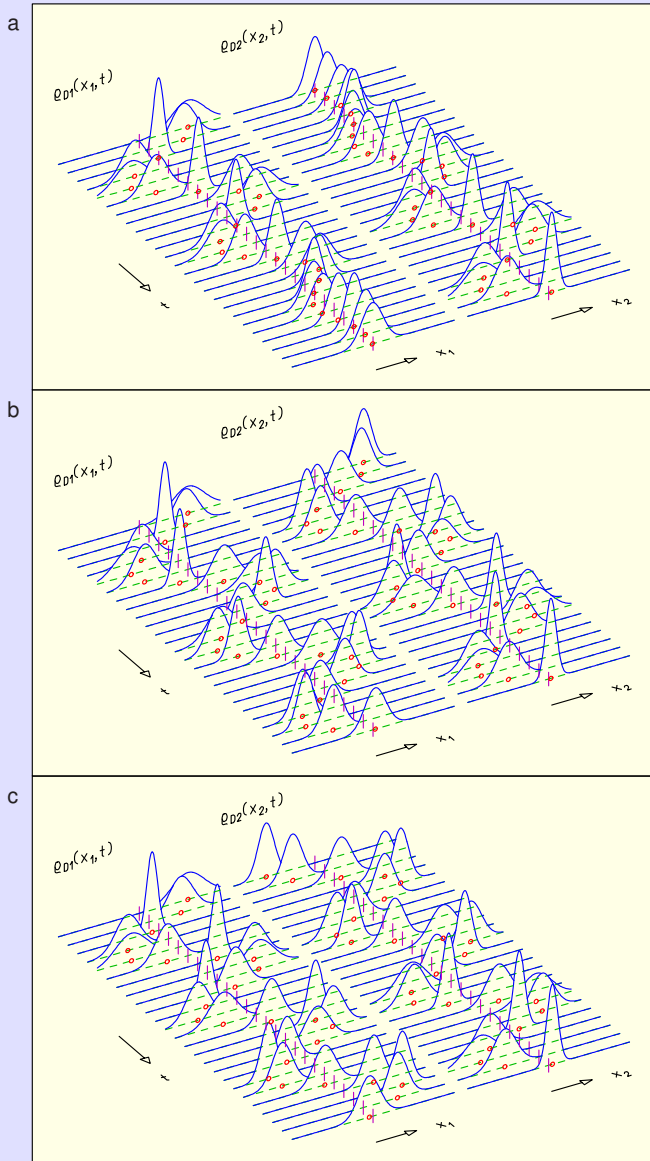


Fig. 8.3. Time development of the marginal distribution $\rho_{D1}(x_1, t)$ on the left and marginal distribution $\rho_{D2}(x_2, t)$ on the right for a system of coupled oscillators. The classical positions of the two distinguishable particles are plotted on the two axes as circles for particle 1 and particle 2. They coincide with the expectation values computed with marginal distributions. (a) The initial position expectation value of particle 2 is zero. (b) The particles are excited in a normal oscillation in which the center of mass oscillates and there is no relative motion. (c) The particles are excited in a normal oscillation in which there is relative motion and the center of mass is at the rest. In all three cases the initial momentum expectation values of the two particles are zero.

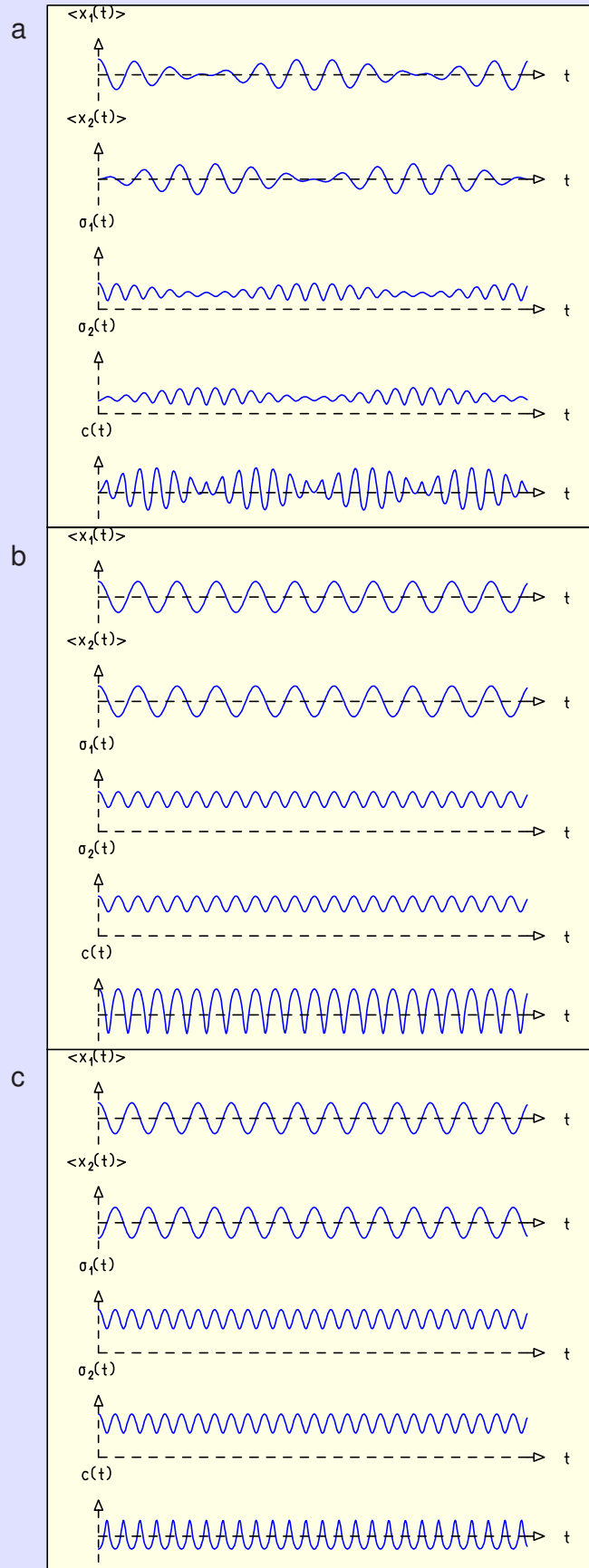


Fig. 8.4. Time dependences of the expectation values $\langle x_1(t) \rangle$, $\langle x_2(t) \rangle$, the widths $\sigma_1(t)$, $\sigma_2(t)$, and the correlation $c(t)$ for a system of coupled harmonic oscillators. (a) Rather general initial conditions were chosen. (b) The oscillation of the expectation values corresponds to an oscillation of the center of mass with frequency ω_R . The initial values $\sigma_1(t_0)$, $\sigma_2(t_0)$, and $c(t_0)$ were chosen so that the two widths and the correlation oscillate with frequency $2\omega_R$. (c) The oscillation of the expectation values corresponds to an oscillation in the relative motion with frequency ω_r ; the widths and the correlation oscillate with frequency $2\omega_r$.

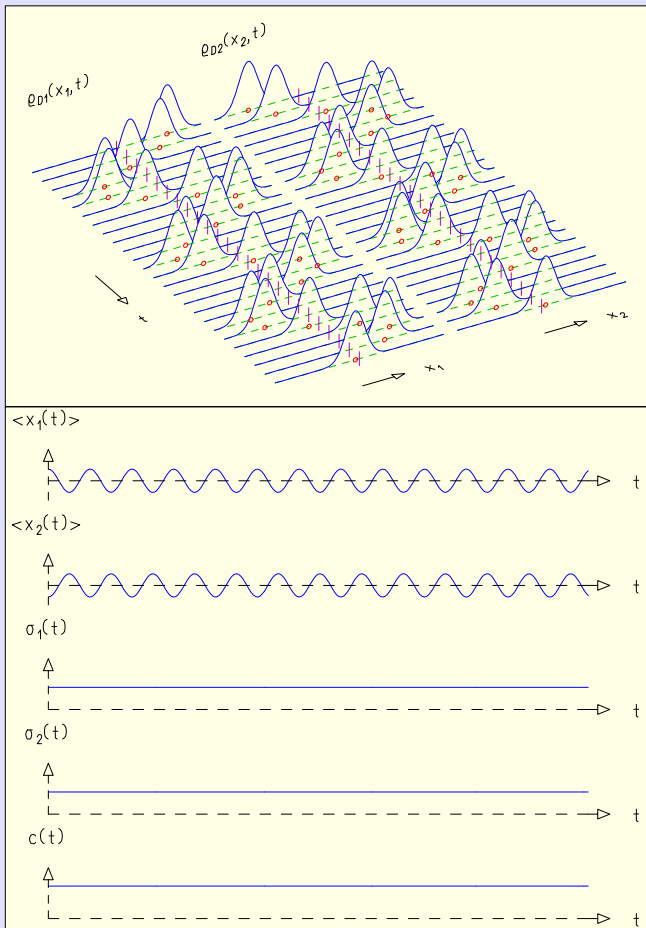


Fig. 8.5. Coupled harmonic oscillators. The initial conditions $\langle x_1(t_0) \rangle$, $\langle x_2(t_0) \rangle$ are the same as in Figure 8.4c, corresponding to an oscillation in the relative motion. The parameters $\sigma_1(t_0)$, $\sigma_2(t_0)$, and $c(t_0)$, however, were chosen so that the widths and the correlation coefficient remain constant independent of time. Top: Time developments of the marginal distributions. Bottom: Time dependences of the quantities $\langle x_1(t) \rangle$, $\langle x_2(t) \rangle$, $\sigma_1(t)$, $\sigma_2(t)$, $c(t)$.

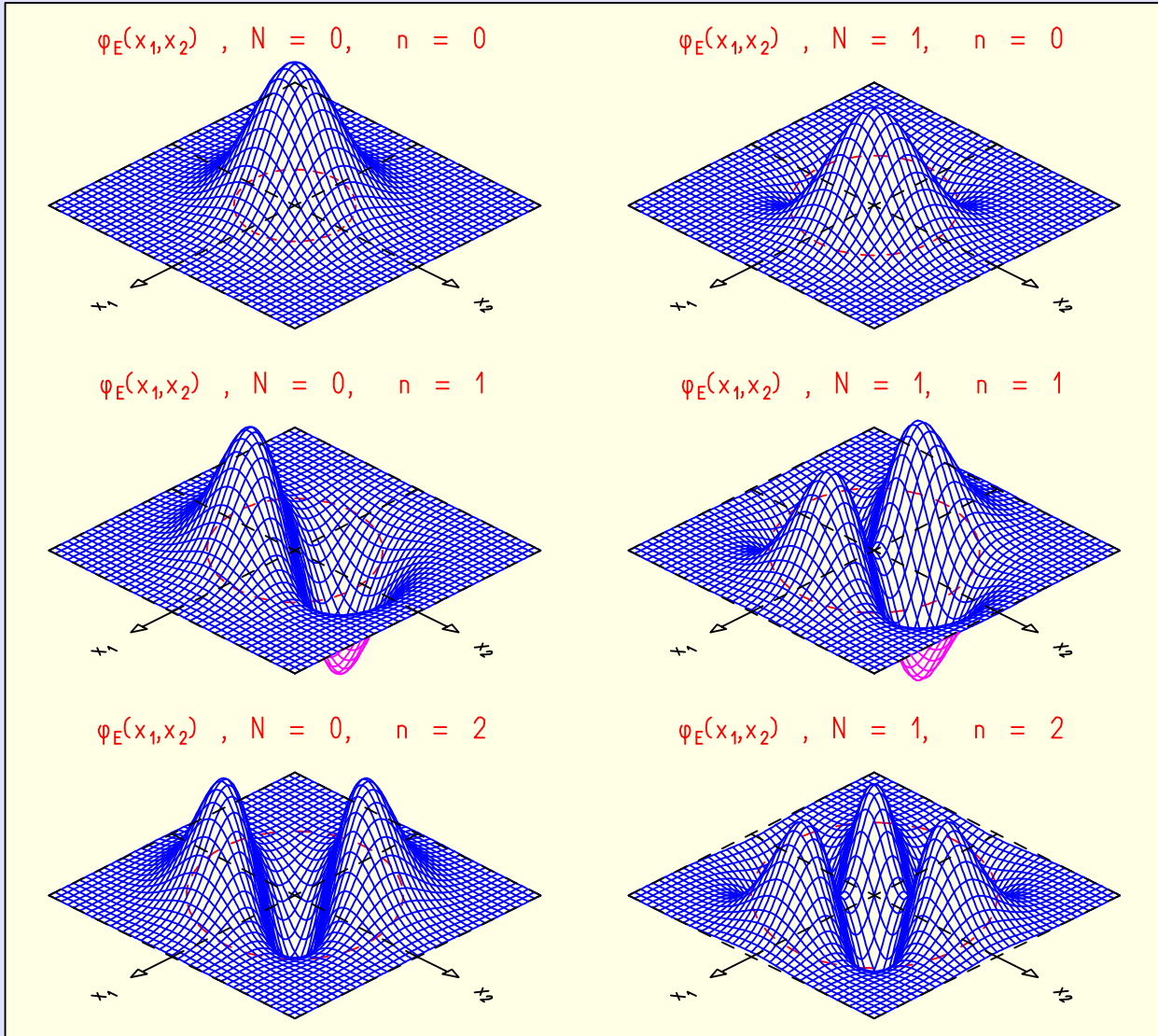


Fig. 8.6. Wave function $\varphi_E(x_1, x_2)$ for stationary states of a system of two coupled harmonic oscillators for low values of the quantum numbers N and n . Note that $\varphi_E(x_1, x_2)$ is symmetric with respect to the permutation $(x_1, x_2) \rightarrow (x_2, x_1)$ for n even and antisymmetric for n odd. The dashed ellipse in the x_1, x_2 plane corresponds to the energetically allowed region for classical particles.