Sylvie Braibant
Giorgio Giacomelli
Maurizio Spurio

Particles and Fundamental Interactions: Supplements, Problems and Solutions

– A deeper insight to particle physics –

January 2012

Springer

Preface

The *Problems* presented here refer to the topics discussed in the corresponding 14 chapters of the *Particles and Fundamental Interactions* textbook. This *Problems, Solutions and Supplements* book is aimed at students in a course of Experimental Particle Physics, not only as a preparation for a written examination, but also as a necessary instrument for a deeper understanding of high energy physics. It contains 170 problems of different difficulty levels. Some of them are traditional, covering most aspects of particle properties and of their fundamental interactions, and some are more advanced. Some problems are derived from our teaching experience to undergraduate students; some are derived from the admission examination to the PhD courses in Italian universities; some are completely original, from our research activities.

Each problem has an identification number and a *title* to facilitate the identification of the subject discussed in the text. Most problems are solved step-by-step, to help both students and teachers to get better acquainted to topics presented in the textbook. We follow the same chapter numeration of the textbook. To avoid confusion when we refer to chapters, equations, figures and tables of *Particles and Fundamental Interactions*, the reference is enclosed in a box. In this way, Fig. 7.2 refers to a figure in this manual, and [Fig. 7.2] to a figure in the textbook. As a general advice, it is useful to try to solve problems only after a first reading of a book. Before facing the more advanced problems, we suggest to read at least up to Chapter 8 of *Particles and Fundamental Interactions*, where the introduction of particle nomenclature and classification, and the presentation of fundamental aspects of the interactions is completed.

In addition to problems and solutions, additional material is presented in form of fifteen *Supplements*. Four of them present the most powerful accelerators, those which produce *cosmic rays*. Cosmic rays were of fundamental importance for the discovery of most long-living particles, the development of particle physics and that of *astroparticle physics*. Three Supplements are devoted to the electronic signals, to data acquisition systems, to the electronic logics and triggers of the experimental apparatuses, ending with the computing effort required for the LHC collider. These issues play also a key role in the contribution that particle physics research provides as a spin-off technology. This is also true for other four supplements, presenting additional information on interactions between charged particles and matter (multiple Coulomb scattering, synchrotron radiation) or the use of radioactive decays for dating old objects. Some problems contain, after the solutions, some comments related to past, running or future experiments (as for instance that for the neutrino beams and neutrino oscillations, the search for proton decay, the study of symmetry violation through the electric dipole moment of the neutron, the measurement of α_S, the study of astrophysical objects using charged particles and/or neutrinos, etc.).

We thank many colleagues, in particular those of the former OPAL and MACRO groups (now, CMS, OPERA and ANTARES) at the University of Bologna, for their cooperation. Finally, we are grateful to many students for their suggestions and questions that allowed us to prepare this work in a way that we hope will be useful for many.

We are responsible for the errors which inevitably could be present in this manual. Some problems contain approximations, or may be solved in different or more straightforward way. We apologize in advance for any mistake that could have survived and that the readers will discover: you are kindly encouraged to inform us.

Bologna, December 2011

Sylvie Braibant, sylvie.braibant@unibo.it
Giorgio Giacomelli, giacomelli@bo.infn.it
Maurizio Spurio, maurizio.spurio@unibo.it

Contents

Problems

1. Historical notes and fundamental concepts.

1.1 **Orders of magnitude**. It is important to become familiar with the orders of magnitude typical of submicroscopic systems. The molecules have dimensions of the order of 10^{-7} cm, the hydrogen atom of the order of 10^{-8} cm, the proton and the neutron of the order of 10^{-13} cm. The size of a quark is less than a hundredth of the proton size. Although the proton can be considered as an almost empty system, it is worth recalling that the vacuum is a complex system.
(a) Let us hypothesize to be able to align the H_2O molecules of a cubic centimeter of water; what would be the length of the very thin line obtained?
(b) The atoms have a size of the order of 10^{-8} cm; an atom is essentially an empty system. If we imagine that the dimension of the proton is 1 mm, at what distance would be the electron in a hydrogen atom?
[A: (a) 20 times the distance Earth-Sun. (b) about 100 m]

1.2 **Natural units**. In the natural unit system ($\hbar = c = 1$), derive the dimensional relations between mass and length and between mass and time.
[A: $[M] = [L^{-1}] = [T^{-1}]$, see Appendix A2].

1.3 **Consequences of $k = 1$**. The Boltzmann constant is $k = 1.38066 \cdot 10^{-23} J\ \mathrm{K}^{-1} = 8.6173 \cdot 10^{-14}$ GeV K^{-1}. Assuming that the Boltzmann constant is $k = 1$ and dimensionless the temperature has the dimensions of energy. Determine the temperature corresponding to 1 eV and to 1 GeV.
[1 eV $= 1.1605 \cdot 10^{4}$ K ; 1 GeV $= 1.1605 \cdot 10^{13}$ K]

1.4 **Planck Mass.** The (dimensionless) gravitation constant can be written as:

$$\alpha_G = \frac{G_N M^2}{\hbar\ c}$$

(different authors insert a 4π factor).
(a) Verify that α_G is dimensionless.
(b) Evaluate α_G for the proton mass $M = m_p$.
(c) Evaluate the *Planck mass* (or Planck energy), M_{Pl}. The Planck mass is, by definition, the mass of the hypothetical particle that would produce $\alpha_G = 1$.
[See solutions]

1.5 **Planck length.** Starting from M_{Pl}, determine a quantity that has the dimension of a length $[L]$. This quantity is defined as the *Planck length.* The Planck mass fixes the energy scale where the unification of the gravitational interaction with the other three interactions (strong, electromagnetic and weak) should occur. In string theory, the Planck length is the natural scale for the string size.
[See solutions]

1.6 **Cross-section $e^+e^- \rightarrow \mu^+\mu^-$.** The cross-section for the $e^+e^- \rightarrow \mu^+\mu^-$ process is:

$$\sigma = \frac{4\pi}{3}\ \frac{\alpha_{EM}^2}{s} \qquad (\text{in GeV}^{-2} \text{ for } \hbar = \text{c} = 1)\ .$$

α_{EM} is the dimensionless electromagnetic coupling constant (called the *fine structure constant* in atomic physics) and $s = E_{cm}^2$. Express the cross-section in the c.g.s system.
[See solutions]

1.7 **Plank units.** The Planck units are units of measurement defined exclusively in terms of five universal physical constants: the Gravitational constant, G_N; the Planck constant, $\hbar$; the speed of light in vacuum, c; the Coulomb constant, $\frac{1}{4\pi\varepsilon_0}$; and the Boltzmann constant, k (see Appendix 5). Using the dimensional analysis, find the five so-called *base Planck units* of mass, length, time, electric charge and temperature.
[See solutions]

1.8 **Kinetic energy.** Evaluate the kinetic energy in TeV of a 10 mg mosquito, moving with a speed of 10 cm/s.
[A: 0.3 TeV]

1.9 **Lifetime and path length.** Instead of the particle lifetimeτ_0, the distance, $d = c\tau_0$, travelled by the particle during its lifetime is sometime specified. Find the momentum of the particle for which this relation is true.
[See solutions]

1.10 **Energy= mass.** At which value of β the kinetic energy is equal to the particle rest mass m_0?
[See solutions]

Supplement 1.1: Cosmic Rays and astroparticle physics.

---

2. Particle interactions with matter and detectors

2.1 **Atom number density**. In the interaction of particles (or nuclei) with matter, the number of collisions depends on the number of scattering centers per unit volume. Often, the scattering centers are atomic nuclei. Consider for example the case of carbon, which has an atomic mass number $A = 12$ and a density (specific mass)$\rho \simeq 2.265$ g cm^{-3}. Determine:
(a) the number of atoms per cm^3;
(b) the number of atoms per gram.
[See solutions]

2.2 α **particle energy loss.** An α particle with 7.4 MeV kinetic energy crosses a target consisting of a thin copper foil 5 10^{-4} cm thick. Determine:
(a) the ionization energy loss in the copper foil;
(b) the particle kinetic energy and (c) the Coulomb multiple scattering angle when going out of the foil.
Hint: see Supplement 2.1.
[See solutions]

2.3 **Muon Energy loss.** A muon of 100 GeV energy crosses without being absorbed a detector whose mass is mainly due to the hadronic calorimeter and to the muon detector. The thickness of the crossed material can be considered as a layer of 3 m of iron. Determine:
(a) what is the dominant energy loss process;
(b) the average energy loss of the muon inside the detector.
Hint: see Supplement 2.2.
[See solutions]

2.4 **Energy transferred.** Calculate the maximum energy ν_{max} transferred in elastic scattering of a charged particle with mass M and energy $E = T + Mc^2$ to an electron at rest:
(a) in the non relativistic case ($T \ll Mc^2$);
(b) in the relativistic case with $M \gg m_e$;
(c) in the general case.

[A: (c) $\nu_{max} = \frac{2m_e c^2(E^2 - M^2c^4)}{M^2c^4 + m_e^2c^4 + 2Em_ec^2}$]

2.5 **Kinematics of the Compton effect.** Using the energy and momentum conservation, describe the kinematics of the Compton effect and derive Eq. (2.19). Calculate the maximum energy of the recoiling electron Eq. (2.21).

2.6 **Electromagnetic shower.** Calculate the average number of particles in an electromagnetic shower initiated by a 50 GeV photon, after 10, 13 and 20 cm of crossed iron.
[See solutions]

2.7 **Muon from pion decay.** Consider a π^+ at rest decaying in $\pi^+ \to \mu^+\nu_\mu$. Calculate the μ^+ kinetic energy and evaluate approximately the μ^+ range in liquid hydrogen (specific mass $\rho = 0.07$ g cm^{-3}).
[See solutions]

2.8 **Neutron discovery.** In his Letter to the Editor of Nature of February 27, 1932 (*Possible Existence of a Neutron*), J. Chadwick described the observation of protons emitted from a target containing hydrogen atoms. The hydrogenated target was exposed to an unknown radiation of strong penetrating power emitted by beryllium when bombarded by α-particles from polonium. See the layout presented in Fig. 2.1. The protons (with mass m_p) were emitted with velocities up to a maximum of nearly 3×10^9 cm/s. Since the penetrating radiation emitted by the beryllium was observed to be neutral, it could consist either of photons or, according to Chadwick's hypothesis, of neutral particles with a mass similar to that of the proton, i.e., the neutrons. Assuming that the neutral radiation emit-

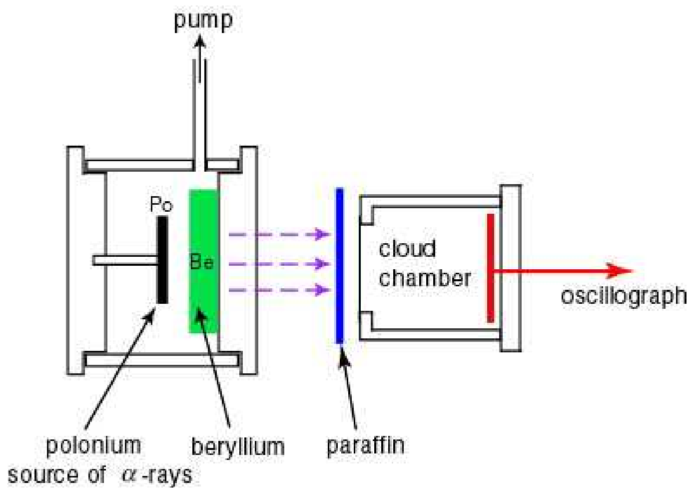

Fig. 2.1. Layout of Chadwick experimental apparatus that lead to the neutron discovery. A beryllium target is exposed to high-energy α rays from a polonium source. A strong penetrating power radiation is emitted from the Be and hit the protons contained in the paraffin layer. The emitted protons are observed in the cloud chamber on the right [2w3]

ted by the Be is composed of photons and that the protons are emitted through the Compton effect induced by these incident photons, calculate the photon energy E_γ. Discuss why this E_γ is inconsistent with the observation. Finally, discuss the reasons that lead Chadwick to formulate the hypothesis of the neutron existence.
[See solutions]

2.9 **Multiple Scattering-1.** Calculate the Coulomb multiple scattering angle in the plane θ^0_{plane} for protons
(a) of 50 MeV/c momentum in 0.1 g cm^{-2} of aluminum;
(b) of 200 MeV kinetic energy in 2 mm of copper.
[See solutions]

2.10 **Multiple Scattering-2.** From considerations based on the Coulomb multiple scattering on nuclei, determine when a target is thin or thick.
[See solutions]

2.11 **Neutron moderation.** Neutrons produced in nuclear reactors are emitted with energies of order of a few MeV and must be slowed down to thermal energies through elastic scattering on nuclei of a *moderator*. Determine the neutron speed variation in each collision assuming that the moderator is (a) hydrogen; (b) carbon; (c) iron. Show that a non-relativistic calculation is sufficient.
[See solutions]

Supplement 2.1: Multiple scattering at small angles

Supplement 2.2: Muon energy loss at high energies

---

3. Particle accelerators and particle detection

3.1 **Energy and momentum.** An on-shell particle with mass m, total energy E and momentum p satisfies the relation $E = +\sqrt{p^2c^2 + m^2c^4}$. For pions with $m = m_{\pi^+} = 140$ MeV/c^2, calculate E for $p = 0.1,\ 1,\ 10$ and 100 GeV/c.
[See solutions]

3.2 **Protons in a magnetic field.** Calculate the curvature radius of the orbit of protons with momentum $p = 10,\ 10^3,\ 10^5$ MeV/c in a magnetic field $B = 1$ Tesla.
[See solutions]

3.3 **Relativistic time dilatation**. Determine the lifetime of a μ^+ with a momentum of 10 GeV/c in the laboratory system; in the muon rest frame $\tau_0 = 2.2$ μs. How far can it go before decaying?
[A: $\gamma = E/m_\mu = 95$; t=$\gamma\tau_0 = 2.1\ 10^{-4}$ s ; d=$c\gamma\tau_0 = 6.3\ 10^6$ cm]

3.4 **Center-of-mass energy.** Calculate the energy available in the center-of-mass (c.m.) system using incident pions with 10 GeV/c^2 kinetic energy in the laboratory against
(a) a proton at rest;
(b) an electron at rest.

3.5 **Threshold energies.** Calculate the threshold energy in the laboratory systems for the production of:
(a) π^0 mesons in the reaction $pp \to pp\pi^0$;
(b) π mesons in the reaction $\pi p \to \pi\pi p$;
(c) K^+ mesons in the reaction $pp \to p\Lambda^0 K^+$;
(d) Σ^+ hyperons in the reaction $pp \to p\Sigma^+$.
In all cases, assume that the target protons are at rest.
(N.B. The reaction $pp \to p\Sigma^+$ is only possible through the weak interaction.)
[See solutions]

3.6 **Antiproton production-1.** Consider the antiproton production

$$p\ +\ p\ \to\ p\,p\,\overline{p}\,p$$

through a beam of protons with a momentum p = 5.5 GeV/c colliding
(a) on protons in a hydrogen target;
(b) on protons in an iron target.

Calculate the threshold energy and the energy in the c.m. system. Is the production possible in both cases?
[See solutions]

3.7 **Antiproton production-2.** (a) What is the threshold energy of the reaction $pp \to ppp\overline{p}$? (b) Determine the kinetic energy available for a target proton bound in a nucleus and moving with a Fermi momentum $\mathbf{p}_F$ (i) towards the incident proton, (ii) in the opposite direction and (iii) orthogonally.
[Hint: see the solution of the previous problem.]

3.8 **Two-body decay.** Consider the decay $\Delta^+ \to \pi^0 p$. Determine the energy and momentum of the two particles in the c.m. system.
[See solutions]

3.9 **Three-body decay.** Consider the decay $K^0 \to \pi^0\pi^+\pi^-$. Determine:
(a) the minimum and maximum values of the π^0 energy and momentum in the K^0 rest system;
(b) the maximum value of the momentum in the lab. system, assuming a K^0 with a momentum $p_K = 100$ GeV/c.
[See solutions]

3.10 ***J/ψ* production.** The J/ψ particle is a meson with a mass $m_{J/\psi} =$ 3.096 GeV and is made of a $c\overline{c}$ quark-antiquark pair. This particle was discovered (Chapter 9) both in proton-proton collisions and in electron-positron collisions.
(a) A proton beam collides on a target of protons at rest; calculate the incident proton beam energy required for the reaction:

$$pp \to ppJ/\psi$$

(b) In the case of electrons, the particle was discovered in a particle collider in which the e^+ and e^- beams had the same energy but opposite momenta. Calculate the beam energy necessary for the J/ψ production:

$$e^+e^- \to J/\psi$$

The J/ψ decays with a very short lifetime, $\tau \sim 10^{-20}$ s.
[See solutions]

3.11 **e^+e^- pair.** A positron and an electron produced in a process of pair production in the laboratory system have four-momenta $P_+ = (E_+, \mathbf{p}_+)$ and $P_- = (E_-, \mathbf{p}_-)$. What is the energy of each particle in the system in which the e^+e^- pair has momentum equal to zero?
[See solutions]

3.12 ***pe***$^-$ **collisions.** At the HERA collider in Hamburg, 820 GeV protons collide frontally with 30 GeV electrons.
(a) Calculate the relativistic invariant $\sqrt{s}$.
(b) Guess the reason why the protons have an energy of 820 GeV and the electrons only of 30 GeV.
(c) What would be the c.m. energy using 850 (=820+30) GeV protons colliding with electrons at rest? Why would this situation be less desirable?
[See solutions]

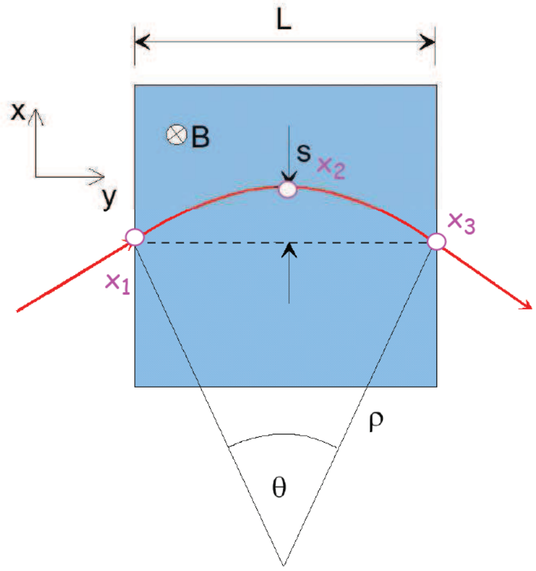

Fig. 3.1. Electrically charged particle moving in a magnetic field **B**. L is the detector dimension, ρ the radius of curvature and s the sagitta of the particle orbit

3.13 **Momentum measurement in a magnetic field.** The variables needed for the measurement of a fast electrically charged particle moving in a magnetic field are shown in Fig. 3.1. L is the detector dimension, **B** the (constant) magnetic field assumed perpendicular to the beam direction, ρ the radius of curvature and s the sagitta (see also Fig. 3.9c) of the particle orbit; p_t is the momentum projection in a plane orthogonal to the magnetic field. Neglecting Coulomb multiple scattering, determine:
(a) the relation between p_t and s;
(b) the precision on the measurement of the transverse momentum if $p_t = 1$ GeV/c, $B = 10$ kGauss, $L = 1$ m and $\Delta s = 200\,\mu$m (where Δs is

the error on the sagitta).
[See solutions]

3.14 **π, K discrimination.** Consider a magnetic spectrometer which selects positively charged particles with momentum $p = 0.5$ GeV/c in a beam mainly made of π^+ and K^+. The discrimination between the two particles is made using the time-of-flight technique. For this purpose, two scintillation detectors are placed 3 m apart. Each plastic scintillator has a thickness of $\Delta x = 2$ cm, density $\rho = 1.03$ g cm^{-3} and radiation length $X_0 = 40$ cm. Determine:
(a) the π and K velocities;
(b) the energy loss in the first scintillator;
(c) the average deflection angle due to Coulomb multiple scattering for the π^+ and K^+ after the first detector.
[See solutions]

3.15 **Luminosity at the Tevatron.** Calculate the luminosity at the Fermilab Tevatron collider. (a) In the interaction region called B_0, assuming the following parameters: circumference $C = 2\pi R = 6.28$ km; number of protons per bunch $N_p = 6 \cdot 10^{10}$; number of antiprotons per bunch $N_{\overline{p}} = 2 \cdot 10^{10}$; number of bunches $N_B = 6$; correction factor $G = 0.9$, to take into account the finite bunch length of about 50 cm; average transverse radius of each beam $r_{B_0} = 43$ μm. (b) Calculate the luminosity in the interaction region E_0, where the average transverse radius is $r_{E_0} = 380$ μm.
[See solutions]

3.16 **Beam attenuation.** The interactions between particles in the beam pipe of an accelerator with the residual gas present in the beam pipe lead to the attenuation of the beam. This effect reduces the permanence lifetime of particles inside the beam pipe, which must be re-filled.
(a) Calculate the half-life of a proton beam circulating in a storage ring of 100 m radius. The vacuum in the beam pipe corresponds to 10^{-6} mm Hg. Assume that the residual gas is hydrogen and that the total pp cross-section is 40 mb.
(b) Evaluate the beam half-life if the vacuum is 10^{-9} mm Hg.
[See solutions]

3.17 **Reconstruction efficiency.** Cosmic ray tracks can be measured using spark chambers: each chamber has an efficiency of 93%. To reconstruct a track, at least 3 points (and therefore the use of at least 3 chambers) are required. What is the track reconstruction efficiency for a system made of 4 chambers? And for a system of 5 chambers?

3.18 **Synchrotron radiation at LEP.** The LEP collider was a storage ring of 27 km circumference where e^+, e^- beams circulated in opposite direction.
(a) Determine the energy loss due to the synchrotron radiation for each revolution of an electron of 50 GeV energy (LEP-Phase 1) and of 100 GeV energy (LEP-Phase 2).
(b) Determine the intensity of the magnetic field needed to keep on orbit the electrons and positrons for both energies of 50 and 100 GeV; assume a uniform magnetic field along the ring.
(c) Determine the energy loss variation if the e^+, e^- beams are replaced with proton and antiproton beams (assuming they have the same energy as the e^+, e^- beams).
[See solutions]

3.19 **Synchrotron radiation at LHC.** The LEP has been substituted, in the same tunnel, by a pair of storage rings where two proton beams can circulate in opposite directions (the Large Hadron Collider, LHC). The protons are deflected by a set of 1230 magnets, each 14.4 m long. The LHC design foresees that each proton beam shall reach a maximum energy of 7 TeV.
(a) What should be the absolute value of the magnetic field to bend the protons with the maximum energy?
(b) What will be be the energy irradiated from a proton per lap; and per second?
[See solutions]

3.20 **Muon factory.** To reduce the energy loss due to the synchrotron radiation of electrons, it was suggested to build a muon storage ring (muon factory). The muons produced in pion decays are collected and directed into the ring. To obtain a high efficiency, it is necessary that the muon direction does not change appreciably from the direction of the charged particle which initially decays.
(a) Determine the muon maximum angle of deflection in the laboratory system with respect to the direction of a pion decaying with a momentum of 20 and 200 GeV/c.
(b) Write the formula of the maximum angle as a function of energy.
[See solutions]

3.21 **Particle-antiparticle pair production.** Consider the production of a particle-antiparticle pair from a high energy positron beam colliding on electrons assumed at rest.
(a) Calculate the minimum positron beam energy necessary to produce a pion pair ($m_\pi = 140$ MeV/c^2).
(b) In the laboratory system, determine the maximum diffusion angle of the pions with respect to the direction of a 150 GeV energy positron beam.

(c) Demonstrate that, for beam energies much larger than the production threshold, the maximum opening angle of the created pair, in the laboratory system, becomes independent of the energy. Calculate the value of this angle for a pair of pions. (If θ_1 is the diffusion angle of the particle and θ_2 that of the antiparticle, the opening angle θ is is defined as $\theta = \theta_1 + \theta_2$).

Supplement 3.1: Synchrotron radiation

---

4. The paradigm of interactions: the electromagnetic case

4.1 **Yukawa range of weak interactions.** The weak interaction is due to the exchange of $W^{\pm}$ (or Z^0) bosons. In the framework of a Yukawa-like model describing the weak interaction, and assuming that the W has a mass of $M = 80$ GeV/c^2, calculate the range of the weak interaction and compare the obtained value with the size of a nucleon.
[A: R=$\hbar/Mc \simeq 3 \times 10^{-16}$ cm]

4.2 **Free electron emission.** Show that the decay $e \rightarrow e\gamma$ (Fig. 4.2a) cannot satisfy, at the vertex, the energy and momentum conservation laws if the initial electron and the particles in the final state are free (this process is therefore prohibited).
[See solutions]

4.3 **Impact parameter and elastic scattering angle.** Using classical kinematics, demonstrate, for the Coulomb elastic scattering (see Fig. 4.9b), the relation $tg\frac{\theta}{2} = \frac{zZe^2}{2E_c b}$ between the scattering angle θ, the kinetic energy $E_c = (p^2/2m)$ of the incident particle and the impact parameter b.

4.4 **Lifetime and path length.** A π^- meson interacts in a bubble chamber, in a point of coordinates $x_1 = -50$ cm, $y_1 = 50$ cm, $z_1 = 20$ cm, $t_1 = 0$ s producing some mesons, including a K^- (vertex 1). The K^- meson travels at a constant speed until it interacts in a region containing counters and chambers (vertex 2 at the position $x_2 = 88$ cm, $y_2 = 48$ cm, $z_2 = 25$ cm, $t_2 = 5.31$ ns). Calculate for the K^-:
(a) the travelled distance, (b) the flight time (corresponding to its lifetime measured in the laboratory system), (c) the velocity, (d) the K^- lifetime in the frame in which the particle is at rest, (e) the probability that the K^- decays along the travelled distance.
[See solutions]

4.5 **Radiocarbon dating.** The half-life, $t_{1/2}$, of the carbon 14 isotope, ${}^{14}_{6}C$, is 5730 years. The concentration of this isotope, in the atmosphere, with respect to the stable isotope ${}^{12}_{6}C$ is ${}^{14}_{6}C/{}^{12}_{6}C = 1.0\ 10^{-12}$.
(a) What is the ratio ${}^{14}_{6}C/{}^{12}_{6}C$ in an object which ended its life 12000 years ago?
(b) An artifact made of wood has a concentration of ${}^{14}_{6}C$ corresponding to 58% of the concentration in a similar object made of freshly cut wood. Determine the age of the original sample.
[See solutions]

4.6 **Rutherford scattering.** The α particles are helium nuclei. A flux of $\Phi = 5.0\ 10^7$ particles/s of 8.0 MeV energy collides on a gold target 4.0 μm thick. A sector of a circular detector, concentric with the beam, is placed at a distance of 3.0 cm after the target (see Fig. 4.9a). The smaller radius of the circular sector from the beam axis is $r_{min} = 5.0$ mm while the larger radius is $r_{max} = 7.0$ mm. Determine the number of particles reaching the detector per second. Discuss and comment the case in which $r_{min} \to 0$.
[See solutions]

4.7 **Electromagnetic transition probability.** Using the second Fermi golden rule, Eq. (4.28), show that the annihilation probability of positronium (bound state e^+e^-) in the singlet state $^1S_0$ (i.e., the state in which the electron and positron spins are antiparallel) is given by :

$$W(^1S_0 \to \gamma\gamma) = \frac{4\pi\alpha^2}{m_e^2}|\psi(0)|^2 \tag{3.1}$$

where $|\psi(0)|^2$ represents the probability that the electron and the positron are in the same region of space. Draw the Feynman diagram of the annihilation process into two photons.
[See solutions]

4.8 **Positronium annihilation.** In the hydrogen atom, the wave function of the state $^1S_0$ of the electron moving around the proton, is known from atomic physics:

$$|\psi(r)| = \frac{1}{\sqrt{\pi}a^{3/2}}e^{-r/a} \tag{3.2}$$

where a is the Bohr radius of the hydrogen atom:

$$a = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c\alpha} = 0.5\ 10^{-10}\ \text{m} \tag{3.3}$$

Using Eq. (3.1), determine the lifetime of the positronium in the $^1S_0$ state.
[See solutions]

4.9 **Dirac theory-I.**[1] Dirac proposed an equation that would satisfy the energy-momentum relativistic relation ($E^2 = p^2 + m^2$), but containing only first order space-time derivative. Show that the eigenvalue equation:

$$E\psi = i\hbar\frac{\partial}{\partial t}\psi = H\psi \tag{3.4}$$

[1] For the following four problems, the reader should know the content of Appendix A4

can be written as:

$$i\hbar\gamma^0 \frac{\partial}{\partial t}\psi + i\hbar\boldsymbol{\gamma} \cdot \boldsymbol{\nabla}\psi - m\psi = 0 \ . \tag{3.5}$$

where the γ matrices are defined in Appendix A4.
[See solutions]

4.10 **Dirac theory-II.** Prove that the Dirac equation Hamiltoninan:

$$H = \boldsymbol{\alpha} \cdot \mathbf{p}c + \beta mc^2 \tag{3.6}$$

commutes with the total angular momentum operator

$$\mathbf{J} = \hbar\mathbf{L} + \frac{1}{2}\hbar\boldsymbol{\sigma} \ . \tag{3.7}$$

[See solutions]

4.11 **Dirac theory-III.** Prove that the Dirac equation Hamiltonian:

$$H = \boldsymbol{\alpha} \cdot \mathbf{p}c + \beta mc^2 \tag{3.8}$$

commutes with the generalized helicity operator (A.27):

$$\Lambda = \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p} & 0 \\ 0 & \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p} \end{pmatrix} \tag{3.9}$$

[See solutions]

4.12 **Dirac theory-IV.** Show that the 4-component wave function

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} e^{i(\mathbf{p}\cdot\mathbf{r} - Et)} \tag{3.10}$$

where ϕ and χ are 2-component spinors, satisfies the Dirac equation if:

$$\phi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E - m}\chi \quad \text{and} \quad \chi = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m}\phi \ . \tag{3.11}$$

[See solutions]

Supplement 4.1: Radiocarbon dating

---

5. First discussion of the other fundamental interactions

5.1 **Neutron and antineutron.** How do you distinguish a neutron from an antineutron? And how can you distinguish a neutrino from the corresponding antineutrino?
[See solutions]

5.2 **Gravity and electric forces.** Estimate the size of a hypothetical hydrogen atom whose electron and proton are bound only by the gravitational force.
[See solutions]

5.3 **Feynman diagrams.** Draw the Feynman diagrams that illustrate the following reactions; indicate the involved fundamental interaction.
(a) $e^- \; p \; \rightarrow \; e^- \; p$
(b) $e^+ \; e^- \; \rightarrow \; \nu_e \; \bar{\nu}_e$
(c) $e^- \; p \; \rightarrow \; \nu_e \; n$
(d) $u \; d \rightarrow \; u \; d \; d \; \bar{d}$
[See solutions]

5.4 **Tau decay.** Consider the decay $\tau^- \rightarrow e^- + X$.
(a) Which neutral particles form the system X ?
(b) Draw the lower order Feynman diagram for this process.
[See solutions]

5.5 **Δ^{++} decay.** Estimate the decay fraction ratio

$$\Gamma(\Delta^{++} \rightarrow pe^+\nu_e)/\Gamma(\Delta^{++} \rightarrow p\pi^+)$$

[See solutions]

5.6 **Neutron star.** A neutron star is produced by the gravitational collapse of stars more massive than $8M_\odot$, where $M_\odot$ is the mass of the Sun. In the center of the star remains a sort of giant nucleus, consisting of only neutrons, held together by its own gravitational force. Calculate the radius of a neutron star assuming that: (i) the mass of the nucleus is $1.4M_\odot$ and (ii) the density of the neutron star is constant.
[See solutions]

5.7 **Σ^+ decay.** Explain the decay chain of the $\Sigma^+(1383)$ baryon. In terms of quarks, the Σ^+ is made of quarks $[uus]$.
[See solutions]

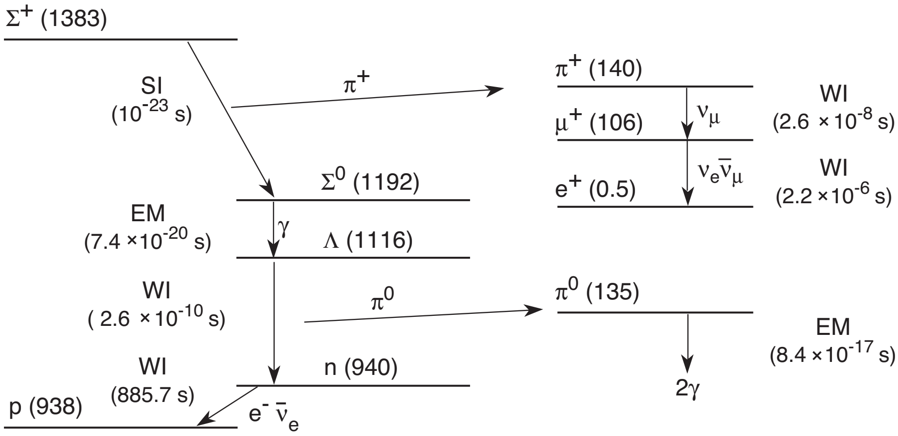

Fig. 5.1. Decay chain of the $\Sigma^+(1383)$ baryon. The lifetime of each decay channel and are reported in parentheses

---

Supplement 5.1: Baryon number conservation: the search for proton decay

---

6. Invariance and conservation principles

Problems

6.1 **Electric dipole of the neutron.** Show that if a particle (for instance, the neutron) has a non vanishing electric dipole moment $\mathbf{d}_E$, both parity and time reversal invariance are violated.
[See solutions]

6.2 **CP of neutron decay.** Consider the β neutron decay, $n \rightarrow p + e^- + \overline{\nu}_e$, and apply the parity operator; does the resulting process exist in nature? Then, apply the charge conjugation operator. What kind of process do you obtain? What can be concluded about the CP operator in β decay?
[See solutions]

6.3 η^0 **decay.** pseudoscalar meson The pseudoscalar meson η^0 has quantum numbers $J^{PC} = 0^{-+}$, mass 548 MeV, full width $\Gamma = 1.30$ keV. The main decay modes (from the PDG) and corresponding *branching ratios* (BR) are:

Decay	BR
$\eta^0 \rightarrow \gamma\gamma$	0.393
$\eta^0 \rightarrow \pi^0\pi^0\pi^0$	0.326
$\eta^0 \rightarrow \pi^0\pi^+\pi^-$	0.227

There are also upper limits for the following decays:

Decay	BR
$\eta^0 \rightarrow \gamma\gamma\gamma$	$< 1.6\ 10^{-5}$
$\eta^0 \rightarrow \pi^0\pi^0$	$< 3.5\ 10^{-4}$
$\eta^0 \rightarrow \pi^0\gamma$	$< 0.9\ 10^{-6}$
$\eta^0 \rightarrow e^+\mu^-$ or $e^-\mu^+$	$< 6\ 10^{-6}$

(a) Determine the lifetime of the η^0 decay and which interaction induces the decay.
(b) Explain, using conservation laws, the reason why the decays with the upper limits quoted above were not observed. Remember that the photon has $J^{PC} = 1^{--}$, and the π^0 has $J^{PC} = 0^{-+}$.
[See solutions]

6.4 Σ **decay.** Explain why the Σ^0 hyperon decays into $\Lambda^0\gamma$ instead of $n\pi^0$.
[See solutions]

6.5 Ω^- **decay.** Discuss the possible decay modes of the Ω^- particle allowed by conservation laws. Show that the weak interaction decay is the only

possible one.
[See solutions]

6.6 **Pion decay.** The π^0 meson has spin zero and mass m_π =135 MeV/c^2, and decays into two photons. Since the measurement of the characteristics of the final state photons provides information on the π^0 spin/parity, determine:
(a) the angular distribution of the emitted photons in the π^0 rest frame;
(b) the shape of the energy spectrum of the emitted photons in the laboratory system;
(c) the maximum and minimum energy of the emitted photons when the π^0 has an energy of 0.8 GeV.
[See solutions]

6.7 **Conservation law rules.** Verify if the following reactions satisfy all conservation laws:
(a) $\overline{K^0} p \to K^- p \pi^+$
(b) $\pi^- p \to K^- \Sigma^+$
(c) $\pi^- p \to \overline{\Sigma}^- \Sigma^0 p$
(d) $\overline{p} p \to \pi^+ \pi^+ \pi^- \pi^-$
(e) $\pi^+ p \to K^+ \Sigma^+$
[See solutions]

7. Interactions of hadrons at low energies and the static quark model

7.1 **Range of the nuclear force.** Despite the fact that the quarks within a neutron or a proton interact via the exchange of gluons, the strong interaction between a proton and a neutron can be viewed as due to the exchange of a pion. If the π has a mass of 140 GeV/c^2, calculate the distance at which the strong force is effective.
[See solutions]

7.2 Λ^0 **decay at rest.** For the $\Lambda^0 \to p\pi^-$ decay at rest, calculate the momentum and the energy of the final state particles.
[See solutions]

7.3 π^- **interactions.** A target of liquid hydrogen (density $\rho = 0.06$ g cm^{-3}) has a volume of 100 cm^3. A monoenergetic π^- beam with 300 MeV/c momentum collide on the target. The beam is broad, uniform and with an intensity of $\Phi = 10^7$ π^-m^{-2}s^{-1}. The cross-section for the reaction $\pi^- p \to \pi^0 n$ at 300 MeV/c is $\sigma = 45$ mb.
(a) Calculate the number of γ-rays produced per second (recall that the π^0 meson decays into two photons, $\pi^0 \to 2\gamma$, in a very short time).
(b) Calculate the mean free path of 300 MeV/c π^- in liquid hydrogen.
[See solutions]

7.4 **Antiproton capture at rest.** The antiprotons captured at rest in deuterium give rise to the reaction $\overline{p}d \to n\pi^0$. Determine:
(a) the deuterium binding energy;
(b) the total energy of the emitted π^0 meson;
(c) the process that occurs in terms of valence quarks.
[See solutions]

7.5 **Energy thresholds.** Consider the collisions of $\overline{K}^0$ and K^0 mesons on protons at rest.
(a) Calculate, in the laboratory system, the minimum kinetic energy of the $\overline{K}^0$ and of the K^0, necessary to induce respectively the following reactions:

$$\overline{K}^0 + p \to \Lambda^0 + \pi^+ \tag{6.12}$$

$$K^0 + p \to \Lambda^0 + K^0 + K^+ \tag{6.13}$$

(b) With the help of Tab. 7.3, write the reactions in terms of valence quarks.
[See solutions]

7.6 $\phi(1020)$ **decay.** Explain why the $\phi(1020)$ vector meson cannot decay into two π^0 mesons.
[See solutions]

7.7 ψ **decay.** Explain why the $\psi(3685) \to J/\psi(3097)\pi^0$ decay is not allowed by the strong interaction, while the $\psi(3685) \to J/\psi(3097)\eta$ decay is permitted.
[See solutions]

7.8 Δ^0 **decay.** The $\Delta^0[udd]$ resonance decays mainly in $p\ \pi^-$ with a width $\Gamma \simeq 100$ MeV. Draw the Feynman diagram. Estimate the Δ^0 lifetime. Is there another possible decay channel?
[See solutions]

7.9 **The Δ^{++} resonance.** Calculate the elastic $\pi^+ p$ cross-section for the formation of the Δ^{++} (1232) resonance when
(a) the π^+ in the lab. system has a kinetic energy $T_{\pi_{lab}} = 190$ MeV;
(b) the π^+ in the lab. system has a kinetic energy $T_{\pi_{lab}} = 300$ MeV.
Assume natural units and $(m_p = 938, m_\pi = 140, m_\Delta = 1232, \Gamma = 120)$ MeV.
[See solutions]

7.10 **Mean free path.** For a graphite target ($\rho = 2.265$ g cm^{-3}), calculate the number N_n of carbon nuclei per cm^3, the absorption coefficient μ for a proton beam and the interaction length λ. Use a cross-section value of $\sigma = 0.331 \cdot 10^{-24}$ cm^2.
[See solutions]

7.11 **Isospin-1: pion-proton collisions.** Express, as a function of the isospin, the amplitudes of the following collision processes:
(a) $\pi^+ p \to \pi^+ p$
(b) $\pi^- p \to \pi^- p$
(c) $\pi^- p \to \pi^0 n$
(d) Calculate the ratio between the cross-sections of the above three processes, assuming that the $I = 3/2$ isospin channel is dominant. Explain the physical motivation for this assumption.
Refer to Supplement 7.1 and Figs. 7.14, 7.15, 7.17 and 7.18 for the isospin values of the involved particles.
[See solutions]

7.12 **Isospin-2: Δ resonance formation in pion-proton collisions.** The Δ resonance is an $I = 3/2$ isospin multiplet. Consider the following two formation mechanisms

$$\pi^- p \to \Delta^0$$
$$\pi^+ p \to \Delta^{++}$$

and determine the ratio between the respective cross-sections.
[See solutions]

7.13 **Isospin-3: $K^- p$ collisions.** Calculate the cross-section ratio of the following processes:
(a) $K^- p \to \pi^+ \Sigma^-$
(b) $K^- p \to \pi^0 \Sigma^0$
(c) $K^- p \to \pi^- \Sigma^+$
(d) $K^- p \to \pi^0 \Lambda^0$
[See solutions]

7.14 **Isospin-4: the pd cross-section.** Calculate the cross-section ratio for the reactions $pd \to ^3$He π^0, $pd \to ^3$H π^+ at a fixed energy in the c.m. system.
[A. $\sigma(pd \to ^3\text{He}\ \pi^0)/\sigma(pd \to ^3\text{H}\ \pi^+) \simeq 1/2$]

7.15 **Isospin-5: Strange and Charmed particles.** Indicate which is the isotopic spin of the π^-, K^-, D^0, D_s^+ mesons and of Σ^-, Σ_c^0, Ξ_{cc}^+ and Ω_{cc}^+ baryons.
[Hint: Refer to Figs. 7.17, 7.18]

7.16 **Weisskopf formula for neutral vector meson decay.** Starting from Eq. (3.1):

$$\Gamma = \frac{16\pi\alpha^2}{m_e^2}|\psi(0)|^2 \tag{6.14}$$

derive the Weisskopf formula given Eq. (7.59) for the decay of the ρ^0, ω^0, ϕ^0 neutral vector mesons into leptons.
[See solutions]

7.17 ρ^0 **Spin-Parity.** Determine the spin-parity of the ρ^0 resonance produced in $\pi^- p \to \rho^0 n$, with the subsequent decay into two pions, $\rho^0 \to \pi^- \pi^+$.
In the above reaction a peak is observed at the invariant mass $m_\rho = m_{\pi^+\pi^-} = 775$ MeV, with a $\Gamma = 149$ MeV width. Explain why no $\pi^0\pi^0$ resonance is observed at the same mass.
[See solutions]

Supplement 7.1: Sum of angular momentum and isospin: the Clebsch–Gordan coefficients

---

8. Weak interactions and neutrinos

8.1 **Universality of weak interactions.** According to the Puppi triangle (see §8.4.3), the weak interaction occurs with the same characteristics and same intensity for beta nuclear decays, for the muon decay and for the nuclear capture of negative muons. Explain the reason, using Feynman diagrams.
[See solutions]

8.2 **Neutron and muon decay.** Taking into account that the neutron lifetime is $\tau_n = 887$ s, and that of the muon $\tau_\mu = 2.2 \cdot 10^{-6}$ s, show that the couplings in these two cases have the same order of magnitude when considering the phase space factor.
[Hint: See §8.4.1]

8.3 **Feynman diagrams.** Draw the Feynman diagrams for the following decay and interaction processes:
(a) $\mu^- \to e^- \overline{\nu}_e \nu_\mu$;
(b) $\tau^+ \to e^+ \nu_e \overline{\nu}_\tau$;
(c) $\nu_e e^- \to \nu_e e^-$;
(d) $\mu^+ e^- \to \overline{\nu}_\mu \nu_e$.
[See solutions]

8.4 **Axial and vector couplings.** Using the data for the β decay of ${}^{14}_{8}O$ and of ${}^{6}_{2}He$ reported in Tab. 8.1, determine the ratio $|g_A/g_V|$.
[See solutions]

8.5 **Sargent rule from $\Sigma^\pm$ decays.** The Sargent rule, Eq. 8.18:

$$W = \frac{(\Gamma_i/\Gamma)}{\tau} \simeq G_F^2 E_0^5 \simeq G_F^2 \Delta m^5 \ . \qquad (8.15)$$

can be tested through the $\Sigma^\pm$ decay.
(a) Show in terms of quark content that the Σ^+ is not the antiparticle of the Σ^-.
(b) Draw the Feynman diagrams and calculate, using the particle masses reported in the *Review of Particle Physics* [P10], the ratio between the lifetimes of the following semileptonic decays:

$$\Sigma^+ \to \Lambda^0 e^+ \nu_e \quad ; \quad \Sigma^- \to \Lambda^0 e^- \overline{\nu}_e$$

(c) Using the lifetimes and branching ratios reported in [P10] for the above semileptonic decays, compare the measured ratio of decay fractions with the expected one.
[See solutions]

8.6 **Strong and weak interaction lifetimes: the ρ^0 and K^0 decays.** The ρ^0 and K^0 mesons decay mainly in $\pi^+\pi^-$. Explain why the ρ^0 lifetime is of the order of 10^{-23} s and that of the K^0 of the order of 10^{-10} s. Draw the Feynman diagrams for both decays.
[See solutions]

8.7 **Pion decay branching ratios.** Calculate the ratio of the phase space factors for the following decays:

$$\pi^- \to e^- \overline{\nu}_e$$
$$\pi^- \to \mu^- \overline{\nu}_\mu$$

Estimate the two lifetimes and compare them with the experimental values. Determine if the results are consistent with the hypothesis that the lifetime is purely determined by the phase space factor.
[Solution: see §8.10]

8.8 **Strange and charmed particle decay.** Draw the Feynman diagrams, specify the couplings and comment the following decays (in bracket, the composition in terms of the valence quarks):
(a) Λ_c^+ $[cud] \to \pi^+ \Lambda^0$
(b) D^+ $[c\overline{d}] \to \pi^+ \overline{K}^0$
(c) B^+ $[\overline{b}u] \to \pi^+ \overline{D}^0$
(d) $\overline{B}_s^0$ $[b\overline{s}] \to \pi^- D_s^+$
(e) Λ_b^0 $[bud] \to \pi^- \Lambda_c^+$.
[See solutions]

8.9 **Non-leptonic D^0 decays.** Draw the Feynman diagrams of the following D^0 meson decays and estimate the relative amplitudes:
a) $D^0 \to K^- \pi^+$
b) $D^0 \to \pi^- \pi^+$
c) $D^0 \to K^+ \pi^-$
[See solutions]

8.10 **Suppression of $\Delta S = 1$ NC interaction.** It is experimentally found that the NC/CC ratio for the charged K decays is:

$$(K^+ \to \pi^+ \nu \overline{\nu})/(K^+ \to \pi^0 \mu^+ \nu_\mu) < 10^{-8}$$

This is one of the experimental evidence that the weak neutral current decays with $\Delta S = 1$ change in strangeness are suppressed.
(a) Draw the Feynman diagrams of the two decays.
(b) Check that, assuming the existence of the c quark, the transition probabilities induced by a $\Delta S = 1$ neutral current is vanishing.
[See solutions]

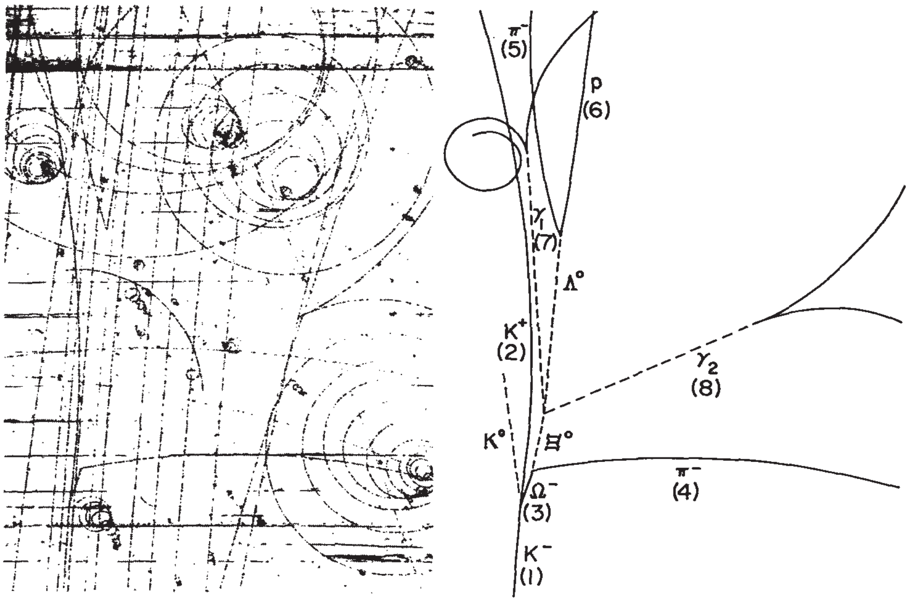

Fig. 8.1. Bubble chamber trace of the first observed Ω^- baryon [8B64]. The event was discovered in 1964 by a team of physicists from the Brookhaven National Laboratory, the University of Rochester and Syracuse University, led by N. Samios of Brookhaven, using the 80-inch bubble chamber

8.11 **The Ω^- decay products.** Fig. 8.1 shows the bubble chamber picture of the first produced Ω^- event. An incoming K^- meson interacts with a proton in the liquid hydrogen of the bubble chamber and produces an Ω^-, a K^0 and a K^+ meson. All these unstable hadrons decay into other particles. To have an idea of the scale of the picture, the length of the Ξ^0 track is 3 cm.
(a) Show that the reaction $K^- p \to \Omega^- K^0 K^+$ is the minimal reaction for the production of a Ω^-;
(b) Evaluate the minimum momentum of the K^- in the laboratory system in order to produce the Ω^-. The K^- of the experimental beam had a momentum of 5 GeV/c.
(c) Discuss the decay of the Ω^- presented in the picture.
[See solutions]

8.12 **Feynman diagrams and couplings.** Draw the Feynman diagrams, specify the couplings and comment the following decays:
(a) $\Lambda^0 \to p e^- \overline{\nu}_e$
(b) $\Xi^- \to \Lambda^0 \pi^-$
(c) $\nu_\mu p \to \mu^- \Delta^{++}$
(d) $D^0 \to K^- \mu^+ \nu_\mu$

(e) $D^0 \to K^+\mu^-\overline{\nu}_\mu$.
[See solutions]

8.13 **Muon and tau decay.** The muon is a charged particle whose mass is 105 MeV/c^2 and lifetime $\sim 2\ 10^{-6}$ s. It decays into an electron (m$_e$=0.5 MeV/c^2), a neutrino and an antineutrino.
(a) The electron is the only observable particle in the muon decay. Explain the characteristic of the electron spectrum necessary to demonstrate that the muon decay at rest is not a two-body decay.
(b) In the case of muon decaying at rest, calculate the maximum electron energy if three particles are present in the final state.
(c) The τ (m$_\tau$=1777 MeV/c^2) lepton decays with the same characteristic of the muon. Estimate the τ lifetime.

8.14 **The CKM matrix.** Using the CKM matrix (data from [P10]):

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347 \pm 0.00016 \\ 0.2252 \pm 0.0007 & 0.97345 \pm 0.00015 & 0.041 \pm 0.001 \\ 0.0086 \pm 0.0003 & 0.040 \pm 0.001 & 0.99915 \pm 0.00005 \end{pmatrix}$$

calculate the decay fraction (or branching ratios, BR) of the W boson decays into all possible quark-antiquark and lepton-antilepton pairs. Remember that the sum of all the BRs must be equal to 1. For the hadronic decays, the color factor $N_c = 3$ must be used.
[See solutions]

8.15 **Bilinear forms.** Regarding to the bilinear forms defined in §8.16.1, prove that:
(a) The scalar $\overline{\psi}\psi$ is a relativistic invariant quantity.
(b) The four-vector current $\overline{\psi}\gamma^\mu\psi$ is a relativistic invariant quantity.
[See solutions]

8.16 **Phase space in neutron decay.** Demonstrate that in the case of the neutron decay, and neglecting the mass of the electron in the final state, the integral of the phase space Eq. (8.9) gives:

$$\int_0^{E_0/c} p_e^2 (E_0 - E_e)^2 dp_e = \frac{E_0^5}{30c^3} \ . \tag{8.16}$$

[See solutions]

8.17 **Tau decay branching ratios.** The decay fractions of the τ^- decay are reported below (data from [P10]):

Particle	Mass (MeV)	Mean life $\times 10^{-15}$(s)	Decay Mode	Decay Fraction (Γ_i/Γ)
τ^-	1776.82 ± 0.16	(290.6 ± 1.0)	$\mu^-\overline{\nu}_\mu\nu_\tau$	$(17.36 \pm 0.05)\%$
			$e^-\overline{\nu}_e\nu_\tau$	$(17.85 \pm 0.05)\%$
			$h^-\nu_\tau$	$(11.61 \pm 0.06)\%$
			$\pi^-\nu_\tau$	$(10.91 \pm 0.07)\%$
			$K^-\nu_\tau$	$(6.96 \pm 0.23) \times 10^{-3}$
			$h^- \geq 1$neutral ν_τ	$(37.06 \pm 0.10)\%$

(a) Explain why the branching ratios for $\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau$ is almost equal to that of $\tau^- \to e^-\overline{\nu}_\mu\nu_\tau$.

(b) Evaluate the expected ratio between decay fractions

$$\frac{\Gamma(\tau^- \to hadrons\ \nu_\tau)}{\Gamma(\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau)} \tag{8.17}$$

and compare the prediction to the measured value.

(c) Compare the measured ratio between decay fractions:

$$\frac{\Gamma(\tau^- \to K^-\nu_\tau)}{\Gamma(\tau^- \to \pi^-\nu_\tau)} \tag{8.18}$$

with the expected one.
[See solutions]

8.18 $\Delta S = 1$ **K decay.** Draw a possible Feynman diagram for the $K^0 \to \mu^+\mu^-$ decay. What can we conclude from the fact that the measured branching ratio of this reaction is $< 10^{-7}$?
[See solutions]

8.19 **WI decay with spectator quarks.** Show that in the hadron model in which a quark undergoes a decay and the others act as "spectators", the lifetimes of the $D^+(c\overline{d})$, $D^0(c\overline{u})$ and $D_s^+(c\overline{s})$ mesons are almost equal.
[See solutions]

Supplement 8.1: Signals, data transmission and electronics

---

9. Discoveries in electron–positron collisions

9.1 **The strong coupling constant.** Using the data shown in Fig. 9.4 and the potential given in Eq. (9.16), estimate the value of the strong coupling constant α_s at the energy of charmonium formation. Assume that $m_c c^2 = 1550$ MeV.
[See solutions]

9.2 **Event rate in e^+e^- collider.** In a small electron-positron collider of $R = 10$ m radius, each beam has a current intensity of $I = 10$ mA and a transverse area of $S = 0.1$ cm^2.
(a) Assuming that each of the two e^- and e^+ beams is contained in one single bunch and that the beams collide head-on twice per revolution, calculate the collider luminosity in cm^{-2} s^{-1}.
(b) The cross-section for the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ at the peak of the ω resonance is $\sigma = 1.5$ μb. Calculate the number of observed events per hour for this process.
[See solutions]

9.3 **Beam attenuation in the beam pipe.** The vacuum in the pipe of an electron accelerator has a pressure of $p = 3 \cdot 10^{-4}$ tor. The electron beam corresponds to an average current of $I = 60$ mA. If a one meter long (l =1 m) section is considered, assuming that the residual gas inside the vacuum pipe is made of hydrogen atoms and knowing that the $e^-p \rightarrow e^-p$ cross-section is $\sigma = 1$ μb, calculate the number of beam-gas collision events expected in one second.
[Suggestion: see Problem 3.16]

9.4 **Design LEP luminosity.** Determine the theoretical luminosity (see §3.3) of the LEP accelerator using the following parameters: $n_p = n_a = 4$, number of circulating particle/antiparticle bunches; $f = 11240$ Hz, revolution frequency; $I = 1$ mA, intensity of the circulating current per beam; $\sigma_h = 300$ μm, beam width; $\sigma_v = 8$ μm beam height.
[See solutions]

9.5 **LEP luminosity from the forward detector.** Determine the LEP luminosity at $\sqrt{s} = 91$ GeV using the experimental information of a forward detector able to measure elastic $e^+e^- \rightarrow e^+e^-$ collisions. The forward detector covers the solid angle: 40 mrad$< \theta <$150 mrad, $0 < \varphi < 2\pi$, and the measured $e^+e^- \rightarrow e^+e^-$ event rate is $R = 0.20$ s^{-1}.
[A: $\frac{(8\pi\alpha^2\hbar^2c^2)}{E^2}(\frac{1}{1-\cos\theta_{min}} - \frac{1}{1-\cos\theta_{max}}) = 3 \cdot 10^{30}$ cm^{-2}s^{-1}]

9.6 **Radiative return to the Z^0.** In the case of an e^+e^- collider, the effective c.m. energy $\sqrt{s}'$ is smaller than the initial one, $\sqrt{s}$, if a photon is emitted from the primary positron or electron. Assuming that the photon energy is E_γ, calculate s'. How does an event with two hadronic jets in the final state look like in this case?
[See solutions]

9.7 J/ψ **resonance.** The intrinsic width of the J/ψ resonance is smaller than the experimental resolution (which was about 2 MeV in the first experiments). It can be indirectly obtained from quantities that do not depend on the resolution. Consider the $e^+e^- \to J/\psi \to e^+e^-$ reaction. The measured cross-section integrated over the resonance is $\int_{resonance} \sigma_{e^+e^-} d\sqrt{s} \simeq$ 790 nb MeV. Derive the value of the J/ψ intrinsic width ($\Gamma_{J/\psi}$) knowing that the mass of the resonance is $M_{J/\psi} \simeq 3097$ MeV and that the branching ratio for the decay into e^+e^- is $BR\ (J/\psi \to e^+e^-) \simeq 0.06$.
[See solutions]

Supplement 9.1: Electronic logic and trigger

10. High energy interactions and the dynamic quark model

10.1 **De Broglie wavelength.** Using the de Broglie relation, determine the momentum p of a probe particle needed to solve the structure of:
(a) an iron nucleus (A =56);
(b) a nucleon.
(c) Estimate the particle energy needed to probe the size r_q of quarks, knowing that the experimental upper limit is $r_q < 10^{-16}$ cm.
[A: (a) 270 MeV/c; (b) 1 GeV/c ; (c)> 1.2 TeV/c]

10.2 **Electron inelastic scattering.** An electron with a $E = 20$ GeV kinetic energy collides inelastically on a proton at rest. The electron is scattered at an angle $\theta = 5°$ with respect to its original direction and with an energy $E' = 12$ GeV. Calculate the effective mass of the final hadronic system.
[See solutions]

10.3 **Structure function.** The momentum distribution of the u-type quark in the proton can be parameterized by the formula $F_u(x) \simeq xu(x) = a(1-x)^3$. Determine the constant a with the assumption that the u quarks carry 33% of the proton momentum.
[See solutions]

10.4 **Quark distribution.** The distributions of u quarks in the proton and of $\overline{d}$ antiquark in the antiproton can be assumed to be represented by the functions: $F_u(x) = xu(x) = a_1(1-x)^3$, $F_{\overline{d}}(x) = x\overline{d}(x) = a_2(1-x)^3$, where x is the Bjorken variable, i.e., the fraction of the nucleon momentum carried by quarks. Assuming that the quarks contribute to half of the nucleon momentum, calculate the constant a_1 and a_2.
[A: $a_1 = 4/3$, $a_2 = 2/3$].

10.5 **Gluon structure function.** It is believed that the structure function describing the distribution of the gluon momentum inside the nucleons, $g(x)$, strongly increases with decreasing x. Estimate the number of gluons that would be possible to resolve with deep inelastic $e + p \to e + X$ collisions at $Q^2 = 10^4$ GeV^2 at low x values (in the intervals $0.0001 \div 0.001,\ 0.001 \div 0.01,\ 0.01 \div 0.1$). Assume that at these Q^2 values the distribution function of the gluons is $g(x) = 0.36\ x^{-0.5}$.
[See solutions]

10.6 **$W^{\pm}, Z^0$ production at the S$p\overline{p}$S CERN collider.** The UA1 and UA2 experiments at the CERN S$p\overline{p}$S collider led to the discovery of

the $W^{\pm}, Z^0$ vector bosons of the weak interaction. At the CERN S$p\overline{p}$S collider, protons and antiprotons were made to collide with a total c.m. energy of 540 GeV (later, 630 GeV).
Discuss in terms of quark interactions at which energy the $W^{\pm}, Z^0$ production is obtained.
[See solutions]

10.7 **Neutrino/antineutrino cross-section ratio.** The ratio $R = \sigma_{\overline{\nu}}/\sigma_{\nu}$ between the cross-sections of neutrinos and antineutrinos colliding on an isoscalar target (e.g., a target with an equal number of protons and neutrons) is $R \simeq 0.5$ (see Fig. 10.12). This experimental result cannot be easily accounted for in the framework of the static quark model of hadrons, since it requires that a small fraction of the nucleon momentum be carried by antiquarks. In the dynamic quark model, many sea quark-antiquark pairs are present inside the nucleon (see Fig. 7.21).
Using the measured value of R, and taking into account the coupling given in Eq. (10.57) of neutrinos and antineutrinos with quarks and antiquarks, determine the ratio between the fraction of the momentum carried by quarks and antiquarks in the nucleon.
[See solutions]

10.8 **Quark and antiquark content of ordinary matter.** Using the integral of the quark and antiquark distribution functions given in Eq. (10.67), determine the ratio between the fraction of the momentum carried by quarks and antiquarks in isoscalar nuclei. Compare with the result obtained in the previous problem.
[See solutions]

10.9 **Neutrino beams-1.** Muon neutrino beams are extensively used in many experimental situations, for instance in deep inelastic scattering as well as in long and short baseline neutrino oscillation experiments (Chapter 12).
To create a muon neutrino beam, the decays of π^+, K^+ mesons are used. A high energy proton interacts with a target nucleon, producing many charged and neutral particles. A magnetic system (the *horn*) selects π^+, K^+ with a defined momentum (narrow-band neutrino beam) or with a large momentum range (broad-band neutrino beam).

In a narrow-band neutrino beam, π^+, K^+ are selected with momentum p =200 GeV/c. The mesons are driven in a 1 km long vacuum tube, where they can decay. Determine:
(a) the π^+, K^+ mean free path;
(b) the fraction of π^+, K^+ decays at the end of the vacuum tube;

(c) the maximum energy in the laboratory system of the neutrinos produced in the π^+ and K^+ decays.
(d) Discuss the narrow-band neutrino beam flux shown in Fig. 8.10.
[See solutions]

10.10 **Neutrino beams-2. Contaminations.** Suppose that only positively charged particles are collected by the magnetic device (*horn*). The main source of neutrinos is the decay $\pi^+ \to \mu^+\nu_\mu$. In this muon neutrino beam, there is an irreducible background of ν_e and $\overline{\nu}_\mu$.
(a) Write the processes which produce ν_e and $\overline{\nu}_\mu$ and give a simple estimate of their relative number with respect to the ν_μ as a function of the length L of the secondary particles decay tunnel.
(b) Evaluate the ν_e, $\overline{\nu}_\mu$ contamination for long baseline experiments (Chapter 12), assuming $L = 1$ km, $E_\mu = 10$ GeV.
[See solutions]

10.11 **Neutrino beams-3. Off-axis beams.** Assume that a neutrino beam is produced by the decay of high energy pions, with $E_\pi \gg m_\pi$, and that $E_\nu \gg m_\pi$.
(a) What is the characteristic angle θ_C of the decay neutrinos with respect to the direction of the pion in the lab. frame, when the neutrino is emitted at $\theta^* = 90°$ in the pion rest frame?
(b) What is the maximum angle $\theta_{max}(E_\nu)$ between the neutrino and the direction of its parent pion, which can assume any energy?
(c) What is the maximum energy $E_\nu(\theta)$ at which a neutrino can be produced in the decay of a pion if it appears at a given angle θ with respect to the pion direction?
Discuss the consequences of the existence of a maximum energy $E_\nu(\theta)$.
[See solutions]

Supplement 10.1: The computing effort at the LHC collider

---

11. The Standard Model of the Microcosm

11.1 **Lagrangian density for the Klein-Gordon equation.** In a quantum field theory invariant under Lorentz transformations, the fields ϕ_i are functions of the space-time coordinates x_μ and the Lagrange equation (6.1) is written in terms of the Lagrangian density $\mathcal{L}(\phi_i, \partial_\mu \phi_i)$:

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} = 0 \tag{8.19}$$

such that $L = \int \mathcal{L} d^3x$. Show that, for a scalar field ϕ describing spinless particles of mass m and Lagrangian density:

$$\mathcal{L}_\phi = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 \ , \tag{8.20}$$

the Klein-Gordon equation (4.13) is obtained.
[See solutions]

11.2 **Lagrangian density for the Dirac equation.** Similarly to the previous problem, show that for a field ψ describing spin 1/2 fermions with a Lagrangian density:

$$\mathcal{L}_\psi = i\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi \ , \tag{8.21}$$

the Dirac equation (4.16) is obtained.
[See solutions]

11.3 Z^0 **partial width.** Calculate the value predicted by the electroweak model for the partial width Γ_{ν_e} for the decay of a Z^0 boson in a $\nu_e, \overline{\nu}_e$ pair at the energy corresponding to the peak of the Z^0 resonance.
[See solutions]

11.4 **The running α_S.** Using Eq. (11.90):

$$\alpha_S(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)} \tag{8.22}$$

compute $\alpha_S(Q^2)$ for (a) $Q = 8$ GeV, (b) $Q = 90$ GeV and (c) $Q = 500$ GeV. For each energy, the number N_f of active flavors must be considered. Assume $\Lambda_{QCD} = 200$ MeV (actually Λ_{QCD} varies discontinuously when it exceeds the threshold corresponding to the mass of a new quark).
[See solutions]

11.5 α_s **at the charm threshold.** Compare the result of the calculation of the α_S constant obtained using Eq. (8.22) at energies corresponding to the *charmonium* production, with the result obtained in problem 9.1.
[See solutions]

11.6 **Non-abelian theory.** Explain the meaning of a non-abelian theory from a physical point of view.
[See solutions]

11.7 **SU(2) and U(1) symmetry groups.** Explain the meaning of the SU(2) and U(1) groups. Say whether they are or not abelian groups (for the definition of abelian group, see the solution of the previous problem).
[See solutions]

12. CP–violation and particle oscillations

12.1 **Neutron-antineutron mixing.** Neutron and antineutron are each the antiparticle of the other; they are neutral, just as the K^0 and $\overline{K^0}$. Explain why a mixing between K^0 and $\overline{K^0}$ can occur, while the mixing between neutron and antineutron is forbidden?
[See solutions]

12.2 $K^0, \overline{K}^0$ **mixing-1.** Describe how to produce a pure K^0 meson beam. The initial K^0 beam evolves during propagation in a mixed K^0 and $\overline{K^0}$ state (see §12.2). The mass difference between the two mass eigenstates K_1^0 and K_2^0 is $\Delta m = m_2 - m_1 \simeq 1/\tau_1$, with $\tau_1 = 90 \times 10^{-12}$ s.
(a) Evaluate Δm;
(b) Write the K^0 and $\overline{K^0}$ intensities as a function of proper time;
(c) Draw a graph of the intensity of the K^0 and $\overline{K^0}$ beams as a function of the proper time τ_1.
[A: See §12.2.1]

12.3 $K^0, \overline{K}^0$ **mixing-2.** A K^0 beam propagating in vacuum can decay. At a distance d corresponding to 20 times the K_1 lifetime ($d = 20c\tau_{K_1}$) there is a target that absorbs 10% of the incoming K^0 beam. If the interaction cross-section for $\overline{K^0}$ is three times larger than that of the K^0, calculate the relative amplitudes of K_1 and K_2 in the beam:
(a) At $t = 0$;
(b) Immediately before the target;
(c) Immediately after the target.
Assume low-energy kaons, and neglect relativistic effects.
[See solutions]

12.4 ϵ, ϵ' **CP-violation parameters.** The quantities η_{+-}, η_{00}, defined in Eqs. (12.16) and (12.17), are related to the CP violation parameters ε and ε' through the relations:

$$\eta_{+-} = |\eta_{+-}|e^{i\varphi_{+-}} = \varepsilon + \varepsilon' \qquad (8.23)$$

$$\eta_{00} = |\eta_{00}|e^{i\varphi_{00}} = \varepsilon - 2\varepsilon' \qquad (8.24)$$

Demonstrate that the ratio ε'/ε can be determined by measuring the double ratio R:

$$R = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} \Big/ \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \simeq 1 - 6\frac{\varepsilon'}{\varepsilon} \ . \qquad (8.25)$$

[See solutions]

12.5 K_L **semi-leptonic decay.** Demonstrate that in the K_L semi-leptonic decays, the asymmetry:

$$A_L = \frac{\Gamma(K_L \to \pi^- \ell^+ \nu_l) - \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_l)}{\Gamma(K_L \to \pi^- \ell^+ \nu_\ell) + \Gamma(K_L \to \pi^+ \ell^- \overline{\nu}_\ell)} \,. \tag{8.26}$$

(ℓ indicates the muon or the electron) is related to the CP violation parameter ε through the relation:

$$A_L \simeq 2Re(\varepsilon) = (3.32 \pm 0.06) \cdot 10^{-3} \tag{8.27}$$

Show that the experimental result of (8.27) is consistent with that obtained in Eq. (12.23-12.24) for non-leptonic decays.
[See solutions]

12.6 B^0 **meson tagging.** The B^0 meson *tagging*, as shown in Fig. 12.8, relies on the presence of a μ^+ amongst the decay products. The branching ratio for the decay into a positive charged lepton is $BR(B^0 \to \mu^+\nu_\mu + anything) = 10.3\%$. The decay into $e^+\nu_e$ has the same BR.
Explain the reason why a positive charged lepton is expected from the B^0 decay with the quoted BR.
[See solutions]

12.7 **Solar neutrino detection.** An experiment devoted to detect neutrinos produced by the Sun is carried out in a mine using the reaction

$$\nu_e + {}^{37}Cl \to {}^{37}Ar + e^- \tag{8.28}$$

The detector contains v $\simeq 4 \cdot 10^5$ liters of tetra-chloroethylene (C_2Cl_4). Estimate the number of ^{37}Ar atoms that would be produced per day using the following assumptions:

1. The solar luminosity is (see Appendix A5) $L_\odot = 3.84\ 10^{26}$ W;
2. 7% of the Sun thermonuclear energy is emitted as neutrinos of average energy $\langle E\rangle \simeq 1$ MeV;
3. Only 0.1% of these neutrinos have enough energy to trigger the ($\nu_e +$ Cl) reaction;
4. The interaction cross-section, for neutrino able to induce the reaction on the ^{37}Cl nucleus, is $\sigma = 10^{-43}\text{cm}^2$;
5. The isotopic abundance of ^{37}Cl is 25%;
6. The density of C_2Cl_4 is $\rho = 1.5$ g cm^{-3} and its molecular weight is $P = 164$ g mole^{-1}.

[See solutions]

12.8 **Atmospheric neutrino oscillation-1.** Using the two flavor oscillation formula (12.49), determine the value of the muon neutrino energy E_ν

which gives a 100% disappearance probability for:
(a) neutrinos crossing the Earth atmosphere ($L \sim 20$ km);
(b) neutrinos crossing the Earth diameter ($L \sim 13000$ km).
Comment the result. Use the best fit value of Δm^2 and $\sin^2 2\theta$.
[A: (a) 40 MeV, below the muon production threshold; (b) 23 GeV]

12.9 **Atmospheric neutrino oscillation-2.** The MACRO [12A98] experiment observed atmospheric muon neutrino from below. The neutrinos, interacting below the detector, produce upgoing muons whose zenith direction θ was measured. The zenith angle is correlated with the neutrino path length L through the relation:

$$L = R_T \cos\theta + \sqrt{(R_T \cos\theta)^2 + 2R_T h + h^2} \qquad (8.29)$$

where R_T= 6370 km is the Earth radius and $h \simeq 20$ km the atmospheric depth.
(a) Draw the oscillation probability for $\nu_\mu \to \nu_\tau$ as a function of $\cos\theta$ for $E_\nu = 20, 50, 100, 200$ GeV;
(b) Assuming that the MACRO signal from the vertical direction is induced by ν_μ with energy between 5 and 100 GeV, evaluate the survival probability $P(\nu_\mu \to \nu_\mu)$
[See solutions]

12.10 **Atmospheric neutrino oscillation-3.** Atmospheric neutrinos contain both $\nu_\mu, \overline{\nu}_\mu$ from $\pi^\pm, K^\pm$ decays. Evaluate the ratio $R = (\nu_e + \overline{\nu}_e)/(\nu_\mu + \overline{\nu}_\mu)$ for atmospheric neutrinos assuming $L = 20$ km and for $E_\mu = 0.2, 20, 200$ GeV.
The measurement of an anomaly in the ratio R at low energy was, at the end of the 1980s, the first indication of an anomaly in the atmospheric neutrinos, which led to the discovery of the neutrino oscillations [10K04].
[See solutions]

12.11 **Neutrino oscillation matrix**. Show that the 3×3 unitary matrix (12.51) representing the mixing between flavor and mass eigenstates can be parameterized as:

$$U_{fj} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (8.30)$$

Remember that the symbol f represents one of the *flavor eigenstate* ($f = e, \mu, \tau$). The symbol j stands for one of the *mass eigenstate* ($j = 1, 2, 3$). Thus, e.g., the matrix element:

$$U_{\mu 3} = s_{23}c_{13} = \sin_{23}\cos_{13}$$

12.12 **Measurement of the small θ_{13} mixing angle**.
(a) Using the neutrino mixing matrix given in (12.51), show that for long baseline experiments (as well as for the atmospheric neutrinos), the contribution from Δm^2_{12} terms can be neglected and that the oscillation probabilities can be expressed in a simplified manner as:

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Phi_{23} \quad (8.31a)$$
$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \Phi_{23} \quad (8.31b)$$
$$\begin{aligned} P(\nu_\mu \to \nu_\mu) &= 1 - P(\nu_\mu \to \nu_e) - P(\nu_\mu \to \nu_\tau) \quad (8.31c) \\ &= 1 - \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \Phi_{23} \\ &\quad - \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2 \Phi_{23} \end{aligned}$$
$$P(\nu_e \to \nu_\tau) = \sin^2 2\theta_{13} \cos^2 \theta_{23} \sin^2 \Phi_{23} \quad (8.31d)$$
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2 \Phi_{23} \quad (8.31e)$$

where $\Phi_{23} = \Delta m^2_{23} L/4E_\nu \simeq \Delta m^2_{13} L/4E_\nu$.
(b) Describe the best way to experimentally measure θ_{13} with present technology.
[See solutions]

Supplement 12.1: Analogy for the neutrino mixing

Supplement 12.2: Dirac or Majorana neutrinos: the double β decay

13. Microcosm and Macrocosm

13.1 **Neutrinos from neutron decay.** It is believed that cosmic rays are accelerated through an iterative mechanism (whose theoretical model is due to E. Fermi), which consists of a sequence of collisions of charged particles with the shock wave produced by the explosion of a supernova. In each collision, a particle gains a small amount of energy. Due to the deflection caused by magnetic fields, charged particles have a low probability to escape the acceleration region. The situation is different for neutral particles such as neutrons; they are not subjected to magnetic fields and can elude the acceleration region.
(a) Give a process in which neutrons can be produced.
(b) What is the minimum energy necessary for a neutron to escape the acceleration region with a probability $\geq 1/e$ ($\tau_n = 887$ s)? Assume that the region size is the order of one light year.
(c) What would be the maximum value of the angle formed by the final state electron (or neutrino) with the flight direction of the neutron?
[See solutions]

13.2 **Search for the proton decay.** The Kamiokande detector had a fiducial volume of 1000 tons of water. Calculate the number of protons in the detector fiducial volume. If the proton has a lifetime of 10^{32} years, how many protons would decay in 1000 tons of water each year?
[See solutions]

13.3 **Indirect estimate of the proton lifetime.** Geophysical measurements show that the Earth emits approximately 40 TeraWatt of energy (see also Problem 14.14). Assuming that all this energy is due to the proton decay process, whose rest mass transforms into heat, estimate the proton lifetime.
[See solutions]

13.4 **Search for massive exotic particles in Cosmic Rays.** New exotic heavy particles X (e.g., with mass $m_X = 100$ GeV) can be searched for studying the arrival time of hadrons in showers produced in high energy cosmic ray collisions with atmospheric nuclei (see Supplement 1.1). Assuming that these particles travel 2 km before decaying, calculate:
(a) the threshold energy to produce a pair of X particles;
(b) the energy of a particle in the laboratory system for a X produced at rest in the c.m. system;
(c) the time delay of the massive X particle with respect to the lighter hadrons in the showers, moving at the light speed c;

(d) the time delay if the particle has mass $m_X = 1$ TeV.
[See solutions]

13.5 **Gravitational binding energy.**
(a) Show that the gravitational potential energy of a spherical mass M with uniform density and radius R is $V_G = -3G_N M^2/5R$.
(b) Calculate the gravitational potential energy of a mass of material equivalent to one solar mass with uniform density and with a radius R equal to
1. 1 light-year (*protostar*)
2. 1 solar radius (*star*)
3. 1000 km (*white dwarf*)
4. 10 km (*neutron star*).

[See solutions]

13.6 **Supernovae and neutrinos.** The 1987A supernova (SN) was located in the Large Magellanic Cloud at around 170000 light-years from Earth. During a SN event, a neutrino burst lasting $\sim$ 2 s is expected. About 10 neutrino interactions from the SN1987A in 1000 t of water were observed in a time interval Δt of less than 10 seconds (assume the value $\Delta t = 3$ s in the calculation). The average energy of supernova neutrinos is about 12 MeV (the energy range is between $E_{min} = 5$ MeV and $E_{max} = 20$ MeV). If massive, the neutrinos with higher energy are expected to arrive on Earth before the lower energy neutrinos. Estimate:
(a) an upper limit for the neutrino mass from the measured neutrino arrival time interval Δt;
(b) the total energy released in neutrinos by the SN;
(c) the gravitational binding energy $3G_N M^2/5R$ of the star, taking into account that the neutron star formed in the explosion has mass $1.4 M_\odot$ and radius R = 10 km.
[See solutions]

13.7 **Propagation of protons in the cosmic microwave background radiation: the GZK effect.** The Universe is filled with a cosmic microwave background radiation (CMBR) with a black body spectrum and an average temperature of ∼2.7 K ($\overline{E} = 10^{-3}$ eV). The average density of CMBR photons is ρ_γ=400 cm^{-3}. The high energy cosmic ray protons, originated from extragalactic sources, can interact with the microwave cosmic background radiation through the resonant reaction:

$$\begin{aligned} p^+\gamma \to \Delta^+ &\to \pi^+ n \\ &\to \pi^0 p \end{aligned}$$

(a) Find the threshold energy that protons must have to induce this reaction.

(b) Calculate the average distance traveled by a proton before undergoing the resonant reaction, knowing that the cross-section of the process is $\sigma_{p\gamma} = 250\mu$b.
This process limits the size of the Universe from which ultra-high energy protons can arrive. This is the so-called Greisen-Zatsepin-Kuzmin (GZK) cut-off named after the physicists that first hypothesized it.
[See solutions]

13.8 **γ-rays attenuation in the CMBR.** High energy gamma-rays can interact with lower-energy photons through the process $\gamma + \gamma \to e^+ + e^-$. This process has a cross-section $\sigma = (8\pi/9)r_e^2$ where $r_e = 2.8 \times 10^{-15}$ m is the classical electron radius.
(a) Find the threshold energy of the high energy gamma-rays so that the reaction can take place through the interaction with
1. the cosmic microwave background radiation;
2. infrared photons (~ 0.1 eV);
3. optical photons (~ 2 eV).
(b) For these three cases, calculate the average distance travelled by the high energy gammas before being converted. Compare the results with the size of the Universe.
[Hint: use the solution of Problem 13.7]

13.9 **Neutrino telescopes.** A neutrino telescope (NT) detects secondary particles produced in neutrino interactions as $\nu_\mu N \to \mu X$ (see Supplement 13.1). The neutrino-induced muons travel in a volume of 1 km^3 of ice or water, where a number N_{pmt} of photomultipliers are plunged. Assume that:
1. The muon track length is 1 km;
2. A muon emits 350 Cherenkov photons per cm of water (§2.11);
3. The PMT have a 10" diameter (1 inch = 2.54 cm) and a quantum efficiency $\epsilon_{pmt} \simeq 0.25$;
4. The PMTs are inside optical module, with light collection transparency of 80%;
5. The water absorption length is $\lambda_{abs} = 50$ m in the 400-500 nm range (100 m for the ice);
6. The number of detected photoelectrons necessary to reconstruct a muon track is $N_{p.e.} = 100$.

Estimate the number of optical sensor N_{pmt} needed to track a muon.
This number is one of the major impact factors on the cost of an experiment.
[See solutions]

13.10 **Cosmic Rays in galactic magnetic field.** The Galaxy is filled with magnetic fields with average modulus of $B = 3\ \mu$G and directions coher-

ent for a length of $1 \div 10$ pc. Evaluate the curvature radius of a cosmic ray proton with $10^{12}, 10^{15}$ and 10^{18} eV in the galactic magnetic field. Compare the curvature radius with the Galaxy dimensions (radius ∼15 kpc, thickness ∼200 pc).
[Hint: see Problem 3.2. A: 3 10^{-4}, 0.3, 300 pc.]

13.11 **Cosmic accelerators of Cosmic Rays.** Using dimensional arguments, estimate the maximum energy of a charged particle (with charge Ze) accelerated in a strongly varying magnetic field. Apply the relation to the case of a neutron star rapidly rotating around its axis (*pulsar*) with mass $M = 1.4 M_\odot$ and magnetic field $B = 10^7$ T. Explain how such magnetic field can be generated and estimate the pulsar angular velocity.
[See solutions]

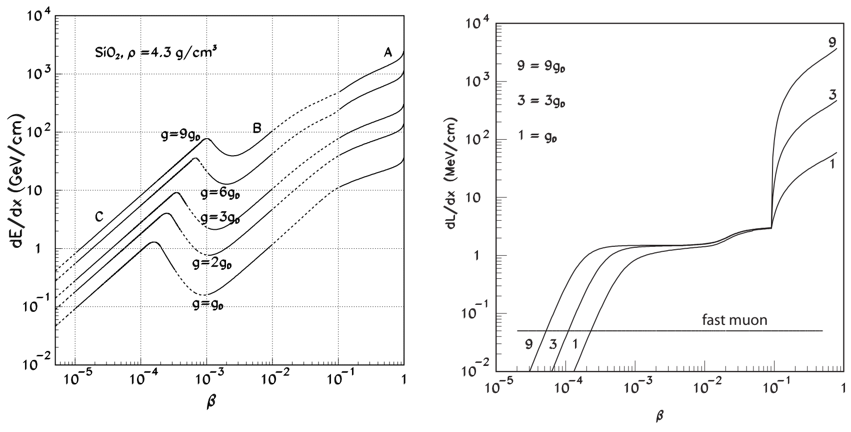

Fig. 13.1. (a) Energy loss of magnetic monopoles with magnetic charge $g = 1, 2, 3, 6, 9g_D$ as a function of $\beta = v/c$ in Si (density $\rho = 4.3$ g cm^{-3}; this is a relatively high density, similar to that present inside the earth). (b) Energy loss in form of visible light dL/dx (in MeV/cm) produced by a magnetic monopole with magnetic charge $g = g_D,\ 3g_D,\ 9g_D$ and for a relativistic muon in a scintillator as a function of β

13.12 **Search for magnetic monopoles.** A muon with energy $E = 10$ GeV and a (hypothetical) magnetic monopole (MM) with magnetic charge $g = 68.5e = g_D$ and speed $v_1 = 0.01c$ cross a 25 cm thick layer of liquid scintillator (for instance that of the MACRO experiment at Gran Sasso laboratory, with density $\rho = 0.85$ g cm^{-3}). Evaluate for the muon and for the (hypothetical) MM:

(a) The total energy lost in the liquid scintillator;
(b) The energy lost in the scintillator that produces light;
(c) The total energy lost for a MM with $v_2 = 0.3c$. Show that MM with $\beta > 0.1$ behave as particles with an equivalent electric charge $Ze = g\beta$.
Refer to the energy loss given in Fig. 2.2b) for charged particles and that given in Fig. 13.1 for particles with magnetic charge.
[See solutions]

Supplement 13.1: Cosmic accelerator

---

14. Fundamental aspects of nucleon interactions

14.1 **Half-life and lifetime.** Derive the value of the half-life $t_{1/2}$ of a radioactive element when its lifetime τ is known.
[A. $t_{1/2} = \tau \ln 2$]

14.2 **Hydrogen isotopes.** The natural hydrogen is a mixture of two stable isotopes, hydrogen and deuterium. The deuterium nucleus has a binding energy of 2.23 MeV. The atomic mass of the natural hydrogen is 940.19 MeV. Calculate the relative abundance of the two isotopes in the natural hydrogen.
[See solutions]

14.3 **Nuclear fusion.** The deuterium nucleus ${}^2_1H$ has a binding energy of 2.23 MeV. The tritium nucleus ${}^3_1H$ has a binding energy of 8.48 MeV. Calculate the energy necessary to bring two deuterium nuclei to the distance of $r = 1.4\ 10^{-13}$ cm. Estimate the corresponding temperature. These conditions are necessary to activate the fusion reaction:

$${}^2_1H +{}^2_1 H \rightarrow{}^3_1 H + X$$

Indicate which particle X is produced in the final state and calculate the energy released in the fusion reaction.
[See solutions]

14.4 **Probability in radioactive decays.** The probability of a radioactive atom to decay in 1 second is equal to 5×10^{-11}. What is the probability that 5 decays take place in 1 second in a statistical sample made of 9×10^{10} atoms? And what is the probability for 15 decays?
[See solutions]

14.5 **Radioactivity.** The α-decay of the radium isotope (${}^{226}_{88}Ra$) has a half-life $t_{1/2} = 1602$ years. The unit of activity (1 Curie =1 Ci) is defined as the number of disintegrations per second of a gram of radium.
(a) Write the decay reaction.
(b) Calculate the number of nuclei in one gram of ${}^{226}_{88}Ra$.
(c) Calculate the number of disintegrations per second corresponding to the activity of 1 Ci.
[See solutions]

14.6 **Carbon isotopes.** The natural carbon contains 98.89% of ${}^{12}_{6}C$ and 1.11% of ${}^{13}_{6}C$, respectively having atomic masses $M({}^{12}_{6}C) = 12.000$ u and $M({}^{13}_{6}C) = 13.003$ u.

(a) Calculate the atomic mass of the natural carbon.
(b) A living organism contains a small fraction r of ${}^{14}_{6}C$ with respect to the natural carbon. This fraction is $r = 1.3\ 10^{-12}$. The ${}^{14}_{6}C$ is a radioactive isotope which undergoes β^--decays with a half-life $t_{1/2} = 5730$ years. Calculate the activity of a gram of carbon in a living organism.
(c) The activity of a fossil of mass $m = 5.000 \pm 0.005$ g is measured. If $n = 3600$ decays are recorded in 2 hours, calculate the age of the fossil. Also estimate the error on the measurement.
[See solutions]

14.7 **Nuclear reactor.** A nuclear reactor produces 2×10^9 Watts.
(a) Calculate the number of fission events occurring per second from the reaction

$$^{235}U \rightarrow^{A1}_{Z} N \ + \ ^{A2}_{92-Z}N + neutrons + 200 \text{ MeV} .$$

(b) Estimate how many kilograms of uranium are consumed in one year knowing that that the proton mass is $m_p = 938$ MeV and that the ^{235}U binding energy is ∼8 MeV/nucleon.
[See solutions]

14.8 **Fermi momentum.** Calculate the Fermi momentum p_F and energy E_F of nucleons in the ${}^{16}_{8}O$ nucleus. Assume a spherical nucleus with radius $R = 1.25A^{1/3}$ fm. The binding energy of the nucleus is 128 MeV. Calculate the depth of the potential well in the Fermi gas model. (Neglect the mass difference between proton and neutron).
[A. $p_F = 240$ MeV/c; $E_F = 30$ MeV; $U = 38$ MeV. See §14.3.1]

14.9 **Neutron moderation in nuclear reactors.** A nuclear reactor has a graphite moderator. The carbon nuclei can be effectively considered free to recoil when hit by fast neutrons. A fast neutron (1 MeV of kinetic energy) collides elastically against a nucleus of carbon 12.
(a) What are the initial speeds of the two particles in the center-of-mass system?
(b) In the c.m. system, the direction of the velocity of the carbon nucleus changes by 135° after the collision. What are the direction and kinetic energy of the neutron in the laboratory system after the collision?
(c) How many elastic collisions are needed on average for the neutron, assuming that the angular deviations are uniformly distributed in the c.m. system, so that its energy in the laboratory system is reduced from 1 MeV to 1 keV? Assume an average energy loss as the mean between the minimum and maximum values.

14.10 **Nuclear radioactive chain.** In a nuclear radioactive chain, τ_1 is the lifetime of the parent nucleus (type 1) in the decay $nucleus_1 \rightarrow nucleus_2$.

The daughter nucleus (type 2) subsequently decays with a lifetime τ_2. Assuming that at the initial time (t =0), the daughter nuclei are absent ($N_2(0) = 0$) and $N_1(0) = N_0$, determine the condition and the time necessary in order that the two nuclear activities become equal.
[See solutions]

14.11 **Nuclear binding energy-I.** Using the Weizsacker formula (14.15) for the nuclear binding energy:

$$BE = a_0 A - a_1 A^{2/3} - a_2 \frac{Z^2}{A^{1/3}} - a_3 \frac{(A-2Z)^2}{A} \pm \frac{a_4}{A^{1/2}} \qquad (13.32)$$

calculate the binding energy of isobar nuclei with A = 27: ${}^{27}_{12}Mg$, ${}^{27}_{13}Al$, ${}^{27}_{14}Si$. Determine which is the more stable nucleus and indicate which terms of the binding energy formula make the other isotopes less stable.
[See solutions]

14.12 **Nuclear binding energy-II.** Using the Weizsacker formula (13.32) for the nuclear binding energy:
(a) Show that the ${}^{64}_{29}Cu$ nucleus can decay either through β^+ and β^-; write the decay reactions.
(b) Calculate the maximum energy for the positron and the electron in each reaction.
(c) Which decay occurs with the largest probability?
[See solutions]

14.13 **Geiger-Nuttal law.** The isotopes of thorium ${}_{90}Th$ decay by α emission (m_α = 3727.38 MeV) to radium ${}_{88}Ra$ isotopes. The measured lifetimes of three of these decays are reported in the last column of the following table. The binding energy (BE), the total angular momentum J and parity P of the nucleus before and after the decay are also indicated.

Z	A	BE (MeV)	J^P	Z	A	BE (MeV)	J^P	τ (s)
90	230	1755.22	0^+	88	226	1731.69	0^+	$3.4\ 10^{12}$
90	229	1748.43	$5/2^+$	88	225	1725.30	$3/2^+$	$3.3\ 10^{11}$
90	228	1743.19	0^+	88	224	1720.41	0^+	$8.7\ 10^{7}$

Calculate the kinetic energy, momentum and angular momentum of the emitted α particles in the above thorium decays. Compare the obtained values with the energies and lifetimes in the Geiger-Nuttal plot, Fig. 14.9. One of the decays has lifetime about two orders of magnitude larger than the extrapolation of other data. Explain qualitatively why.
[See solutions]

14.14 **Measurement of geo-neutrinos.** Geophysical measurements show that the Earth emits approximately 40 TeraWatt of energy. Models pre-

dict that approximately 40% of this energy outflow is due to the decay of radioactive nuclei, 90% of which being due to the uranium and thorium decay chains. (A ^{238}U nucleus induces a cascade of 8 α and 6 β transitions and the chain ends in the stable ^{206}Pb isotope. The ^{232}Th induces a cascade of 6 α and 4 β transitions that terminates in the ^{208}Pb isotope).
(a) Evaluate the flux on Earth surface of neutrinos emitted in the β decays of the U and Th chains (geo-neutrinos), assuming that neutrinos (with average energy of $\sim$1 MeV) carry 10% of the uranium/thorium released energy.
(b) Guess which reaction and detector are needed to detect geo-neutrinos.
[See solutions]

14.15 **Nuclear muon capture.** Explain and discuss the process of nuclear capture of a μ^- in a hydrogen nucleus $\mu^- p \to n\nu_\mu$, and in a heavier nucleus, for example aluminum ($Z = 13$, $A = 27$). Consider that the lifetime is $\tau = 2.16$ μs for a free μ^-, while it is $\tau_{\rm Al} = 0.88$ μs in aluminum. Determine the μ^- mean free path in Al.
[See solutions]

14.16 **μ-mesic atom radius.**
(a) In the framework of Bohr's atomic theory, determine the radius R_0 of a mesic atom consisting of a proton and a μ^-.
(b) For a mesic atom consisting of a nucleus and a μ^-, calculate the atomic number Z for which the nuclear radius equate the size of the mesic atom (use Fig. 14.2 for the relationship between A and Z).
[A. (a) $R_0 = 256$ fm; (b) $Z \simeq 50$]

Supplement 14.1: Nuclear collisions of Cosmic Rays during propagation in the Galaxy.

Supplement 14.2: Quantum Mechanics and Nuclear Physics → White Dwarfs and Neutron Stars

---

References

[Note] This *Problems and solutions* book refers to *Particle and Fundamental Interactions* [B11], and the bibliography reported therein should also be considered. Below there is a list of texts, reviews and some specialized works which were explicitly considered in this book. The more general bibliography is referred to within square brackets and ordered by year (the last two characters). The first character is that of the first author surname. Those more specific, used for a particular Problem or Supplement, are divided between chapters, whose number is denoted by the first characters. We frequently refer to web pages: in this case, they are indicated by the lower case character [w], followed by a progressive number. The Nobel lectures are all available at: http://nobelprize.org/nobel_prizes/physics/laureates/

[**General Bibliography**]

[L87] W. R. Leo. *Techniques for nuclear and particle physics experiments.* Springer–Verlag, Berlin (1994). ISBN: 978-3540572800

[P87] D. H. Perkins. *Introduction to High Energy Physics.* 4th edition. Cambridge University Press, (2000). ISBN: 978-0521621960

[A89] I. J. R. Aitchison, A. J. G. Hey. *Gauge theories in Particle Physics.* Third edition. Taylor& Francis, New York and Philadelphia (2002). ISBN: 978-0750308649

[G90] T. K. Gaisser. *Cosmic Rays and Particle Physics.* Cambridge University Press, Cambridge (1991). ISBN: 978-0521339315.

[H91] I. S. Hughes. *Elementary Particles.* Third edition. Cambridge University Press, (1991). ISBN: 978-0521407397

[L92] M. S. Longair. *High Energy Astrophysics.* Second edition. Cambridge University Press, Cambridge (1992). ISBN: 978-0521387736 (Vol. I); ISBN: 978-0521435840 (Vol. II)

[P95] B. Povh, K. Rith, C. Scholz, F. Zetsche. *Particles and Nuclei. An introduction to the physical concepts.* Sixth edition. Springer–Verlag, Berlin (2008). ISBN: 978-3540793670

[J99] J. D. Jackson. *Classical Electrodynamics.* Wiley, New York (1999). ISBN: 978-0471309321.

[C03] F. Ceradini. *Appunti del corso di Istituzioni di Fisica Nucleare e Subnucleare* (in Italian). http://webusers.fis.uniroma3.it/∼ceradini/efns.html.

[S04] P. Sokolsky. *Introduction to Ultrahigh Energy Cosmic Ray Physics*. Westview Press (2004). ISBN: 978-0813342122.

[P10] K. Nakamura *et al.* (Particle Data Group). *2010 Review of Particle Physics* J. Phys. G 37, 075021 (2010). Physics Lett. B667, 1 (2008). Available at: http://pdg.lbl.gov/ .

[B11] S. Braibant, G. Giacomelli , M. Spurio. *Particle and fundamental interactions.* Springer (2011). ISBN 978-9400724631.

[Chapter I]

[1F49] E. Fermi, *On the origin of cosmic rays.* Phys. Rev. 75, 1169 (1949) 15.
E. Fermi, *Galactic magnetic fields and the origin of cosmic radiation.* Astrophys. J. 119, 1-6 (1954).

[1B95] P.L. Biermann, T.K. Gaisser, T. Stanev, *Origin of galactic cosmic rays.* Phys. Rev. D 51, 3450 (1995).

[1H06] A.M. Hillas, Recent progress and some current questions. astro-ph/0607109. The figure is due to T. Gaisser.

[1K11] K. Kotera, A. V. Olinto. *The Astrophysics of Ultrahigh-Energy Cosmic Rays.* Annu. Rev. Astron. Astrophys. (2011) 49:119-53

[Chapter II]

[2G01] D.E. Groom, N.V. Mokhov, and S.I. Striganov, *Muon stopping-power and range tables: 10 MeV-100 TeV.* Atomic Data and Nuclear Data Tables 78, 183-356 (2001). More extensive printable and machine-readable tables are given at http://pdg.lbl.gov/AtomicNuclearProperties/.

[2w3] *The Internet Seminar (Microscopic World-3 The World of the Atomic Nucleus)* http://www.kutl.kyushu-u.ac.jp/seminar/MicroWorld3_E/.

[Chapter III]

[3W03] Helmut Wiedemann. *Synchrotron Radiation.* Springer–Verlag, Berlin (1994). ISBN: ISBN 978-3-540-43392-7

[Chapter IV]

[4B90] Sheridan Bowman. *Radiocarbon Dating (Interpreting the Past)* University of California Press. ISBN: 978-0520070370

[4K04] S.G. Karshenboim: *Precision Study of Positronium: Testing Bound State QED Theory.* Int.J.Mod.Phys. A19 (2004) 3879. `arXiv:hep-ph/0310099 [hep-ph]`.

[4R04] J. Reimer Paula et al. (2004). *INTCAL04 Terrestrial Radiocarbon Age Calibration, 0-26 Cal Kyr BP.* Radiocarbon 46 (3): 1029-1058.

[Chapter V]

[5P84] D.H. Perkins. *Proton Decay Experiments.* Annual Review of Nuclear and Particle Science 34 (1984) 1-50.

[5K09] H. Nishino et al. (Super-K Collaboration). *Search for Proton Decay via* $p \to e^+\pi^0$ *and* $p \to \mu^+\pi^0$ *in a Large Water Cherenkov Detector*. Physical Review Letters 102 (2009) 141801.

[Chapter VIII]

[8B64] V. E. Barnes et al. *Observation of a Hyperon with Strangeness Number Three.* Physical Review Letters 12 (1964) 204.

[8C97] F. Reines, C. Cowan, Jr. *Reines-Cowan Experiments: Detecting the Poltergeist*. Los Alamos Science n.25 (1997). http://lascience.lanl.gov/lascience25.shtml

[8T10] Stefaan Tavernier. *Experimental Techniques in Nuclear and Particle Physics*. Springer–Verlag, Berlin (2010). ISBN: 978-3642008283

[Chapter IX]

[9A92] Aida X. El-Khadra et al.: *Determination of the strong coupling constant from the charmonium spectrum*. Phys. Rev. Lett. 69, 729-732 (1992).

[9D95] C. T. H. Davies. *The Heavy Hadron Spectrum*. Lectures at Schladming Winter School 1997. `arXiv:hep-ph/9710394v1`.

[9B96] P. N. Burrows et al. *Prospects for the Precision Measurement of* α_S. `arXiv:hep-ex/9612012v2`.

[Chapter X]

[10S85] S. van der Meer. *Stochastic cooling and the accumulation of antiprotons.* Rev. Mod. Phys. 57, 689-697 (1985).

[10M01] Kirk T. McDonald. *An Off-Axis Neutrino Beam.* `arXiv:hep-ex/0111033`

[10T11] K. Abe et al. *The T2K Experiment.* Nucl. Instr. Meth. A659 (2011) 106-135. Also on `arXiv:1106.1238`

[10w2] http://lcg.web.cern.ch/lcg/public/default.htm.

[10w3] ROOT: A Data Analysis Framework. http://root.cern.ch.

[10w4] http://www-off-axis.fnal.gov/

[Chapter XII]

[12K89] B. Kaiser. *The Physics of Massive Neutrinos.* World Scientific Publishing Press (1992). ISBN: 9971-50-661-0.

[12A98] M. Ambrosio *et al.* (MACRO Coll.) *Measurement of the atmospheric neutrino–induced upgoing muon flux using MACRO.* Phys. Lett. 434 (1998) 451

[12K00] For an extensive review see: H.V. Klapdor-Kleingrothaus, H. Pas, and A.Y. Smirnov. *Neutrino mass spectrum and neutrinoless double beta decay.* Phys. Rev. D63 (2001) 073005. `[hep-ph/0003219]`.

[12C03] M. Apollonio et al., *Search for neutrino oscillations on a long base-line at the CHOOZ nuclear power station.* Eur. Phys. J. C27, 331 (2003).

[12K04] Takaaki Kajita. *Atmospheric neutrinos.* New J. Phys. 6 (2004) 194. (http://iopscience.iop.org/1367-2630/6/1/194)

[12A08] F. T. Avignone, S. R. Elliott, J. Engel. *Double Beta Decay, Majorana Neutrinos, and Neutrino Mass.* Rev. Mod. Phys. 80 (2008) 481-516. `arXiv:0708.1033 [nucl-ex]`

[12A11] K. Abe et al. (T2K Collaboration). *Indication of Electron Neutrino Appearance from an Accelerator-Produced Off-Axis Muon Neutrino Beam.* Phys. Rev. Lett. 107, 041801 (2011).

[12C11] E. Andreotti et al. (Cuoricino Coll.) ^{130}Te *neutrinoless double-beta decay with CUORICINO.* Astropart. Phys. 34 (2011) 822-831.

[Chapter XIII]

[13H97] M. Herant, S. A. Colgate, W. Benz, C. Fryer. *Neutrinos and Supernovae.* Los Alamos Science Number 25,1997. http://lascience.lanl.gov/lascience25.shtml

[13O02] M. Ostrowski. *Mechanisms and sites of ultra high energy cosmic ray origin*, Astrop. Phys. 18 (2002) 229-236.

[13A08] F. Aharonian et al. *High energy astrophysics with ground-based gamma ray detectors*. Rep. Prog. Phys. 71 (2008) 096901.

[13D08] A. de Angelis et al. *Very-high-energy gamma astrophysics*. Riv. Nuovo Cimento 31 (2008) 187.

[13F09] A. Abdo et al. (Fermi LAT Coll.) *Fermi Large Area Telescope Bright gamma-rays source list*. Astrophys.J. Suppl.183 (2009) 46-66. `arXiv:0902.1340 [astro-ph.HE]`.

[13C10] T. Chiarusi, M. Spurio. *High-energy astrophysics with neutrino telescopes.* Eur. Phys. J. C65(2010) 649-710.
`arXiv:0906.2634 [astro-ph.HE]`

[13w3] http://icecube.wisc.edu.

[13w4] http://www.km3net.org.

[**Chapter XIV**]

[14E90] Engelmann, J. J. et al. *Charge composition and energy spectra of cosmic-ray nuclei for elements from Be to NI. Results from HEAO-3-C2*. Astronomy and Astrophysics, vol. 233 (1990) 96-111.

[14S90] R. Silberberg, CH. Tsao. *Spallation processes and nuclear interaction products of cosmic rays*. Phys Rep. 191 (1990) 351-408.

[14K01] K. Ferriere. *The interstellar environment of our galaxy*. Rev. Mod. Phys. 73 (2001) 1031-1066.

[14B10] Borexino Collaboration *Observation of Geo-Neutrinos*, Phys. Lett. B687:299-304,2010 . Also: `arXiv:1003.0284v2 [hep-ex]`.