Slide supporting material

Lesson 4: Access Protocols: Aloha, CSMA, and Token Ring; Exercises

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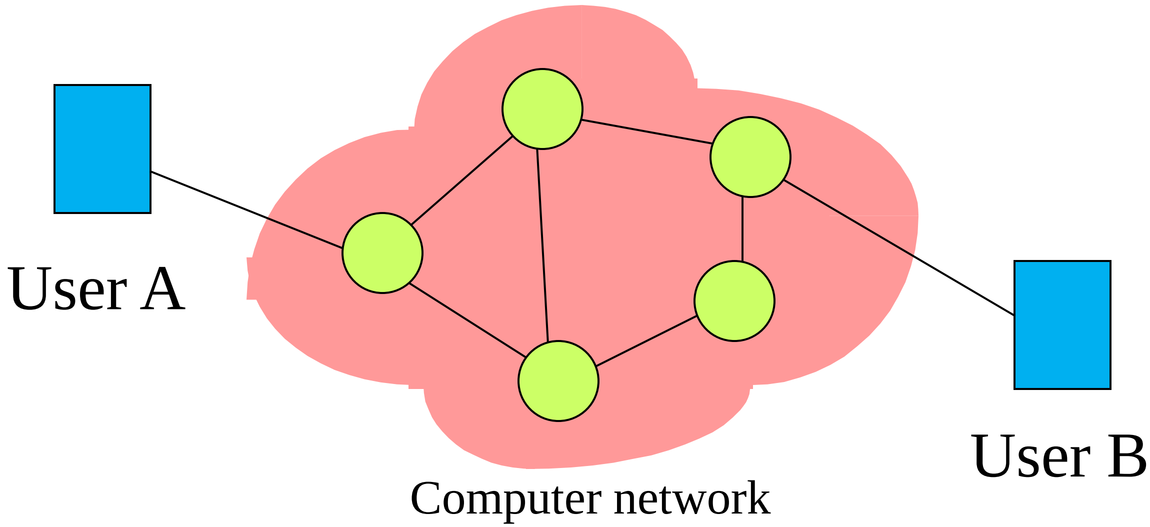
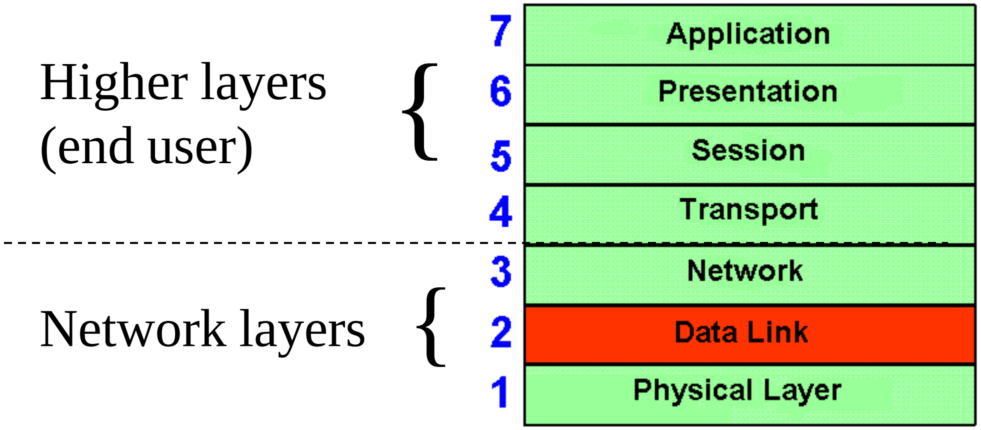
Queuing Theory and Telecommunications: Networks and Applications
2nd edition, Springer

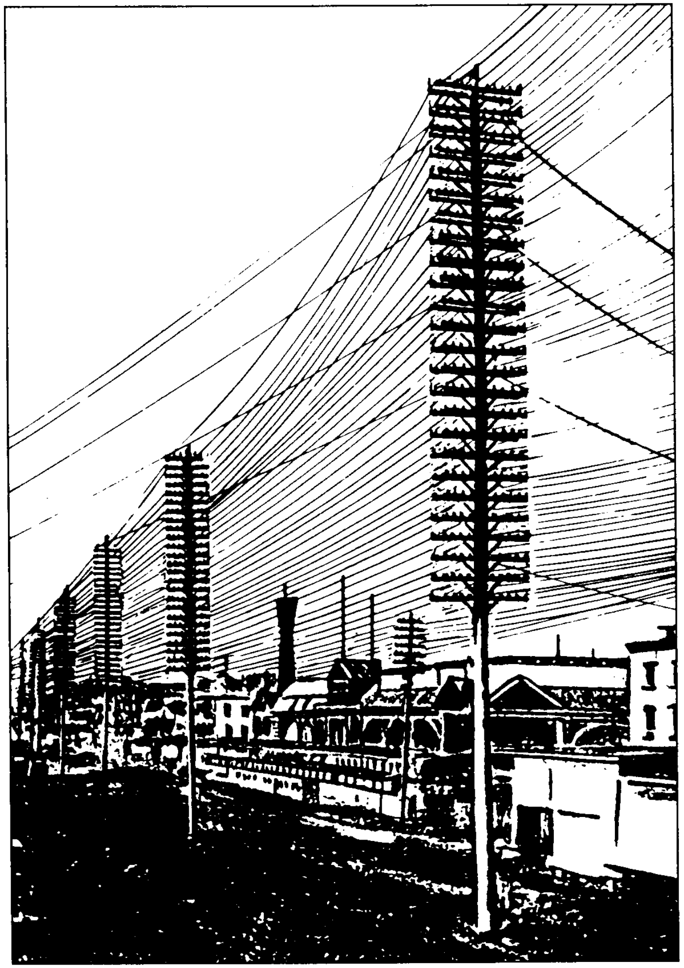
The OSI Protocol Stack: Reference Model





The Need of Multiple Access Schemes

- This picture shows the technique adopted to transport phone signals at the beginning of 1900: different wires for different users.
- The introduction of **multiplexing** and **multiple access** allows that a transmission resource is shared among different users.

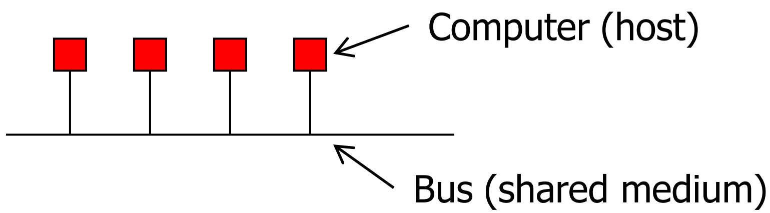


The MAC Layer

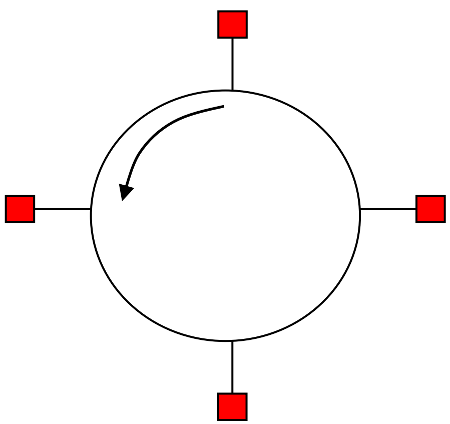
- The access to the shared medium in Local Area Networks (LANs) is managed by the Medium Access Control (MAC) protocol at **layer 2**.
- MAC protocols depend on
  - Physical medium
  - Network topology and related transmission medium.
- One of the first examples of **packet data transmissions** is based on a random access protocol named Aloha (or Slotted Aloha), developed at the beginning of '70 (in parallel to the first Internet experiments of the DARPA project).

Networks and Topologies

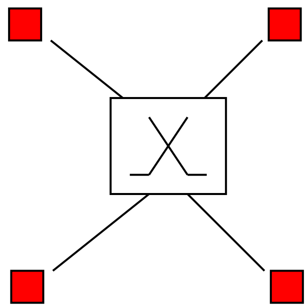
- **Bus** topology (broadcast)



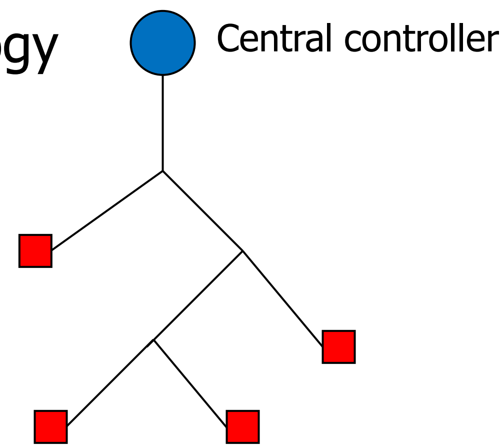
- **Ring** topology with point-to-point wiring (token ring)



- **Start** topology (wired or wireless)



- **Tree** topology



MAC Protocols: Basic Requirements

- MAC protocols have to meet the following requirements:
  - Managing different traffic classes with suitable priority levels and **Quality of Service (QoS)** requirements,
  - **Fair sharing of resources** within a traffic class,
  - Guaranteeing a **prompt *access* to resources** for real time and interactive traffics,
  - Allowing a **high utilization** of radio resources,
  - Guaranteeing **protocol stability.**

Taxonomy of MAC Protocols

- **Fixed access protocols** that grant permission to transmit only to one terminal at once, avoiding collisions of messages on the shared medium. Access rights are statically defined for the terminals.
  - Examples: Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), Code Division Multiple Access (CDMA), and Orthogonal Frequency Division Multiple Access (OFDMA).

- **Contention-based protocols** that may give transmission rights to several terminals at the same time. These policies may cause two or more terminals to transmit simultaneously and their messages to collide on the shared medium. Suitable collision resolution schemes (backoff algorithms) have to be used.
  - Examples: Aloha, Slotted-Aloha, Carrier Sense Multiple Access (CSMA), CSMA-CA of WiFi.

- **Demand-assignment protocols** that grant the access to the network on the basis of requests made by the terminals. Resources used to send requests are separated from those used for information traffic. The request channel can be contention-based or adopt a piggybacking scheme.
  - Examples: polling method, token ring and token bus, Reservation-Aloha for radio systems.

Performance Indexes for MAC Protocols

- **Throughput** (MAC level): percentage of time for which the shared channel is busy owing to the correct transmission of packets (analogous to traffic intensity in stable conditions).
  - At transport layer, the throughput has a slightly different meaning, concerning the traffic (bit-rate) injected by a source in the network.

- **Mean packet delay**: mean time needed from packet generation (arrival) to the correct packet transmission or delivery.

Survey of Analytical Methods

Analytical Methods

- Analysis is needed for the following typical QoS problems with MAC layer protocols:
  - Access protocol performance analysis (uplink), taking into account the propagation delay: mean delay, throughput
  - Queuing analysis for downlink transmissions (scheduling scheme): mean delay.
- Available approaches for access protocols analysis:
  - The traditional **S-G analysis**
  - Imbedded **Markov chains** (time-division Markov chains); see next Lessons No. 6 and 7 for definitions of chains and queues.
  - **Equilibrium Point Analysis (EPA)**.

S-G Classical Approach for Uplink Analysis

- The traditional S-G analysis was widely used in 1970's-1980's to study the throughput and delay performance of both slotted and non-slotted multiple access protocols such as Aloha and CSMA.
- This analysis assumes that an infinite number of nodes collectively generate traffic equivalent to a Poisson source with an aggregate mean arrival rate of S packets per slot; moreover, aggregate new transmissions and retransmissions are approximated by a Poisson process with mean arrival rate of G packets per slot.
- **This is a simplified approach, because there is no consideration of the buffer size on terminals.**

L. Kleinrock, S. S. Lam, "Packet Switching in a Multiaccess Broadcast Channel: Performance Evaluation", *IEEE Transaction on Communications*, Vol. 23, No. 4, pp. 410-423, April 1975.

Markov Chain Model for Uplink Analysis

- An imbedded Markov chain model is developed for the system.
- The chain describes the MAC behavior of a terminal or of all the terminals.
- Imbedding points are suitable instants in time depending on the PHY-MAC characteristics (e.g., end of slots).
- The state space depends on the different conditions of the MAC protocol of the terminal (e.g., empty, backoff, transmission, etc…) or of a group of terminals.
- Transition probabilities between states need to be formally derived.

G. Bianchi, “Performance Analysis of the IEEE 802.11 Distributed Coordination Function”, *IEEE Journal Sel. Areas. in Comms.*, Vol. 18, No. 3, pp. 535-547, March 2000.

Markov Chain Model for Uplink Analysis (cont'd)

- The Markov chain is solved by stating equilibrium conditions for each state and using a normalization condition. More details on the solution of Markov chains are provided in Lesson No. 6.

- The space of states of the Markov model increases with the complexity of the protocol.

- This study typically differentiates between saturated and non-saturated cases:
  - **Saturation** is a special condition according to which there is always a packet in the terminal buffer ready to be transmitted. This assumption is valid for studying and optimizing the MAC throughput, but it is not suitable to analyze the mean packet delay, because it entails an unstable MAC queue (fully-loaded system).
  - **Non-saturated study** is needed for the analysis of the mean packet delay on the basis of the queuing theory. The Markov chain transitions need to account for the terminal queue dynamics.

EPA Approach for Uplink Analysis

- EPA allows a Markov chain-like approach:
  - **One state diagram** has to be considered modeling the behavior of a terminal at suitable imbedding instants.
  - **One equilibrium equation** can be written for each state of the diagram, assuming that the state is "populated" by an equilibrium (i.e., mean) number of terminals and assuming a stable behavior.
    - EPA is based on the assumption that **at equilibrium the mean rate of terminals leaving a given state is balanced by the mean rate of terminals entering the same state**.
    - **EPA equations** can be written equalizing arrival and departure rates for any state. A normalization condition is needed considering that the sum of the mean number of terminals in the different states is equal to the total number of terminals in the system.

S. Nanda, D. J. Goodman, and U. Timor, "Performance of PRMA: A Packet Voice Protocol for Cellular Systems", *IEEE Trans. Veh. Technol.*, Vol. 40, pp. 584–598, Aug. 1991.

Further Considerations on Analytical Methods

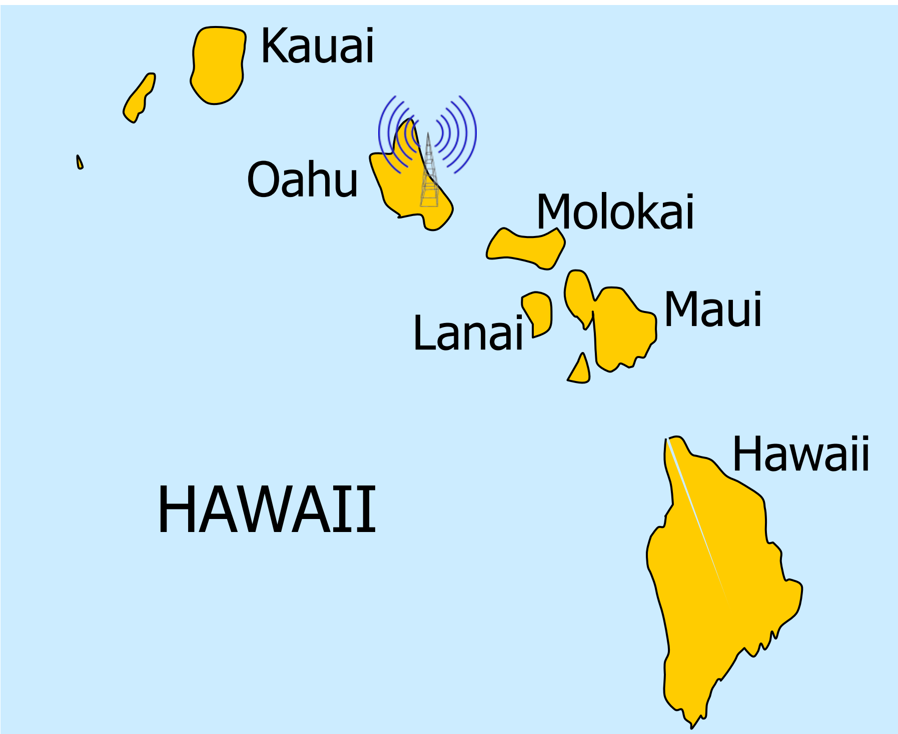
- Both Markov chains and EPA methods typically need **numerical methods to solve non-linear systems** to determine the state probability distribution (Markov chain) and the mean number of terminals in the different states (EPA).

- EPA methods have been used to study PRMA access protocols (contention-based protocols).

- Markov chain methods have been used to study the contention-based access schemes of WiFi and WiMAX.

The S-G Analysis for Aloha and Slotted Aloha Protocols

The Aloha Protocol (Wireless Network, Star Topology)



The Aloha protocol was implemented in '70 also in a satellite network, named ALOHAnet.

The Aloha protocol was proposed at the beginning of '70 by Professor Norman Abramson who needed to connect terminals dispersed among different islands and a central host (= controller) at the Hawaii University in Honolulu (Oahu island).

The main idea is **allowing terminals to transmit to the central controller as soon as they need to do so**.

- Collisions
- Mechanism to reveal collisions (The Aloha protocol is reliable: use of ACKs with a sender-side timer based on the round trip propagation delay or use of a broadcast channel)
- Retransmission attempts after a collision are rescheduled using a random **backoff** time

Note: Aloha is not an acronym, but the classical Hawaiian welcome expression

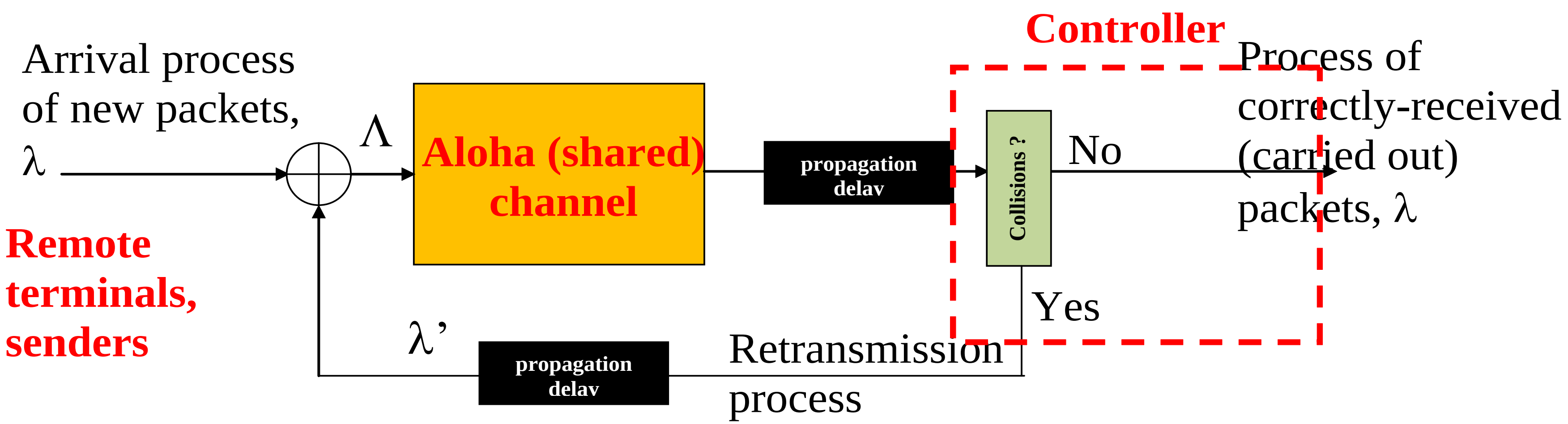
N. Abramson, "The ALOHA System-Another Alternative for Computer Communications", *Fall Joint Computer Conference*, 1970.

Aloha Protocol Analysis

- Hypotheses:
  1. Remote terminals generate packets according to a **Poisson arrival process** with mean rate λ (i.e., sum of an infinite number of elementary and independent sources).
  2. The transmission time of a packet is **constant**, T.
  3. **Asynchronous** transmission of packets.
  5. **Collisions** are detected by broadcast (re)transmissions made by the controller. Let Δ denote the round trip propagation delay (remote terminal – controller).
  6. When a collision occurs, a packet retransmission is re-scheduled after a random delay, called **backoff time** (with an exponential distribution and mean value E[R]).
  7. Collisions (even partial collisions) completely destroy the involved packets (**capture effect is neglected**).

A Model of the Aloha Protocol

- **Assumption and approximation**: the process of packet retransmissions is Poisson with mean rate λ'. This process is independent from new packet generations.



- The total arrival process of packets in the Aloha channel is Poisson with mean rate $\Lambda = \lambda + \lambda'$.

A Model of the Aloha Protocol

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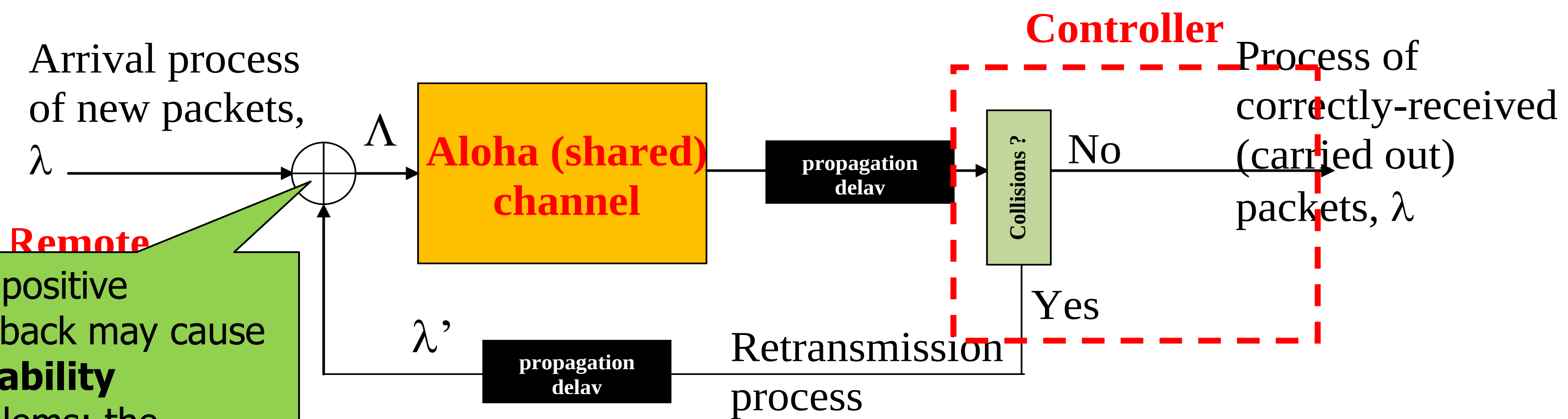

The positive feedback may cause **instability** problems: the throughput of successful packet transmissions goes to zero.

arrival process of packets in the Aloha channel with mean rate $\Lambda = \lambda + \lambda'$.

The Law Modeling the Protocol Behavior

- **The intensity of the traffic (new arrivals) offered to the system is S = λT.**
- The intensity of **the total traffic** (new arrivals + retransmissions) circulating in the system is **G = ΛT**.
- S and G are measured in Erlangs.
- In conditions of **stability** for the access protocol, the mean rate of new packets entering the system, λ, must be equal to the mean rate of packets correctly delivered at destination (and hence leaving the system).
  - S also represents the system throughput, the intensity of the correctly carried traffic.
  - **S/G = Ps**, where Ps is the success probability for a packet transmission attempt.

The Law Modeling the Protocol Behavior

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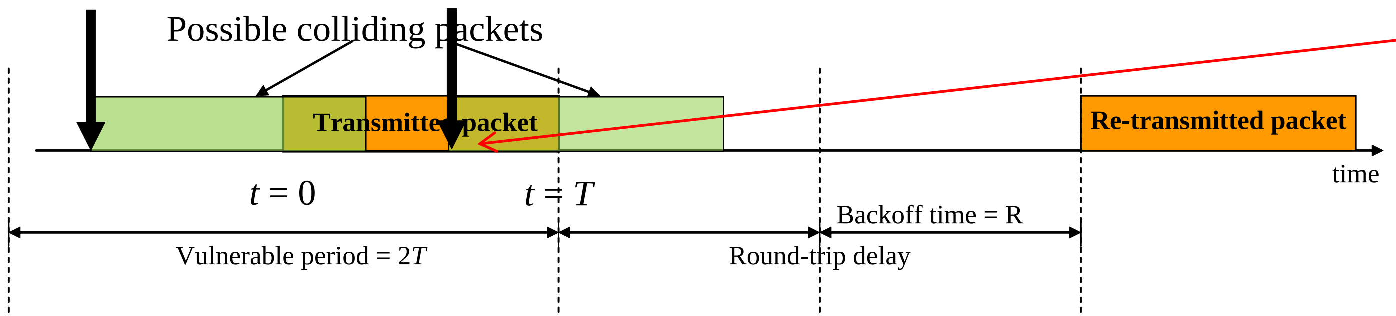
    S/G = Ps, where Ps is the success probability for a packet transmission attempt.

Derivation of Ps

- Let us consider a reference packet starting transmission at time t = 0 and ending transmission at instant t = T.



The presence of the reference packet does not alter the mean rate of the colliding traffic, Λ, because the reference packet is generated by an elementary source and the other elementary sources are infinite

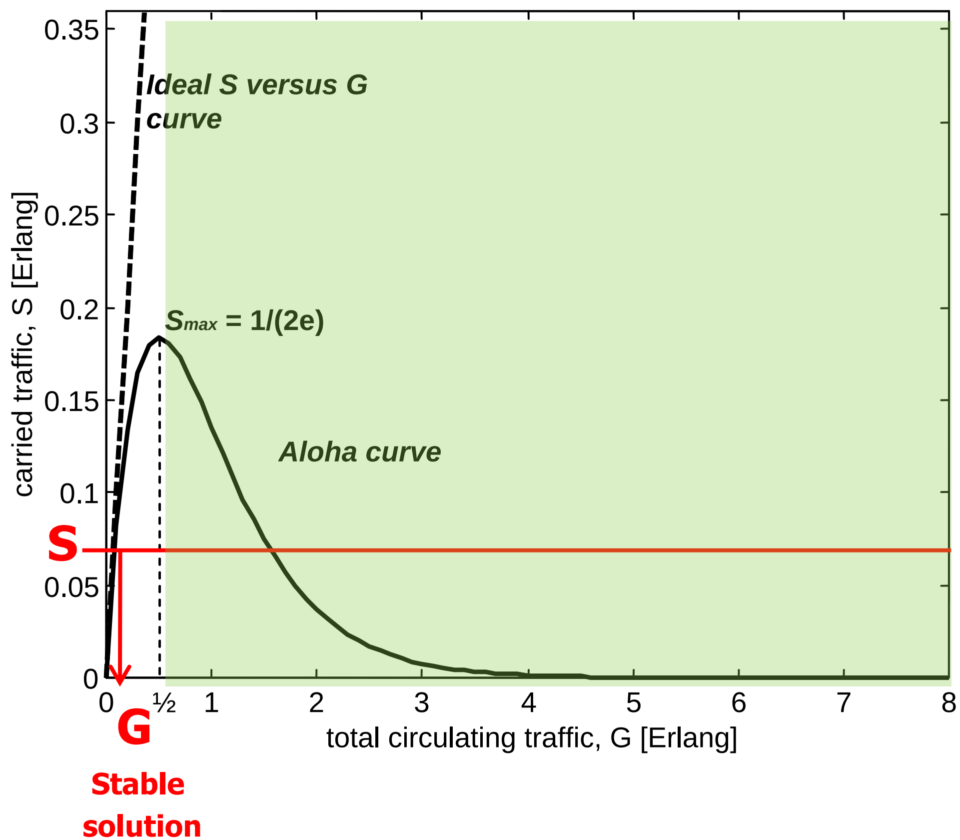
- There are collisions with the reference packet if there are other packet generations (according to the Poisson process with mean rate Λ) in the **vulnerability period with length 2T**.
  - **Ps = no packet generation due to the Poisson process with mean rate Λ in the interval 2T** → $P_s = e^{-2\Lambda T} = e^{-2G}$
  - Note: the distribution of the number of colliding packets in the vulnerability period is Poisson with mean value $2\Lambda T$.

Aloha Protocol Throughput Behavior

- We have obtained the following fundamental relation between S and G for the Aloha protocol:

$$\frac{S}{G} = P_s = e^{-2\Lambda T} \quad \Rightarrow \quad S = Ge^{-2G}$$

- **S intended as the intensity of the traffic offered is the true independent variable** and G is the dependent variable.
- S has a maximum for G = ½ Erl: S_{max} = 1/(2e) ≈ 0.18 Erl. The Aloha protocol reaches the max channel utilization of 18%.
- The function S = S(G) cannot be inverted: given a certain S value (< 0.18), we find two corresponding G values; practically, we consider that only the solution for G < ½ (> ½) Erl is stable (unstable).
  - With a **high (but finite) number of terminals** accessing the Aloha channel and **with a suitable selection of the retransmission interval**, the protocol behavior for a given S (< 0.18 Erl) corresponds only to the stable solution with G = G(S) < ½ (use of the Lambert function in Matlab©).



Mean Number of Transmissions for a Packet

Number of attempts in order to transmit successfully a packet	Time needed to successfully transmit a packet	Probability of a successful transmission attempt
1	$T + \Delta/2$	P_s
2	$T + \Delta + E[R] + T + \Delta/2$	$(1-P_s)\, P_s$
....	....	....
n	$(n-1)(T + \Delta + E[R]) + T + \Delta/2$	$(1-P_s)^{n-1}\, P_s$

- The number of attempts in order to successfully transmit a packet is according to a **modified geometric distribution** with parameter Ps (= e^{-2G}). The mean number of transmission attempts in order to deliver successfully a packet is equal to 1/Ps = e^{2G}.

The Mean Packet Delay for the Aloha Protocol

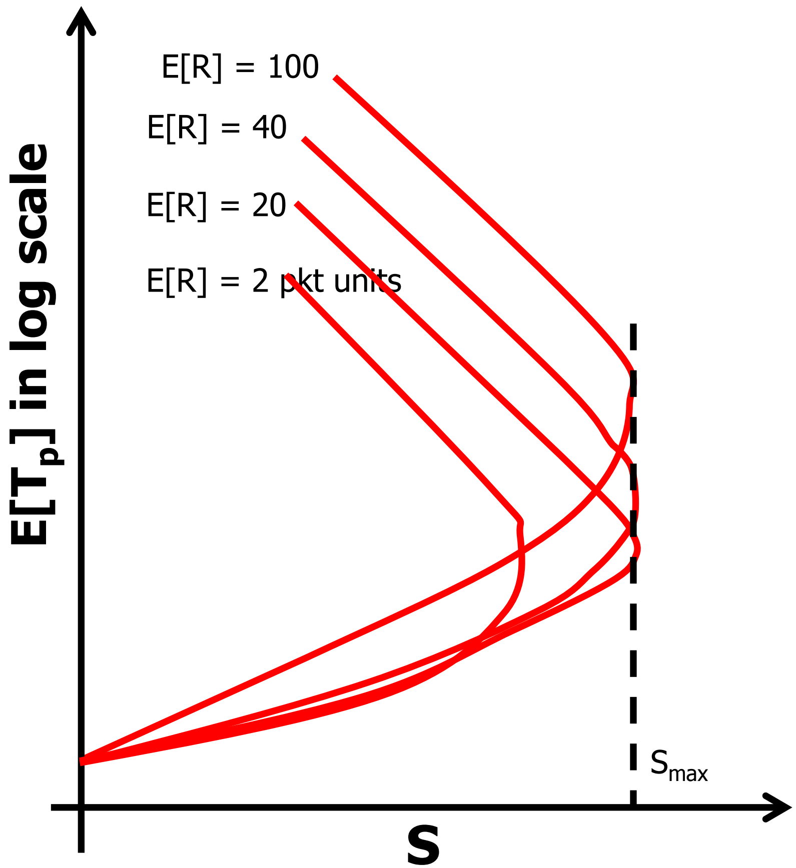
- The mean packet delay $E[T_p]$ with the Aloha protocol is:

$$E[T_p] = \left(\frac{1}{P_s} - 1\right)(T + \Delta + E[R]) + T + \frac{\Delta}{2} =$$

$$= \left(e^{2G} - 1\right)(T + \Delta + E[R]) + T + \frac{\Delta}{2}$$

- **In a real system (finite number of non-elementary sources), the access protocol can be made stable, provided to select a sufficiently-high E[R] value.**

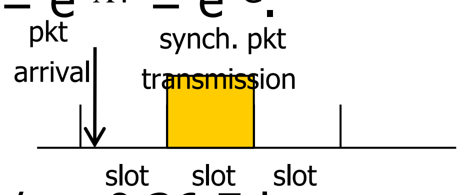
- **E[R] has to be not too small to avoid protocol instability, nor too big to avoid too high packet delays.**



The Slotted Aloha Protocol

- This is a variant of the Aloha protocol proposed by Roberts in 1972.
  - The controller sends **synchronization pulses** to coordinate the transmissions on time slots.
  - Packet **transmissions are made to arrive synchronized with time slots at the controller**: a terminal can send a packet only at regular intervals.
  - We consider that the slot time is coincident with the packet transmission time, T.
- Let us consider the same model adopted for Aloha; in particular, we use the law S/G = Ps, where Ps has to be re-determined.
  - With Slotted Aloha, the **vulnerability interval is reduced to T**: a packet transmission is successful only if there is no other terminal generating a packet in the same slot (mean rate Λ): $P_s = e^{-\Lambda T} = e^{-G}$.

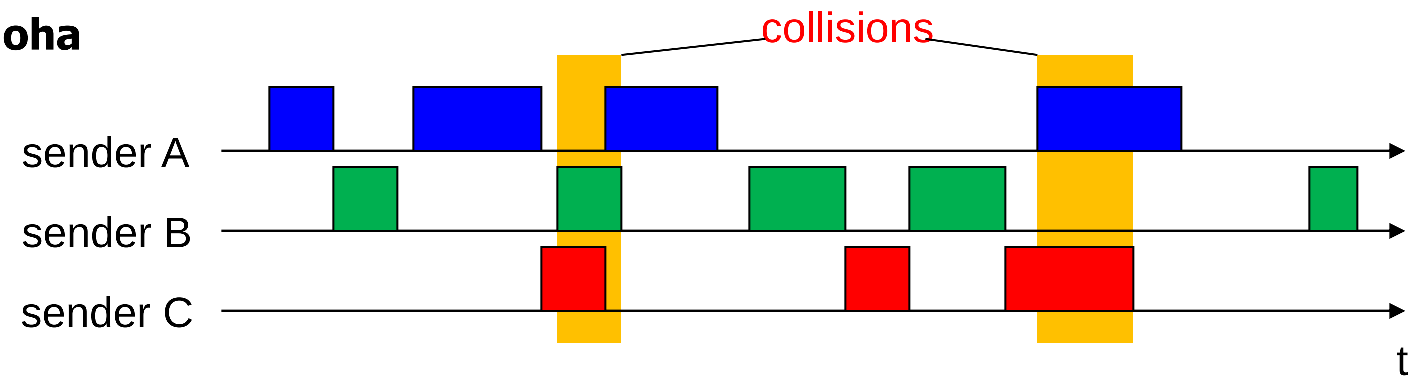
$$\frac{S}{G} = P_s = e^{-\Lambda T} \quad \Rightarrow \quad S = Ge^{-G}$$



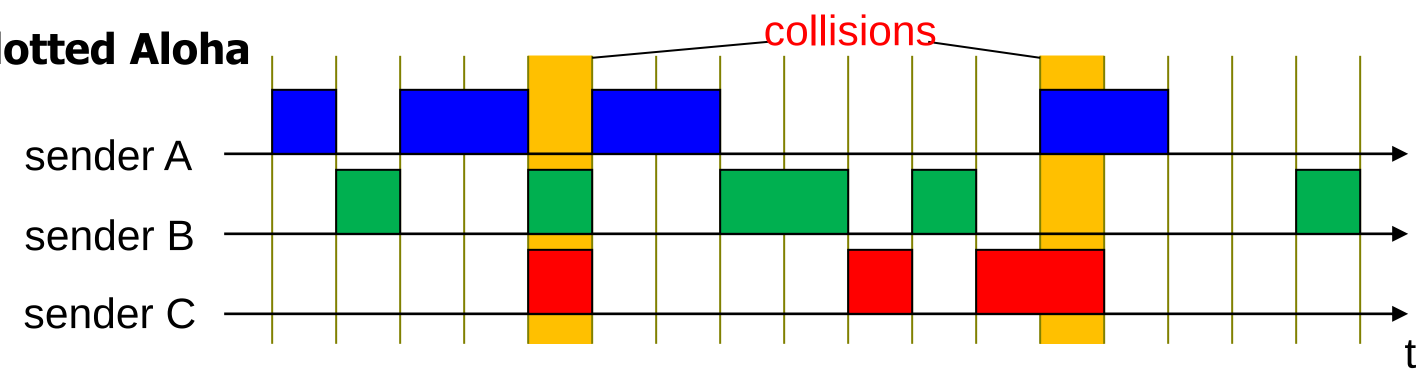
  - S has a maximum for G = 1 Erl and its value is $S_{max} = 1/e \approx 0.36$ Erl: the slotted Aloha protocol **doubles the maximum throughput** with respect to Aloha.

Aloha & Slotted-Aloha

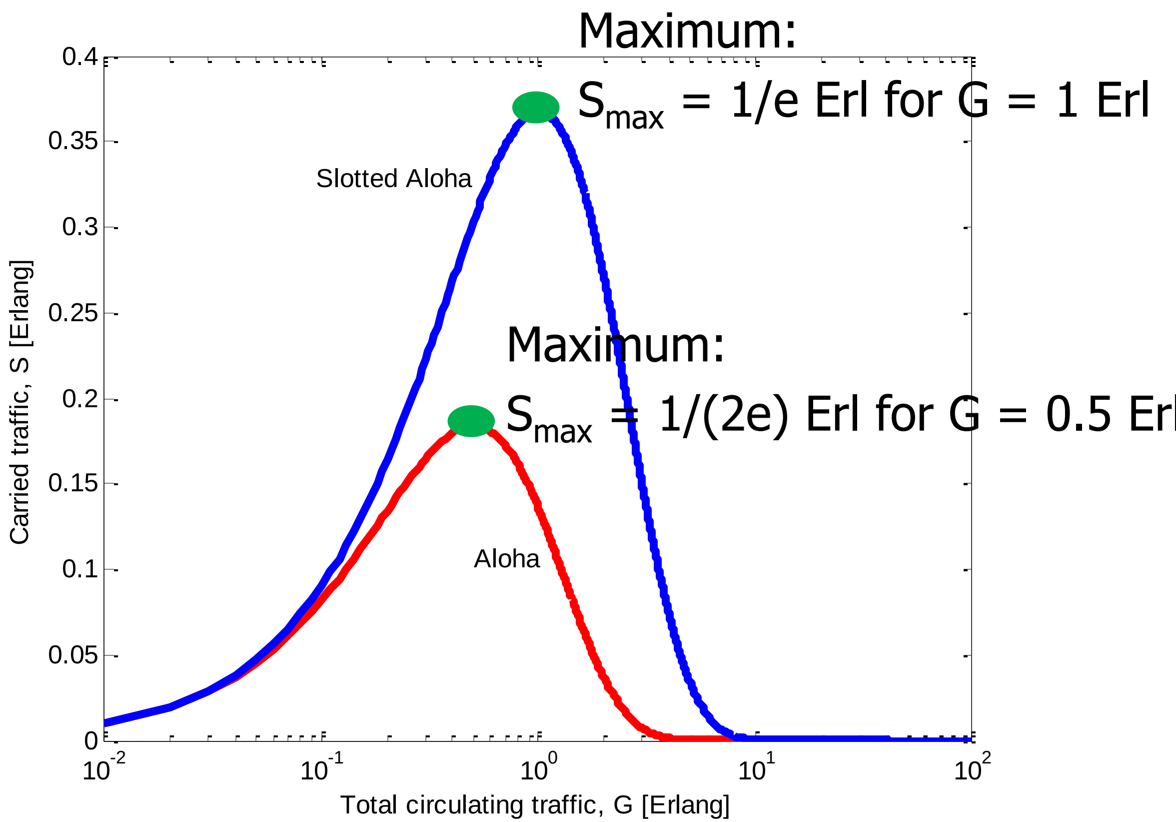
- **Aloha**



- **Slotted Aloha**



Throughput Comparison Aloha vs. Slotted Aloha



This graph is somewhat different from the previous one, because the abscissa is in logarithmic scale.

The Mean Packet Delay for the Slotted Aloha Protocol

- The mean delay needed for the successful transmission of a packet, can be calculated in the same way as for the Aloha case, considering the following aspects:

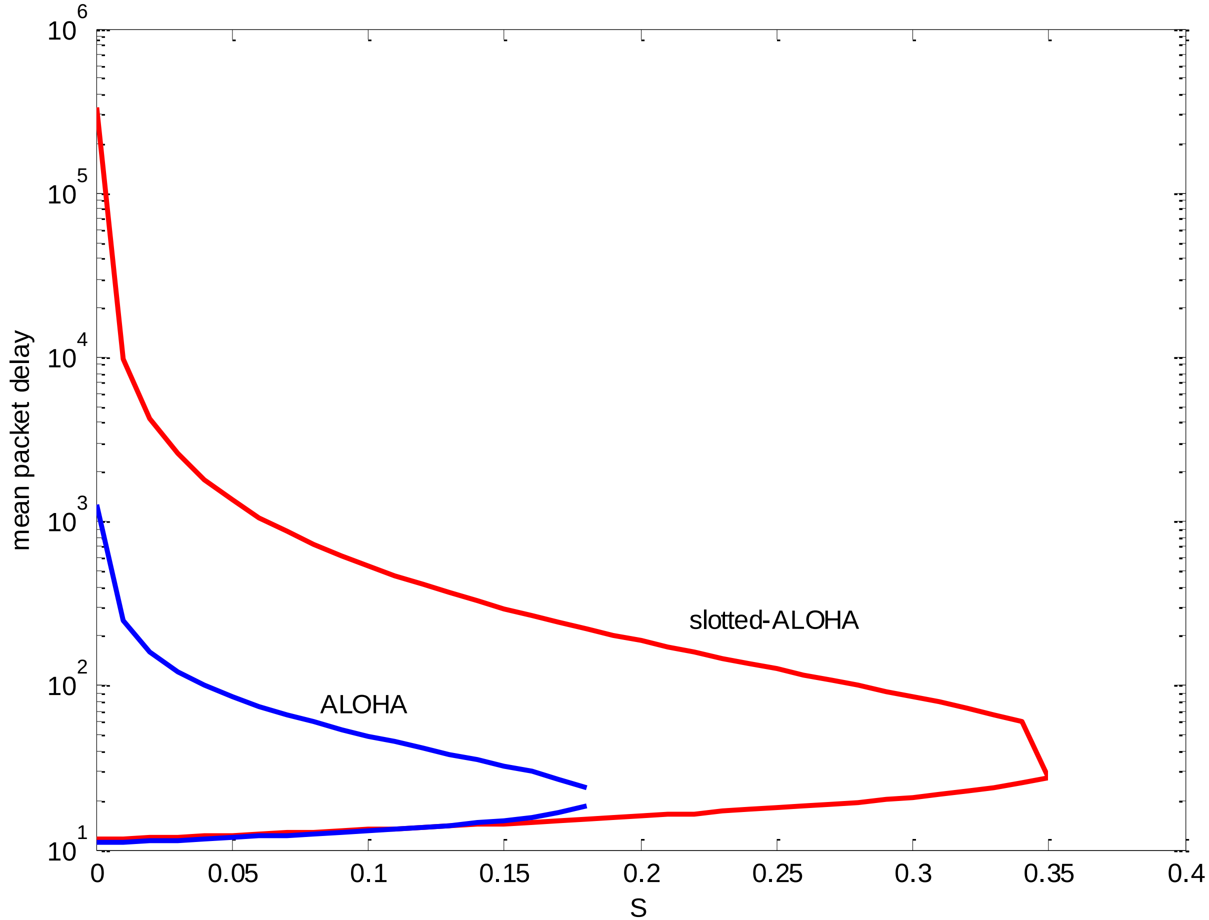
  1. The expression of Ps has changed: $P_s = e^{-G}$
  2. There is a **synchronization delay** due to the time a packet (Poisson arrival process) has to wait (*vacation time*) for the start of the next slot where it is transmitted: the mean synchronization delay is equal to T/2.

$$E[T_p] = \frac{T}{2} + \left(\frac{1}{P_s} - 1\right)(T + \Delta + E[R]) + T + \frac{\Delta}{2} =$$

$$= \frac{T}{2} + \left(e^G - 1\right)(T + \Delta + E[R]) + T + \frac{\Delta}{2}$$

- Qualitatively, the behavior of the mean packet delay as a function of S is similar to that of Aloha, apart for the fact that higher values of S (up to 0.36 Erl) can be supported.

Comparison of Mean Packet Delays as Functions of S



Slotted Aloha with Finite Number of Terminals

- In this study, we consider a finite number N of independent terminals sharing the Slotted Aloha channel. The packet arrival process is binomial (not Poisson) on a slot basis. Let us denote:
  - S_i, the probability to successfully transmit a new packet on a slot by the i-th terminal;
  - G_i, the probability to transmit a (new or collided) packet on a slot by the i-th terminal.

- The total traffic carried out on a slot, S, and the total circulating traffic on a slot, G, can be expressed by assuming that all terminals generate the same traffic load:

$$S = \sum_{i=1}^{N} S_i = NS_i \quad \text{e} \quad G = \sum_{i=1}^{N} G_i = NG_i$$

Slotted Aloha with Finite Number of Terminals (cont'd)

- The probability of a successful packet transmission on a slot $S_i = S/N$ by the i-th terminal can be expressed as the product of the probability that the i-th terminal transmits on the slot, $G_i = G/N$, and the probability that no other terminal transmits on the same slot, $\Pi_j(1-G_j) = (1-G_j)^{N-1} = (1 - G/N)^{N-1}$:

$$\frac{S}{N} = \frac{G}{N}\left(1-\frac{G}{N}\right)^{N-1} \quad \Rightarrow \quad S = G\left(1-\frac{G}{N}\right)^{N-1}$$

- The maximum throughput is achieved for G = 1 Erl and corresponds to $S_{max} = (1-1/N)^{N-1}$ Erl. For $N \to \infty$ (case of infinite independent and elementary sources), the above law S = S(G) can be expressed by means of the following notable limit:

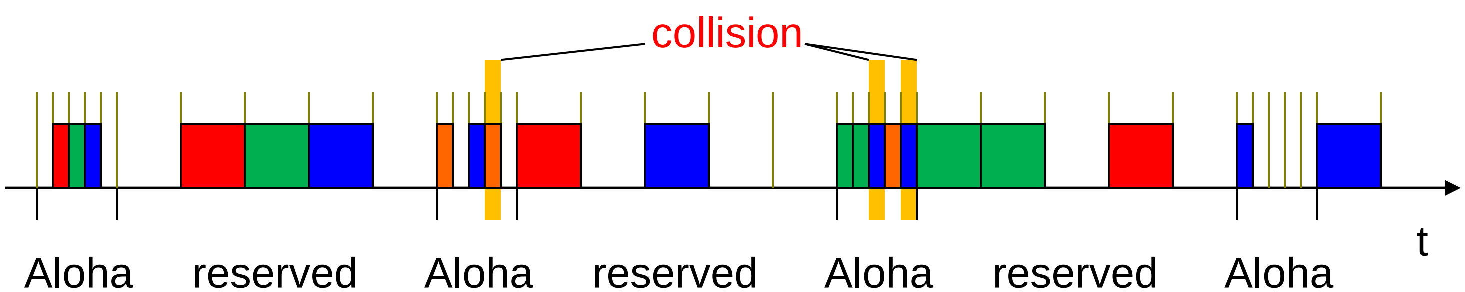
$$\lim_{N\to\infty}\left(1-\frac{G}{N}\right)^{N-1} = e^{-G}$$

- Hence, we re-obtain the classical result of the Slotted Aloha protocol for infinite sources:

$$S = Ge^{-G}$$

Random Access with Reservation: R-Aloha (1973)

- Reservation Aloha (R-Aloha) protocol of the demand-assignment type:
  - **Minislots/minipackets** are used to request the reservation of resources on a frame basis.
  - This protocol has two phases that are organized in time according to a frame:
    - **Contention mode (Aloha type) to acquire a reservation**: minipackets are transmitted on a separated Slotted-Aloha channel with minislots; collisions occur when more minipackets are transmitted on the same minislot.
    - **Reservation mode** for the transmission of data on the reserved slot(s); in this phase there is no collision.
  - A **centralized scheduler** can be used in order to manage the allocation of resources in the frame depending on different priorities.



L. G. Roberts, "Dynamic Allocation of Satellite Capacity Through Packet Reservation", *Proceedings of the National Computer Conference*, AFIPS NCC73 42, 711-716, 1973.

Combinatorial Aspects for the Analysis of R-Aloha

Let us consider an R-Aloha protocol case with m minislots per frame. Let us assume to have k terminals transmitting their requests (minipackets) in the contention part of the frame with m minislots, by selecting a minislot with uniform probability out of m minislots. We consider two extreme cases:

- **Case #1 (without capture effect)**: two transmissions occurring on the same minislot collide destructively (no request can be received).
- **Case #2 (with capture effect)**: among the colliding transmissions on a minislot (if any), one is received successfully.

We adopt the **urn theory** to study the distribution of k minipackets transmitted on m minislots (selected at random).

Case #1: The mean number of successful transmissions per frame, N_1, is equal to the mean number of minislots occupied by only one minipacket:

$$N_1(k,m) = k\left(1-\frac{1}{m}\right)^{k-1}$$

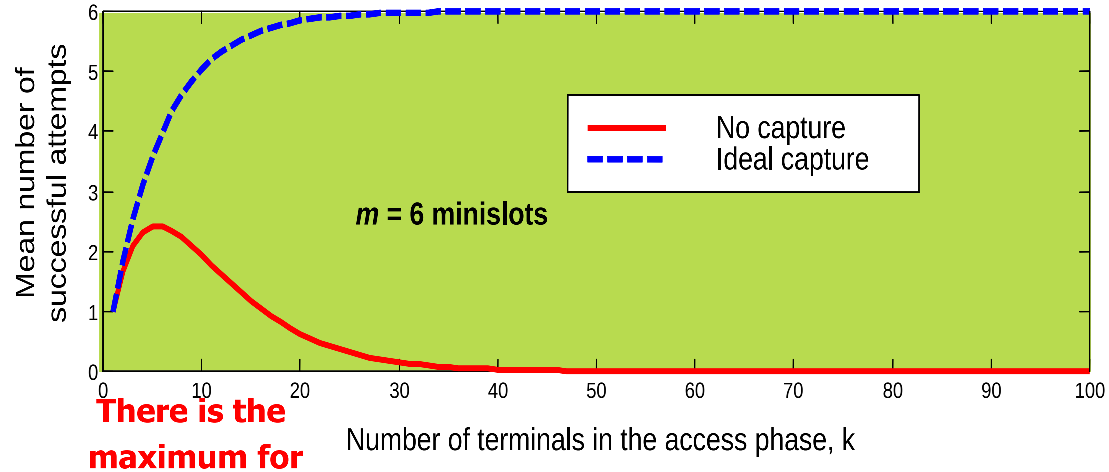
Case #2: The mean number of successful transmissions per frame, N_2, is equal to the mean number of minislots occupied by at least one minipacket:

$$N_2(k,m) = m \times \left[1-\left(1-\frac{1}{m}\right)^{k}\right]$$

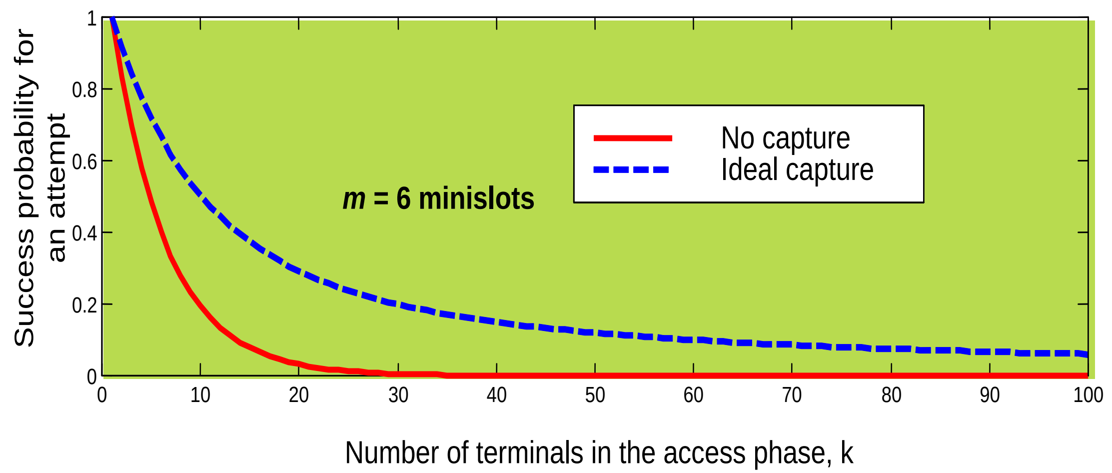
Combinatorial Aspects for the Analysis of R-Aloha (cont'd)

The probability of success for a transmission attempt is :

$$P_s = \frac{N(k,m)}{k}$$



There is the maximum for k = m.



Final Comments on the Aloha Protocol

- The Aloha protocol is the ancestor of the access protocols used in wired (Ethernet) and wireless (WiFi, WiMAX) networks.
- Protocols of the slotted Aloha type (or minislotted, like R-Aloha) are quite commonly adopted in 2G and 3G cellular networks (PRACH channel) as well as in wireless networks like WiMAX (contention-based access for transmission requests of the best effort traffic class).
- **Important references:**

J. F. Hayes. *Modeling and Analysis of Computer Communication Networks*. Plenum Press, NY, 1986.

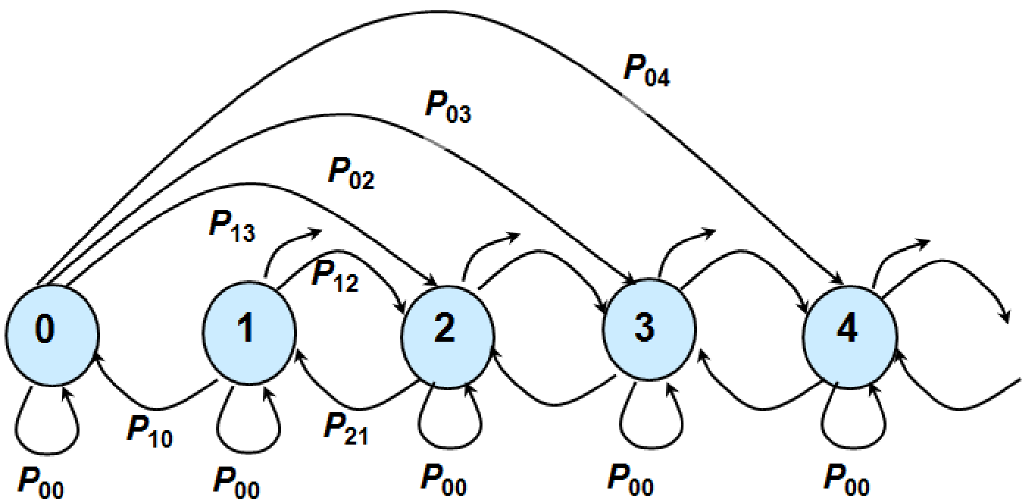
L. Kleinrock, S. S. Lam, "Packet Switching in a Multiaccess Broadcast Channel: Performance Evaluation", *IEEE Transaction on Communications*, Vol. 23, No. 4, pp. 410-423, April 1975.

Other Aloha Analysis Approaches: Markov Chain and EPA

Markov Chain Approach for Slotted Aloha

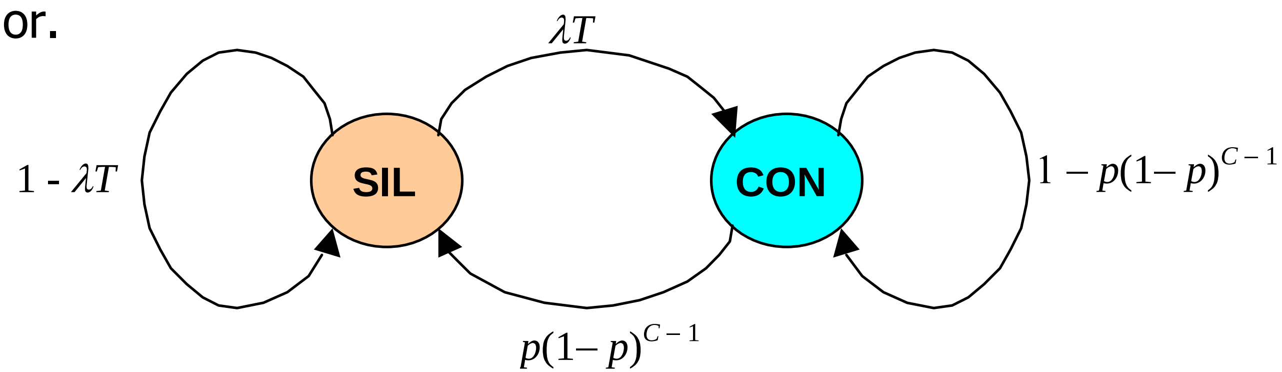
- We consider the state as the **number of contending terminals at the beginning of a slot**. Let M denote the total number of terminals. We model the system by means of the following **imbedded Markov chain** with M + 1 states and where P_{ij} is the transition probability from state i (= i contending terminals, each having one packet to transmit) to state j (= j contending terminals).

- Even for this extremely-simple protocol, the Markov chain analytical approach is quite complex and requires the determination of transition probabilities P_{ij}.



EPA Approach for a Variant of Slotted Aloha

- As soon as a terminal has a new packet ready for transmission it leaves the SIL (inactivity) state enters the CON state, where it can transmit the packet on a slot (duration T) according to a **permission probability p**.
- A terminal cannot generate a new packet until the previous packet has been successfully transmitted.
- Let C $\in$ [0, M] denote the number of terminals in the CON state
  - A given transmission attempt is successful with probability $(1-p)^{C-1}$
- The state diagram of a terminal is shown below where **p and λT can be considered as two control parameters** that influence the protocol behavior.



The state diagram is imbedded at the end of each slot, T.

EPA Approach for a Variant of Slotted Aloha (cont'd)

- Let s (c) denote the equilibrium number of terminals in the SIL (CON) state.
- According to the EPA approach, we may write:
  - The flow balance condition at equilibrium between SIL and CON states and
  - The normalization condition stating that the total (equilibrium) number of terminals in SIL and CON states must be equal to M:

$$\begin{cases} s\lambda T = cp(1-p)^{c-1} \\ s + c = M \end{cases}$$

- This EPA system can be converted into the following equation in the unknown c (unsolvable, in a closed form) with control parameters p and λT and with input parameter M:

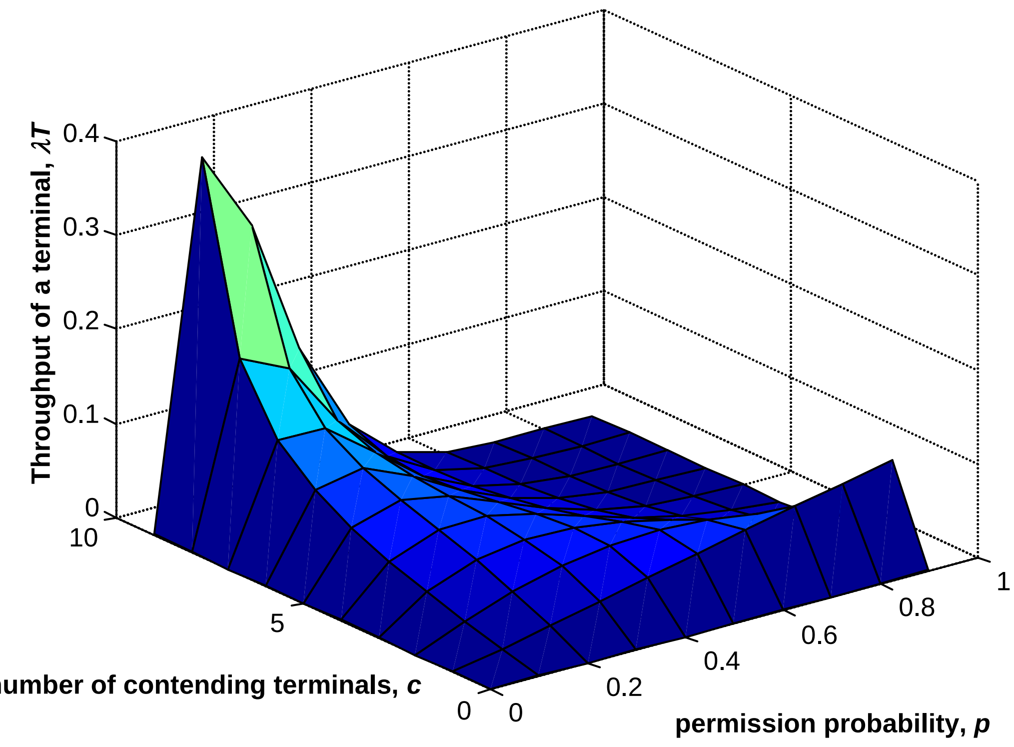
Potential function

$$(M-c)\lambda T - cp(1-p)^{c-1} = 0 \quad \Longleftrightarrow \quad \Pi_{p,\lambda T}(c) = 0$$

G. Giambene, E. Zoli, "Stability Analysis of an Adaptive Packet Access Scheme for Mobile Communication Systems with High Propagation Delays", *International Journal of Satellite Communications and Networking*, Vol. 21, pp. 199-225, March 2003.

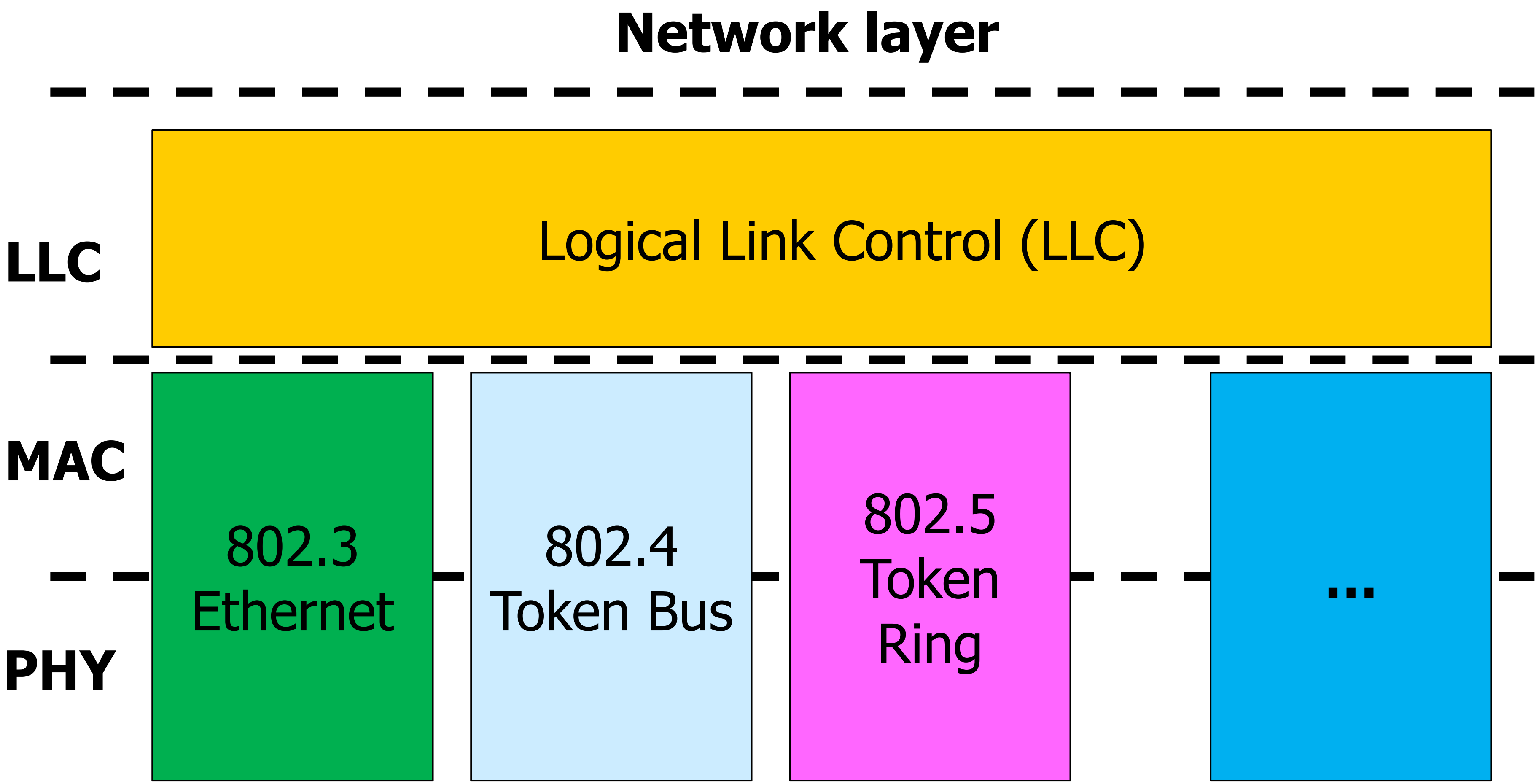
Single or Multiple EPA Solutions for Slotted Aloha

- Depending on the values of the control parameters p and λT, there are one or three EPA solutions (i.e., c values solving the EPA system).
- Situations with **multiple EPA solutions** correspond to cases where the protocol continuously oscillates between an unacceptable congestion case and a low congestion case.
- It is important to determine the configurations of control parameters for which the protocol changes from multiple to single EPA solution.
  - This is the typical case investigated by the **catastrophe theory**.



LAN Access Protocols

A Survey of IEEE 802 Protocols for LANs



CSMA Protocols

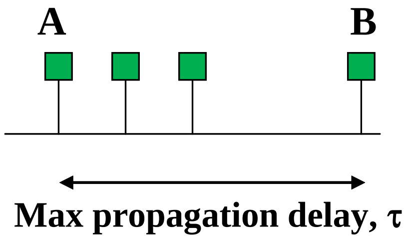
- The performance of random access schemes can be improved if the packet transmission time, T, is **much bigger than** the maximum propagation delay in the network, τ. The following parameter a is used:

$$a = \frac{\tau}{T}$$

- Let us refer to LANs with a **broadcast physical medium** (e.g., a single bus) that permits a remote station (listening to the physical medium) to recognize weather another transmission is in progress or not (**carrier sensing**). If another transmission is revealed, the remote station refrains from transmitting in order to avoid collisions.
  - The protocols of this type are called Carrier Sense Multiple Access (CSMA).
  - CSMA schemes are based on a decentralized control.
  - Both slotted and unslotted versions are available for each of the CSMA protocols.

CSMA Protocols (cont’d)

- For carrier sensing, a special line code must be used in order to avoid that a bit ‘0’ corresponds to a 0-volt level for all the bit duration.
  - The Ethernet standard uses Manchester encoding.
- Since the **medium is of the broadcast type**, a transmitting terminal cannot simultaneously receive a signal, otherwise there is a collision event. Half-duplex transmissions are typical of CSMA protocols.



- **Collisions may occur** with this protocol since a terminal recognizes that another terminal is using the medium only after a (maximum) delay τ.
  - If station A starts transmitting at time t = 0, this signal reaches station B at time $t = \tau$ (worst case). If station B generates a new packet at instant $t = \tau - \varepsilon$ (where ε denotes an elementary positive value), station B can transmit this packet thus causing a collision.
  - Parameter a characterizes the vulnerability to collisions: it is better to have low a values.

CSMA Protocols (cont'd)

- When a terminal recognizes that its packet transmission has been collided, the packet transmission is rescheduled after a random waiting time (backoff).
  - **Truncated binary exponential backoff**:
    - Packet retransmissions occur after a random delay according to a time window that exponentially increases (up to a maximum value) at each new collision of the same packet.
    - The terminal (among the colliding ones) selecting the lower retransmission delay has the higher probability to be successful.
    - There is a maximum number of retransmission attempts after which the packet is discarded.

Non-Persistent CSMA

- When a terminal is ready to send its packet, it senses the broadcast medium and acts as follows:
  - If no transmission has been revealed (i.e., the channel is free), the terminal transmits its packet;
  - If a transmission has been revealed, the terminal reschedules a new check of the channel status (i.e., free or busy) after a random delay (i.e., the same delay adopted to reschedule transmissions after a collision).

1-Persistent CSMA

- When a terminal is ready to send its packet, it senses the broadcast medium and acts as follows:
  - If no transmission has been revealed (i.e., the channel is free), the terminal transmits its packet;
  - If a transmission has been revealed: the terminal waits and transmits the packet as soon as a free medium is sensed.

p-Persistent CSMA

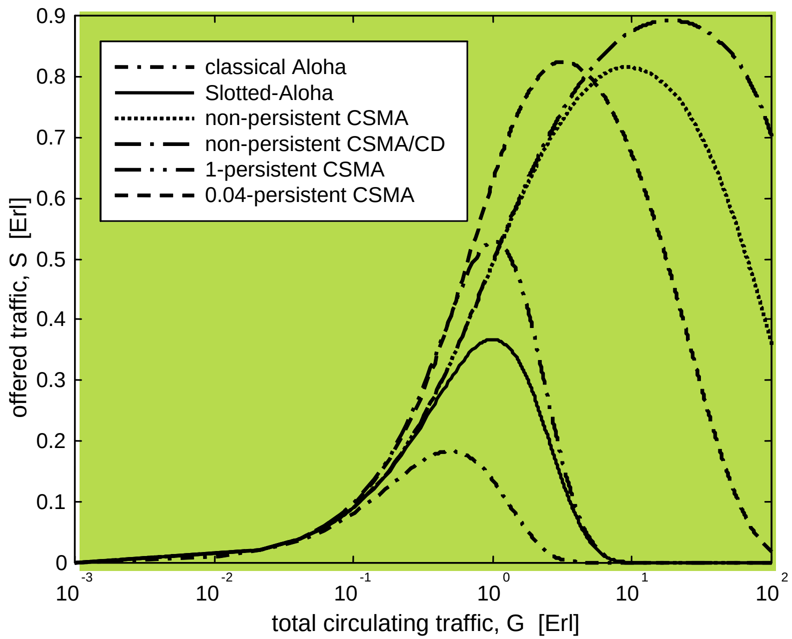
- When a terminal is ready to send its packet, it senses the broadcast medium and acts as follows:
  1. If the medium is empty, the terminal transmits its packet.
  2. If the medium is not empty, then wait until it is empty.
  3. When the medium becomes free, a slotted transmission scheme is adopted being τ the slot duration.
     a. At each new slot, the terminal transmits with probability p and defers the same attempt at the next slot with probability 1 – p, going to next point #b.
     b. If the channel is empty at the new instant, the process at the above point #a is performed; otherwise a random waiting time (as in the case of a collision) is introduced to restart the process from the above point #1.

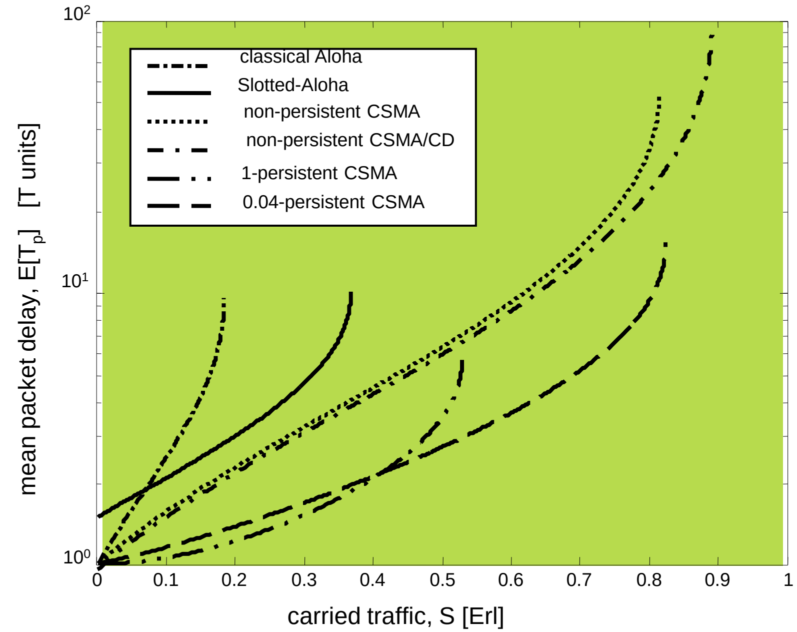
CSMA with Collision Detection

- When a collision occurs, it lasts for the whole packet transmission time T. Thus, there is a significant waste of resources.
- The Collision Detection (CD) mechanism has been added to CSMA.
  - **As soon as a terminal detects that its packet transmission is suffering from a collision, the terminal stops transmitting the packet** and sends a special jam message.
  - All other involved terminals abandon their corrupted frames. Then, the terminal waits for a random time (backoff algorithm for collision resolution) and returns to the initial carrier sensing phase to verify whether the physical medium is free or not.
  - With this protocol the remote terminal listens before and while talking. The CD scheme requires that a terminal reads what it is transmitting: if there are differences, the terminal realizes that a collision is occurring.
  - To ensure that a packet is transmitted without a collision, a terminal must be able to detect a collision before it finishes transmitting a packet; such condition imposes a **constraint on the transmission time of a packet in relation to the maximum round-trip propagation delay 2τ of the network**.

Comparison Among CSMA Schemes

- A less aggressive CSMA protocol means higher throughput, because there are fewer collisions, but also higher delays. The 1-persistent CSMA scheme provides good-enough throughput and packet delay if S < 0.5 Erl.
- As a approaches 1, the maximum throughput achievable by CSMA protocols reduces below the maximum ones of Aloha and Slotted Aloha.





E[R] = 4 [T units], a = 0.01 [T units] and jam message duration = 0.2 [T units]

Ethernet LAN

- The Ethernet LAN was realized in 1976 when Xerox (Robert Metcalfe) adopted the CSMA/CD protocol to implement a network at 1.94 Mbit/s to connect more than 100 terminals.

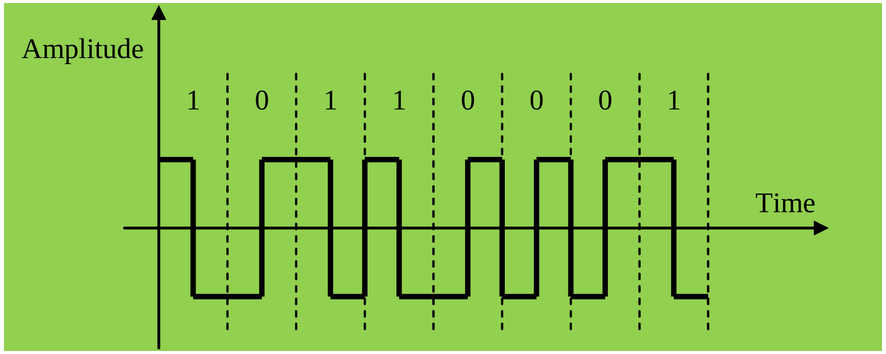
- The IEEE 802 committee started to develop a LAN standard based on CSMA/CD, similar to the Ethernet and was called IEEE 802.3.

Ethernet LAN (cont'd)

- The IEEE standard specifies both physical and MAC layer. Conceptually, the IEEE 802.3 standard is related to a bus topology and a broadcast medium.
- Two modes of operation are allowed by the MAC layer:
  - **Half-duplex transmissions**: stations (terminals) contend for the use of the physical medium by means of the CSMA/CD protocol.
  - **Full-duplex transmissions**: it has been introduced later and can be used when the physical medium is capable of supporting simultaneous transmission and reception without interference. The LAN is formed by point-to-point links. The typical topology for a full-duplex mode is a **central switch with a dedicated connection to each device**. This is the so-called "switched Ethernet"; nowadays, this is the prevailing LAN technology.

Ethernet LAN, Half-Duplex Operation Mode

- The following description is related to the IEEE 802.3 half-duplex operation mode, characterized as follows:
  - **1-persistent CSMA/CD** access protocol with **truncated binary exponential backoff**;
  - **Base-band transmissions of bits with Manchester encoding** (each bit contains a transition in the middle). Thus, the clock can be recovered from the bit stream and the signal has no DC component.



Ethernet LAN, Half-Duplex Operation Mode (cont'd)

- A station has one packet to transmit:
  - If no carrier signal is revealed, the station waits for an InterFrame Gap (IFG) and then transmits (no further carrier sense verification is performed). This is a 1-persistent-like behavior.
  - Whereas, if the medium is sensed busy, the station defers the transmission.
- With CD, if the receiver interface reveals a signal when a station is transmitting (revealing an increase in the average voltage level on the line), a collision event is assumed.
  - According to the CSMA/CD protocol, the transmitting station revealing this collision sends a 32-bit **jam message** (also 48 bit jam messages are possible) to allow that all other involved stations abandon their corrupted frames.
  - Then, a **retransmission procedure** is started on the basis of a truncated binary exponential backoff algorithm. Soon after the first collision, time is slotted; one **slot time** T_s corresponds to the time to transmit a minimum frame of 64 bytes.
  - The **transmission time of the minimum frame must be greater than or equal to the maximum round trip propagation delay 2**τ.

Token Ring Protocol

- Random access protocols do not guarantee fairness or bounded access delays for real-time traffics.
- Other access protocols have been investigated that allow a more regulated access of the terminals to the shared physical medium.
- Two different types of protocols can be considered:
  - Reservation protocols and
  - Token-based (including polling) schemes.

Token Ring Protocol (cont'd)

- This scheme is based on a **cyclic authorization** according to which terminals are enabled to transmit.
- The polled terminal is enabled to transmit the contents of its buffer. Two main techniques can be considered:
  - **Gated technique**: a terminal sends only the packets that are in the buffer at the instant of the arrival of the authorization to transmit.
  - **Exhaustive technique**: a terminal sends all packets in its buffer when it receives the authorization to transmit (i.e., a terminal releases the control only when its buffer is empty).

Token Ring Protocol (cont'd)

- A typical ring topology (either physical or logical) is used for these LANs.
- A **token** rotates around a ring in turn to each node. All nodes (terminals, routers, etc.) copy all data and tokens (input interface), and repeat them along the ring (output interface).
- When a node wishes to transmit packet(s), it grabs the token as it passes and holds the token while it transmits. When the transmission completes, the node releases the token and sends it on its way.
- Two variants of the token ring protocol are possible depending on the adopted policy to release a token on behalf of the station that has completed a transmission.
  - **Release After Reception (RAR)**: A node captures the token, transmits data, waits for data to successfully travel around the ring, and then releases the token. Such approach allows nodes to detect erroneous frames and to retransmit them.
  - **Release After Transmission (RAT)**: A node captures the token, transmits data, and then releases the token so that the next node can use the token after a short propagation delay.

IEEE Token-Based Standards

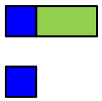
- The IEEE standards for token protocols are:
  - IEEE 802.4 for a **bus** topology (token bus standard)
  - IEEE 802.5 (IBM - 1976) for a **ring** topology (token ring standard) with **RAR approach**.
- In IEEE 802.5, the **token is a small 3-byte packet circulating the ring or contained in the header of a transmitted frame**.
  - The token is composed of a token delimiter (1 byte, where the line encoding scheme is violated to distinguish such byte from the rest of the frame), an access control field (1 byte) and an end of token (1 byte).
  - A free token is a 3-byte message that is used to release the control to the next station according to the cycle order.

IEEE 802.5

- IEEE 802.5 adopts a sophisticated **priority system** that permits certain user-designated, high-priority stations to use the network more frequently than other stations. 8 priority levels are used and specified in the access control field of the token.

- The access control byte has two fields that control priority: the priority field and the reservation field.
  - Only the stations with a priority equal to or higher than the priority value contained in a token can seize that token.
  - After the token is seized and changed to an information frame, only stations with a priority value higher than that of the transmitting station can **reserve the token** for the next pass around the network.
  - When the next token is generated, it includes the higher priority of the reserving station. Stations raising the token priority level must reset the previous priority when their transmission ends.

IEEE 802.5 and the RAR Scheme

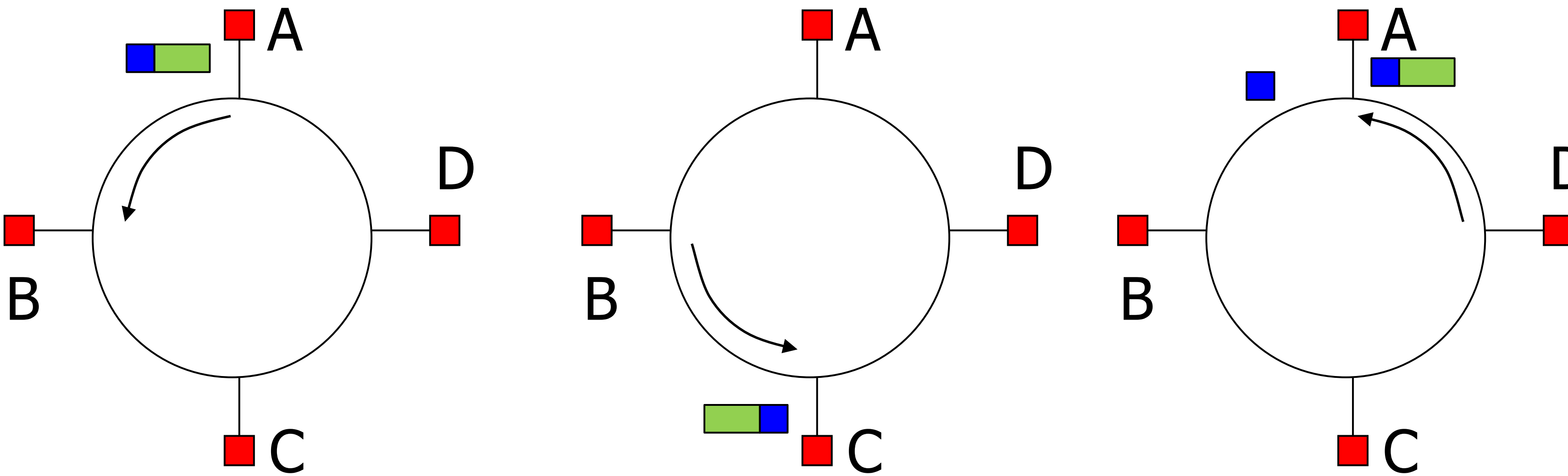


= Frame with header (= token)

= Free token

Message sent by A with the token

The message travels back to the source A where a free token is released in the ring (RAR)



Message received by the destination B

Analysis of CSMA and Token Ring Protocols

Un-Slotted non-Persistent CSMA Analysis

- For the shared medium, a cycle is composed of a busy period (B, during which there are packet transmissions) and the subsequent idle period (I, during which the medium is unutilized).
- We assume a Poisson arrival process for new packets with mean rate λ. The offered traffic intensity (= throughput, under stability assumption) is S = λT Erl; whereas the total circulating traffic intensity (new arrivals plus retransmissions due to collisions, with total mean rate Λ) is G = ΛT Erl.
- Let U denote the time during a cycle that the channel is used to successfully transmit a packet (i.e., without collisions).
- The channel throughput S can be obtained by means of the following formula:

$$S = \frac{E[U]}{E[B] + E[I]}$$

Un-Slotted non-Persistent CSMA Analysis (cont'd)

- **Idle period analysis** - due to the assumption of Poisson arrivals with total mean rate Λ (new arrivals plus retransmissions):

$$E[I] = \frac{1}{\Lambda}$$

- **Useful period analysis** - the transmission of a packet of duration T is successful if there is no other packet generation in the vulnerability window τ (at the beginning of packet transmission); this occurs with the probability of no arrivals in the window τ for the total Poisson process with mean rate Λ:

$$U = \begin{cases} T, & \text{with prob.}\, e^{-\Lambda\tau} \\ 0, & \text{otherwise} \end{cases} \quad \Rightarrow \quad E[U] = Te^{-\Lambda\tau}$$

- **Busy period analysis** - B = T + τ, in case of a successful packet transmission. Note that the packet transmission needs a time T and a further time τ is necessary to have that a free channel condition is perceived by all terminals. While, B > T + τ in a busy period with multiple packet transmissions (there are collisions). In general, we may write: B = Y + T + τ, where Y $\in$ [0, τ] (Y = 0 in case of no collisions).

$$F_Y(x) = \text{Prob}\{Y \le x\} =$$

$$= \text{Prob}\{\text{no arrival in the interval of length } \tau - x\} = e^{-\Lambda(\tau - x)} \quad \Rightarrow \quad E[Y] = \tau + \frac{e^{-\Lambda\tau} - 1}{\Lambda}$$

Un-Slotted non-Persistent CSMA Analysis (cont'd)

- Since $\tau/T = a$ and $\Lambda\tau = Ga$, we conclude:

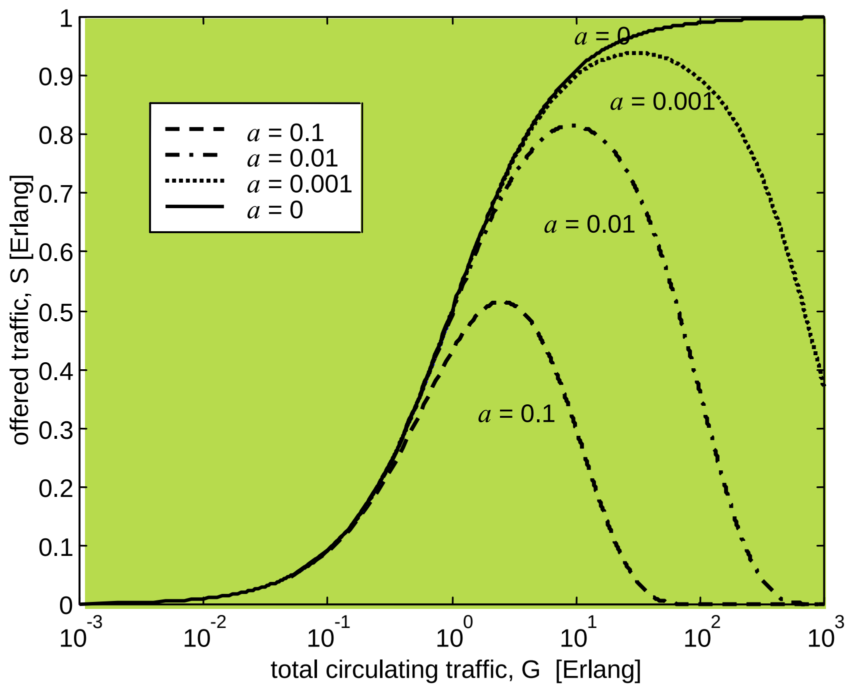
$$S = \frac{E[U]}{E[B]+E[I]} = \frac{Ge^{-Ga}}{G(1+2a)+e^{-Ga}}$$

The peak of the throughput increases as a decreases.

In the limiting (and ideal) case for $a \to 0$, we have the following simple result:

$$S = \frac{G}{G+1} \quad \Leftrightarrow \quad G = \frac{S}{1-S}$$

If a is ideally 0, there are no collisions and the throughput has a steady increase to 1 as G increases (there is no instability phenomenon).



- The mean packet delay can be expressed analogously to the study carried out for Aloha schemes:

$$E[T_p] = \left(\frac{G}{S}-1\right)\{T+\Delta+E[R]\}+T+\frac{\Delta}{2} =$$

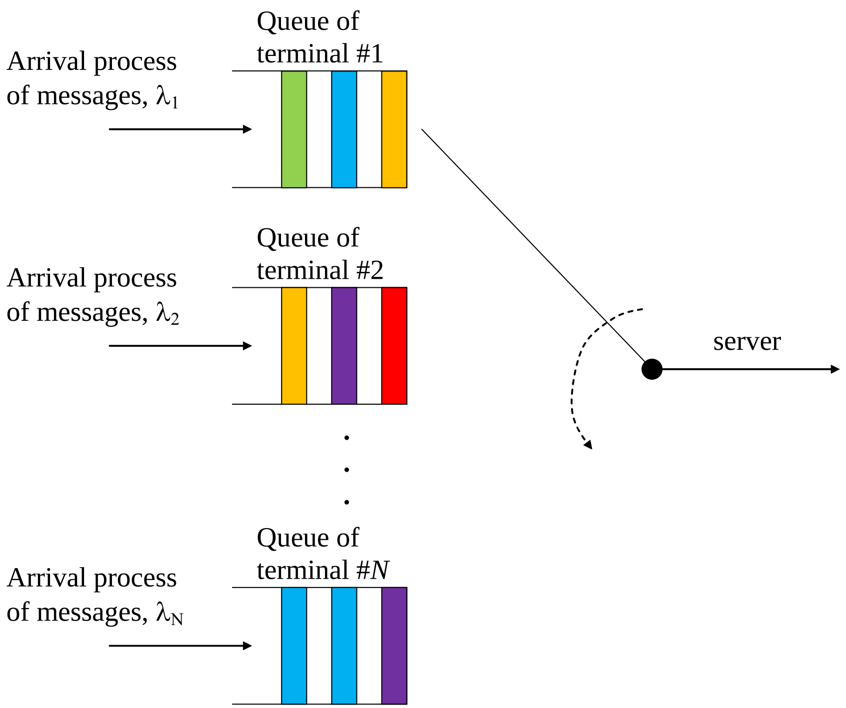
$$= Ge^{Ga}(1+2a)\{T+\Delta+E[R]\}+T+\frac{\Delta}{2}$$

Analysis of the Token Ring Protocol

- The generic i-th station (i = 1, …, N) on the ring has a buffer (= queue) with an input process characterized by a mean message arrival rate λ_i.
- Each message has a random length in packets l_i that, in general, may have a different distribution from queue to queue.
- Let T denote the packet transmission time.
- Let T_i denote the service time for the i-th queue.
- Let δ_i denote the overhead time (deterministic value) to switch the service from the i-th queue to the next (i+1)-th queue according to the service cycle.
- The overhead time depends on both the adopted protocol and the LAN topology.
  - In a token ring network with RAT scheme, δ_i is the propagation delay from terminal i to terminal i+1 (including a synchronization time for terminal i+1). If terminals are at the same distance on the ring and if τ is the total propagation delay on the ring, we have: $\delta_i = \delta = \tau/N$.

Analysis of the Token Ring Protocol (cont'd)

- We are interested in characterizing the cycle time T_c, that is the time interval from the instant when the server starts to service a generic queue to the instant when the server 'comes back' to the same queue (after having completed the cycle).



$$T_c = \sum_{i=1}^{N} (T_i + \delta_i)$$

$$E[T_c] = E\left[\sum_{i=1}^{N} (T_i + \delta_i)\right] \Rightarrow$$

$$E[T_c] = \sum_{i=1}^{N} (E[T_i] + \delta_i)$$

$$E[T_i] = \lambda_i E[T_c] E[l_i] T$$

Analysis of the Token Ring Protocol (cont'd)

$$E[T_c] = \frac{\sum_{i=1}^{N} \delta_i}{1 - \sum_{i=1}^{N} \lambda_i E[l_i] T}$$

- This study is valid only in the case that the total system overhead $\Sigma\delta_i > 0$.
- The value of $E[T_c]$ is finite if the following stability condition is fulfilled:

$$\rho_{tot} = \sum_{i=1}^{N} \lambda_i E[l_i] T < 1 \ [\mathrm{Erl}]$$

If $\rho_{tot} \to 1$ Erl the network becomes congested.

- $E[T_c]/2$ is the mean delay a packet arriving at an empty queue must wait for the arrival of the server.

Analysis of the Token Ring Protocol (cont'd)

- Let us consider that the arrival processes to the different queues are Poisson and independent. Let us assume that the buffers have infinite capacity.
  - Then, the queuing behavior experienced by the messages in the whole token ring network can be described by means of an M/G/1 global queue (see Lesson No. 7) with a corrective term.
  - In the case of constant overhead times ($\delta_i = \delta$) with all N stations having the same traffic characteristics ($l_i = l$, $\lambda_i = \lambda$), the mean message delay, $E[T_m]$, and the mean message transfer delay (from source station to destination station), $E[T_{transf}]$, are obtained as:

$$E[T_m] = \frac{E[T_c]}{2} \times \begin{cases} (1-\lambda E[X]), & \text{exhaustive} \\ (1+\lambda E[X]), & \text{gated} \end{cases} + E[X] + \frac{N\lambda E[X^2]}{2[1-N\lambda E[X]]} \qquad E[T_{transf}] = E[T_m] + \frac{1}{2}\sum_{i=1}^{N}\delta_i$$

where $X = l \times T$ and $E[T_c] = N\delta/(1 - N\lambda E[X])$.

Efficiency Comparison of Ethernet and Token Ring

Ethernet Efficiency Analysis

- The efficiency analysis is carried out considering that the time on the transmission medium is divided between intervals spent to successfully transmit data (useful intervals) and intervals spent to contend for the transmission on the broadcast medium (contention intervals).

- **Saturation study hypothesis:** We consider that after each successful transmission phase, there are always **N stations** that contend for the transmission of their packets.

- By means of the CD scheme, a station knows that its transmission is successful or not within a time 2τ from the starting instant of its transmission, **we ideally consider that the contention interval is (mini)slotted with duration 2τ.**

- Every contending station may decide **to transmit (according to its backoff algorithm) at each slot with probability q** and knows the result (success or collision) within the end of the slot.

Ethernet Efficiency Analysis (cont'd)

- A slot carries a successful transmission attempt of a station with the probability $P_s(N, q)$ that only one station transmits on that slot:

$$P_s(N,q) = Nq(1-q)^{N-1}$$

- $P_s(N, q)$ is equal to 0 for both q = 0 and q = 1. $P_s(N, q)$ has a maximum for q = 1/N; correspondingly, $P_{s,opt}(N, q=1/N) = P_{s,opt}(N)$ results as:

$$P_{s,opt}(N) = \left(1 - \frac{1}{N}\right)^{N-1}$$

- The mean number of slots for the first successful transmission, $E[n_{slot}]$, results as:

$$E[n_{slot}] = \frac{1}{P_{s,opt}(N)} = \left(1 - \frac{1}{N}\right)^{1-N}$$

Ethernet Efficiency Analysis (cont'd)

- The mean length of the contention phase is $E[C] = 2\tau(E[n_{slot}]-1)$, since we have to exclude the last slot where the correct packet transmission starts. The efficiency of CSMA/CD $\eta_{CSMA/CD}$ results as:

$$\eta_{CSMA/CD}(N) = \frac{T}{T+E[C]} = \frac{1}{1+2a\left[\left(1-\frac{1}{N}\right)^{1-N}-1\right]}$$

- The above $\eta_{CSMA/CD}$ can be considered as an estimate of the maximum throughput (with optimized transmission probability q) S in Erlangs that the CSMA/CD protocol can achieve.

- The longer the propagation delay (i.e., a), the lower the efficiency. Moreover, the efficiency decreases with N. The limiting $\eta_{CSMA/CD}$ value for $N \to \infty$ is as follows:

$$\lim_{N\to\infty} \eta_{CSMA/CD}(N) = \frac{1}{1+2a[e-1]} \approx \frac{1}{1+3.43a}$$

Token Ring Efficiency Analysis

- Assumptions:
  - We refer to the **RAR policy**: if a station transmits a frame, it releases the token when it receives the transmitted frame that has propagated on all the ring.
  - Once a ring station acquires the token, it has always to transmit just one packet of fixed length T.
  - There are N equi-spaced stations on the ring. τ denotes the full propagation delay on the ring.
  - **Saturation study hypothesis**: ring resources are used according to a periodic sequence in time of
    - Packet transmission time, including the propagation time back to the originating station to notify the release of the token (busy line interval), B, and
    - Time to propagate the free token to the next station (protocol overhead interval), O_N.
- The Token ring efficiency $\eta_{\text{token ring}}$ is:

$$\eta_{token\ \ ring}(N) = \frac{T}{B + O_N} = \frac{1}{\frac{B}{T} + \frac{O_N}{T}}$$

Token Ring Efficiency Analysis (cont'd)

- **<u>Case with a < 1</u> (i.e., τ < T)**
  - A reference station receives the free token at time t = 0 and starts to transmit a packet. At time t = aT < T, the station starts to receive the packet that has propagated along the ring. At time t = T, the transmission of the packet of our station ends and the station releases the token. The released token reaches the next station in the ring after a time τ/N. Hence, B/T = 1 and O_N/T= a/N and the efficiency is:

$$\eta_{token\ \ ring,a<1}(N) = \frac{1}{1+\dfrac{a}{N}}$$

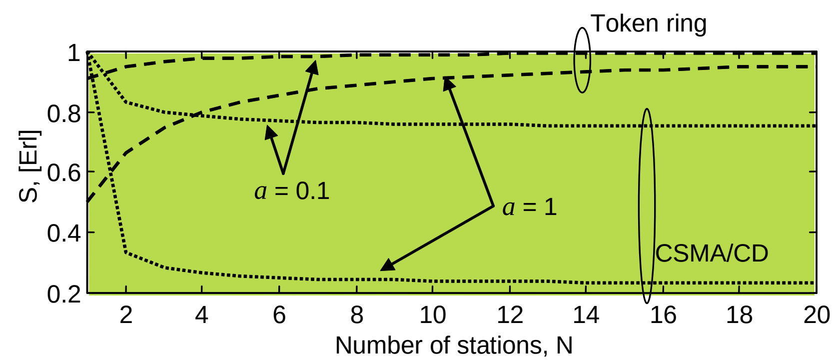
- **<u>Case with a > 1</u> (i.e., τ > T)**
  - A reference station receives the free token at time t = 0 and starts to transmit a packet. At time t = T, the transmission of the packet of our station ends. At time t = aT > T, the station starts to receive the packet that has propagated along the ring and the station releases the token. The released token reaches the next station in the ring after a time τ/N. Hence, B/T = a and O_N/T = a/N and the efficiency is:

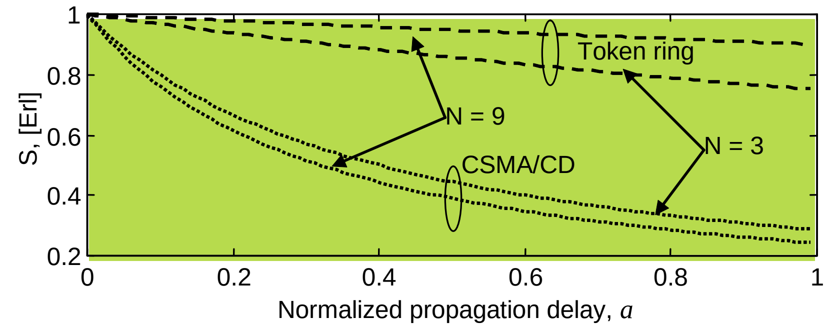
$$\eta_{token\ \ ring,a>1}(N) = \frac{1}{a+\dfrac{a}{N}}$$

For N $\rightarrow \infty$, we have the maximum efficiency equal to 1/max(1 , a).

Comparisons

- We compare the optimal efficiency of CSMA/CD and that of the token ring protocol. In both cases **efficiency corresponds to the maximum S value supported by the protocol**.
- Parameters:
  - Number of stations N
  - Normalized maximum propagation delay a (depending on the physical length of the LAN and the transmission bit-rate).
- Token ring efficiency increases with N due to the reduction in the time to send the token to the next station. Whereas, CSMA/CD efficiency decreases with N due to increased collision rate.
- The efficiencies of both CSMA/CD and token ring decrease with a.





Exercises on MAC Protocols

Exercise #1

- Let us consider a Slotted Aloha system, where packets arrive according to a Poisson process with mean rate λ and are transmitted in a time T. The packet transmission power is selected between two levels (namely P_1 and P_2, with $P_1 >> P_2$) with the same probability. This mechanism allows a partial capture effect, as follows:
  - Two simultaneously-transmitted packets of the same power level class collide destructively (i.e., both packets are destroyed).
  - A packet transmitted at power level P_1 is always received correctly if it collides with any number of simultaneous transmissions with power level P_2 (partial capture effect).
- It is requested to determine the relation between the intensity of the offered traffic, S, and the intensity of the total circulating traffic, G. Can this access protocol support an input traffic intensity of 0.5 Erl ? Finally, it is requested to derive the mean packet delay.

Solution of Exercise #1

- Λ denotes the mean packet arrival rate of the total circulating traffic (i.e., new arrivals and retransmissions). The offered traffic intensity is $S = \lambda T$. The intensity of the total circulating traffic is $G = \Lambda T$.
- S and G are related by the classical formula $S/G = P_s$ where we need to derive the probability of a successful packet transmission P_s.
- When a packet is transmitted, one of the two power levels is chosen at random with equal probability. We have two cases:
  - **Packet transmission at power level P_1**: Such transmission is successful with the probability $P_{s|1}$ that no other type #1 transmission is performed on the same slot. Since transmissions are equally distributed on the two power levels, we have: $P_{s|1} = e^{-\Lambda T/2} = e^{-G/2}$.
  - **Packet transmission at power level P_2**: Such transmission is successful with the probability $P_{s|2}$ that no other transmission is performed on the same slot; we have: $P_{s|2} = e^{-\Lambda T/2} \times e^{-\Lambda T/2} = e^{-G}$.

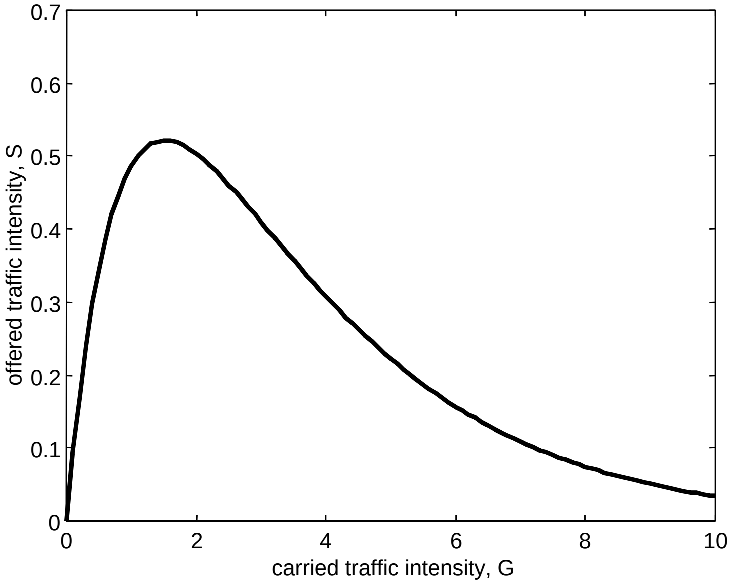
Solution (cont'd)

- We can combine the two above equiprobable cases in order to obtain P_s:

$$P_s = \frac{1}{2} P_{s|1} + \frac{1}{2} P_{s|2} = \frac{e^{-\frac{G}{2}} + e^{-G}}{2}$$

- The corresponding expression of S as a function of G is:

$$S = \frac{Ge^{-\frac{G}{2}} + Ge^{-G}}{2}$$



Solution (cont'd)

- The maximum of the carried traffic S can be obtained by the null-derivative condition for S = S(G). Due to the particular expression of this S = S(G) function, the null-derivative condition has not a solution that can be expressed in a closed form.
- Through numerical evaluations, the maximum S value is about 0.5216 Erl for G $\approx$ 1.5 Erl. Hence, this protocol can support an input traffic intensity of 0.5 Erl.
- The mean packet delay is obtained as:

$$E[T_p] = \frac{T}{2} + \left(\frac{1}{P_s} - 1\right)\{T + \Delta + E[R]\} + T + \frac{\Delta}{2} \quad \text{for } G \leq 1.5$$

where Δ denotes the round-trip propagation delay (from the remote terminal to the central controller and, then, back to the remote terminal), E[R] denotes the mean delay used for each packet retransmission, and $1/P_s$ is obtained from the above S = S(G) expression of this access protocol.

Exercise #2

- We have a LAN adopting the unslotted non-persistent CSMA protocol with N = 10 stations. Each station generates new packets according to exponentially distributed interarrival times with mean value D = 1 s. The packet transmission time is T = 10 ms. The maximum propagation delay is τ = 0.6 ms.
  - Determine the approximate relation between the offered traffic intensity, S, and the total circulating traffic intensity, G.
  - Determine the total traffic intensity generated by the N stations in Erlangs.
  - Study the stability of the non-persistent protocol in this particular case and in general.

Solution of Exercise #2

- The arrival process of new packets is Poisson with mean rate λ = 1/D = 1 pkts/s for each station. The maximum propagation delay τ = 0.6 ms is much lower than the packet transmission time T = 10 ms. In this case, parameter $a = \tau /T$ is close to 0. Correspondingly, the offered traffic S and the total circulating traffic G can be related as:

$$S = \frac{G}{G+1}$$

- The intensity of the traffic offered by the N stations is S = NλT = 0.1 Erl.
- In this study $a \approx 0$ and the non-persistent CSMA scheme is always stable and can support up to 1 Erl of input traffic. This is an optimal situation.
- If in general $a > 0$, S = S(G) curve has a maximum highlighting a maximum input traffic beyond which the non-persistent CSMA scheme becomes unstable.
- With the total input traffic of 0.1 Erl envisaged in this exercise, the access protocol is stable even if a is greater than 0.

Exercise #3

- Let us refer to a ring LAN with M = 6 stations where the token ring protocol of the exhaustive type is adopted. We know that the time to send the token from one station to another is δ = 0.5 ms, equal for all stations. The rate according to which packets of fixed length are sent in the ring is μ = 20 pkts/s. The arrival process of messages at a station is Poisson with mean rate of λ = 1 msgs/s. Messages have a length l_p ($\geq$ 1) in packets according to the following distribution:

$$\mathrm{Prob}\{l_p = n\,\mathrm{pkts}\} = \frac{1}{1-(1-0.3)^5}\binom{5}{n}0.3^n(1-0.3)^{5-n}, \quad n \in \{1, 2, 3, 4, 5\}$$

- It is requested to determine the following quantities:
  - The mean cycle duration,
  - The stability condition for the buffers of the stations on the ring,
  - The mean transfer delay from the message arrival at the buffer of a station to the instant when the message is delivered to another station on the ring. In this case, we have to refer to an exhaustive service policy for the buffers of the stations.

Solution of Exercise #3

- All stations of the ring contribute the same traffic load (i.e., the same message arrival process and the same message length distribution).
- We focus on the distribution of the number of packets per message. This is a binomial distribution truncated because of the removal of the value '0'. The PGF of the message length, $L_p(z)$, results as:

$$L_p(z) = \frac{\sum_{n=1}^{5} \binom{5}{n} 0.3^n (1-0.3)^{5-n} z^n}{1-(1-0.3)^5} = \frac{(1-0.3+0.3z)^5 - (1-0.3)^5}{1-(1-0.3)^5}$$

- By means of the above PGF it is easy to determine both $E[l_p]$ and $E[l_p^2]$ as:

$$E[l_p] = \frac{d}{dz} L_p(z)\Big|_{z=1} = \frac{5 \times (1-0.3+0.3z)^4 \times 0.3}{1-(1-0.3)^5}\Bigg|_{z=1} \approx \frac{5 \times 0.3}{0.83} \approx 1.87 \quad \left[\frac{\text{pkts}}{\text{msg}}\right]$$

$$E[l_p^2] = \frac{d^2}{dz^2} L_p(z)\Big|_{z=1} + \frac{d}{dz} L_p(z)\Big|_{z=1} \approx \frac{5 \times 4 \times 0.3 \times 0.3 + 5 \times 0.3}{0.83} \approx 3.97 \quad \left[\frac{\text{pkts}^2}{\text{msg}}\right]$$

Solution (cont'd)

- The mean duration of a cycle can be obtained as:

$$E[T_c] = \frac{M\delta}{1 - M\lambda \frac{E[l_p]}{\mu}} = 6.83 \quad [\text{ms}]$$

- The stability conditions for the buffers of the stations on the ring is that the total traffic intensity is lower than 1 Erl:

$$M\lambda \frac{E[l_p]}{\mu} = 0.56 < 1 \quad [\text{Erl}]$$

- Finally, we have to determine the mean transfer delay for a message, $E[T_{transf}]$, for the exhaustive discipline:

$$E[T_{transf}] = \frac{E[l_p]}{\mu} + \frac{M\lambda \frac{E[l_p^2]}{\mu^2}}{2\left[1 - M\lambda \frac{E[l_p]}{\mu}\right]} + \left[\frac{M\delta}{1 - M\lambda \frac{E[l_p]}{\mu}}\right] \frac{1}{2}\left(1 - \lambda \frac{E[l_p]}{\mu}\right) + \frac{1}{2} M\delta \approx 0.16 \quad [\text{s}]$$

Thank you!

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