# Chapter 1
# Vector Algebra I: Scalars and Vectors

Dear reader. This study guide will help you to study efficiently with the textbook. Your study periods will alternate between the textbook and this study guide. All you need will be explained in due time. Now start and turn overleaf to page

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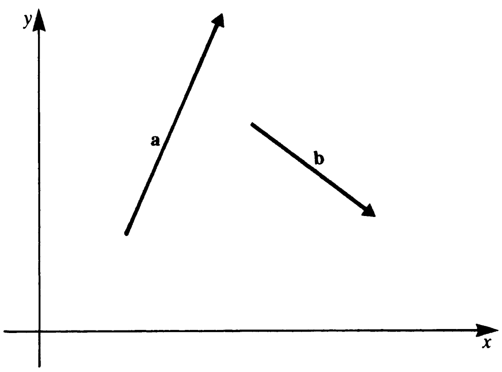
57

Draw the projection of **a** on to **b**.




If you are still experiencing difficulty read the relevant parts of the textbook once more, and then solve this exercise with the help of the construction given in the textbook.

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## 1.1 Scalars and Vectors

2

Your first task is to study in the textbook Sect. 1.1, Scalars and vectors. When you have completed this simple and limited task carry on with this programmed study guide.

The introduction to each section in this study guide will always have the same structure. The objectives are named in catchwords, so you know in advance what you will learn.

Thus, for this section:

**Objective:** Concepts of scalars, vectors, directed segments, line of action, free vector.

**READ: 1.1 Scalars and vectors**
**Textbook pages 1–4**

Afterwards, move on to the next frame in this study guide, i.e. go to frame 3

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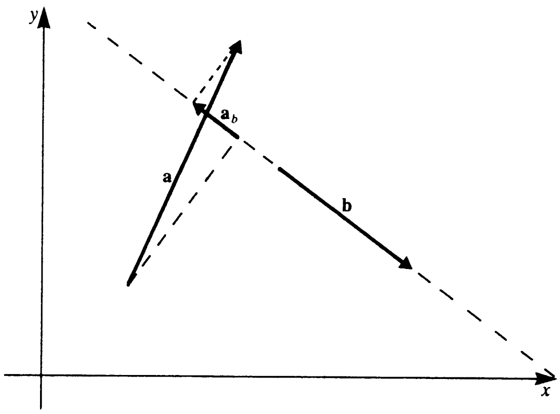
58




The difficulty in this case is that the line of action of **b** crosses that of **a**. But the principle is still the same, namely to drop perpendiculars from the start and tip of **a** on to the line of action of **b**, as shown in the drawing.

------------------▷ 59

3

After you have studied a particular section in the textbook you are asked to answer questions in the programmed study guide. Thus you test your progress. Even if you have understood the text you may not always retain all of it.

You should know and remember the following concepts:

Definition of a scalar quantity:
Scalars are defined by their ............................

Definition of a vector quantity:
Vectors are defined by their ............................ and ............................

Write down your answers in a separate sheet. The dotted lines always indicate that answers are required. You will find the answers on the top of the next frame.
Go to frame 4.

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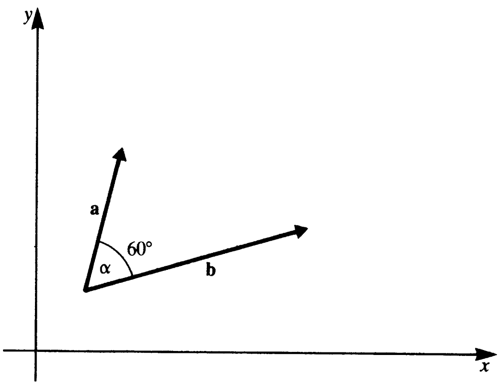
59

Numerical calculation of the projection of a vector:
If $|\mathbf{a}| = 3,\ |\mathbf{b}| = 4$ and $\alpha = 60°$, calculate the magnitude of the projection of **a** on to **b**.




$a_b =$ ............................

*Hint* : $\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 60° = 0.5$

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4

At this place you will always find our answers for you to check your own.

Scalars are defined by their *magnitude*.
Vectors are defined by their *magnitude* and *direction*.

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A vector can be represented geometrically by a ............................
A vector from the origin of a coordinate system to a point P is called ......................
A vector of magnitude 1 is called ............................

When learning, be strong-minded; look up our answers only after you have completed yours!
You certainly know by now that the arrow points to the number of the frame to come.

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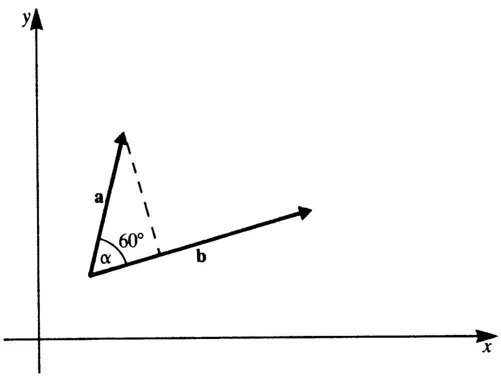
60




$$\begin{aligned} a_b &= a\cos\alpha \\ &= 3\cos 60^\circ \\ &= 3\times 0.5 = 1.5 \end{aligned}$$

Note that the magnitude of **b** is of no importance.

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directed line
position vector
unit vector

5

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Of the following, which are scalars and which are vectors?

| | |
|---|---|
| mass | ............................. |
| temperature | ............................. |
| field strength | ............................. |
| force | ............................. |
| gravitational field strength | ............................. |
| density | ............................. |
| pressure | ............................. |
| time | ............................. |
| displacement | ............................. |
| velocity | ............................. |
| acceleration | ............................. |

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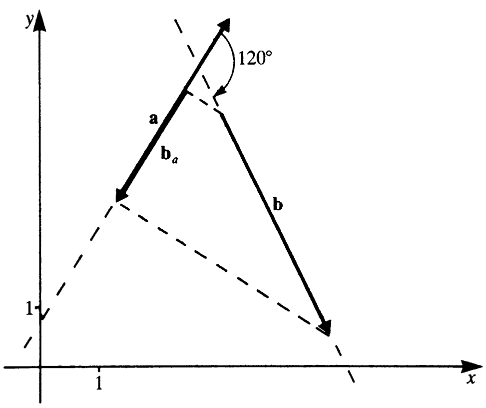
Obtain graphically the numerical value (magnitude) of the projection of **b** on to **a** and check it mathematically: $a = 3,\ b = 4,\ \alpha\ = ?$

61




$b_a =$ .............................

In case of difficulty refer to the textbook.

------------------▷ 62

6

| | |
|---|---|
| mass | scalar |
| temperature | scalar |
| field strength | vector |
| force | vector |
| gravitational field strength | vector |
| density | scalar |
| pressure | scalar |
| time | scalar |

Displacement, velocity and acceleration are all vectors.

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Now decide how you will proceed:

If everything you have done so far was correct then go to --------------------▷ 8

If you made a mistake and thought that pressure was a vector then go to --------------------▷ 7

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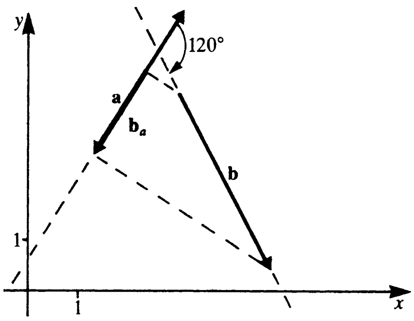
62




$$b_a = b\cos 60^\circ = 4 \times 0.5 = 2$$

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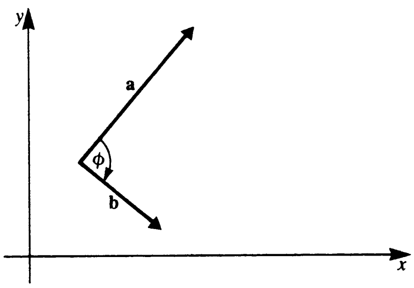


Obtain graphically and mathematically the numerical value of the projection of **a** on to **b**:

$a = 4,\ b = 2,\ \phi = \frac{\pi}{2}$

$a_b = \ldots\ldots\ldots\ldots\ldots\ldots$

--------------------▷ 63

Pressure is a scalar, not a vector, because it has no preferred direction; it acts in all directions. **7**

The scalar quantity *pressure* and the vector quantity *force* are connected in this case by a physical relationship.

Consider a gas inside a cylinder; each point in the cylinder is under the same pressure. It has no direction. But the pressure exerts a force on the wall of the cylinder. The direction of this force is not determined by the pressure but by the direction of the wall, and it always acts in a direction perpendicular to the wall.

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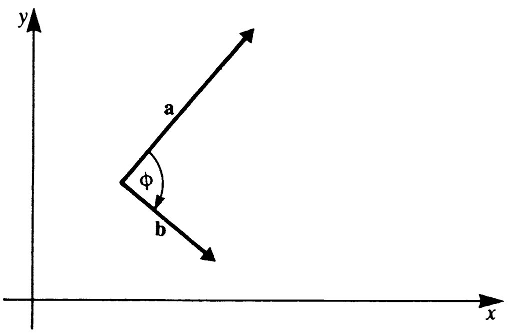


$a_b = 0$, because $\cos\frac{\pi}{2} = 0$ **63**

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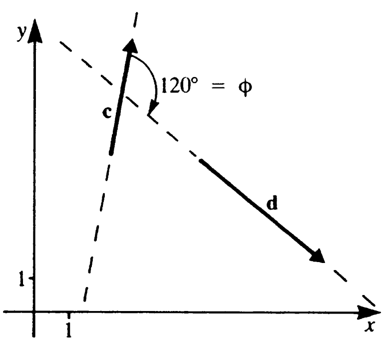


Obtain graphically and mathematically the projection of **c** on to **d**:

$c = 4,\ d = 5,\ \phi = 120°$

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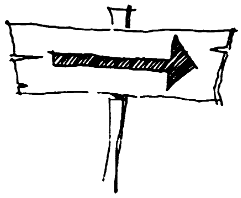
Vector quantities are conveniently represented by means of arrows. With the help of an arrow we can represent ............................ and ............................ of the physical quantity. 8



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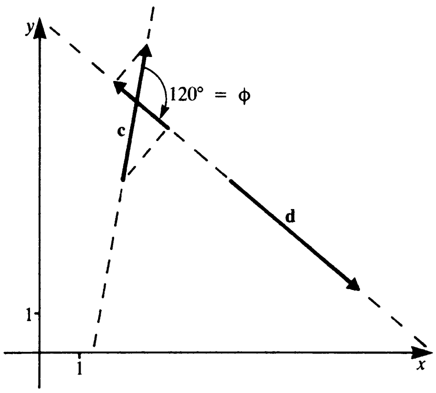
64




$$
\begin{aligned}
c_d &= c\cos 120^\circ \\
&= 4 \times (-0.5) \\
&= -2
\end{aligned}
$$

65

*magnitude* and *direction* 9

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As an exercise in notation, which symbols are used to represent vectors?

☐ $\mathbf{b}$ ☐ $\overrightarrow{PQ}$
☐ $\left|\overrightarrow{PQ}\right|$ ☐ PQ

Tick the appropriate boxes!

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## 1.4 Planning working periods and breaks.

65

All creatures get tired, human beings included.

Occasionally we have to have a break, particularly if we are tired. Should we have a break when we are so tired that our eyelids are drooping? Yes, of course.

But it is important to have a break before that stage is reached. Having breaks in good time can delay a decrease in concentration and therefore in performance and achievement.

Typical findings of psychological experiments are presented on the next page.

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$\mathbf{b}, \overrightarrow{PQ}$

10

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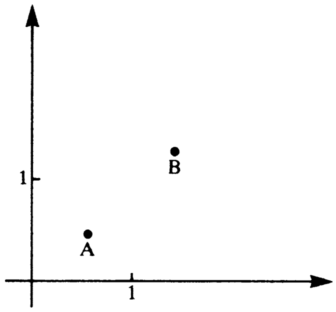


A car travels from A to B. Can this change in position be represented by a vector?
☐ Yes
☐ No

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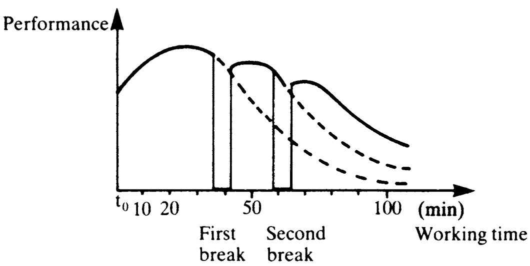
66

The performance of students at study has been determined experimentally and is represented as a function of working time on the graph.




The broken lines indicate the trend without a break.

Breaks delay a loss in performance.

That means you should divide your work into well-defined working periods. In this programmed study guide breaks are suggested. The extent of a working period depends on the difficulty of the content and is not easy to forecast. Hence the working periods in the study guide are chosen to be short rather than long.

The most profitable working periods for total concentration lie between 20 and 40 minutes.

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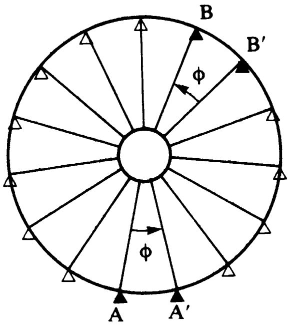
Yes

11




The diagram shows a big wheel often found in fairgrounds. Neil is sitting in gondola A and Mary in gondola B. The wheel rotates through an angle $\phi$. Insert the vectors $\overrightarrow{AA'}$ and $\overrightarrow{BB'}$.

Have both vectors the same direction?
☐ Yes
☐ No

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67

After studying a section you should check what you have retained. It is no use reading something and immediately forgetting the most important aspects.

Before a break write down the keywords from the section just studied; this is most useful. If you find that you have already forgotten what you have just read then read the material in the textbook once more.

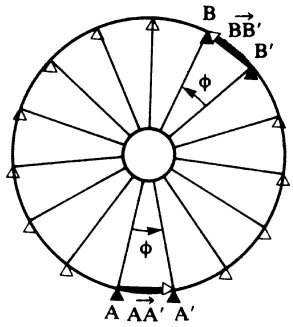
Therefore, before you start a break: *Check that you have achieved the objectives of the section by expressing them in your own words.*

The next frame shows the findings of a psychological experiment, but you may leave it out.

Continue with the programmed study guide ------------------▷ 70

Investigation of active versus passive learning modes ------------------▷ 68

No

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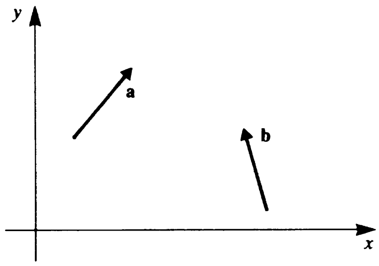


Given two vectors **a** and **b**. Shift **a** and **b** along their respective directions and plot equivalent vectors **a** and **b**.

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**Experimental design:**

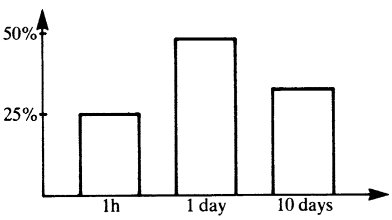
68

Group A: A section in a textbook is read four times.
Group B: The same section in the textbook is read twice and reproduced actively after each reading.

The ability to recall the material is checked after certain periods of time.
**Result:** The diagram shows the *difference* in the ability to recall subject matter between the groups.

Group B (active reproduction) performs better at any time.

**Conclusion:** Actively acquired knowledge is easier to recall than passively acquired knowledge.




Recall performance of group B compared with that of group A.

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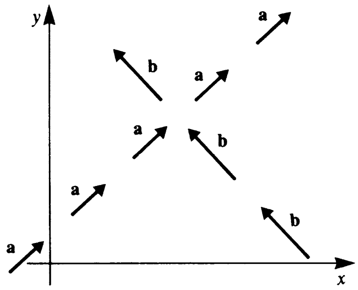


The line which defines the direction of the vectors is called the *line of action*. 13

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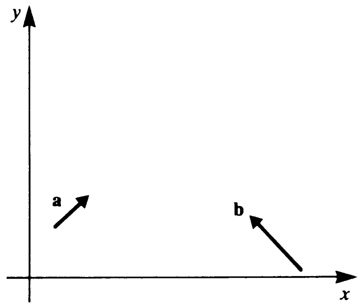


Plot equivalent vectors which are shifted parallel to **a** and **b**.

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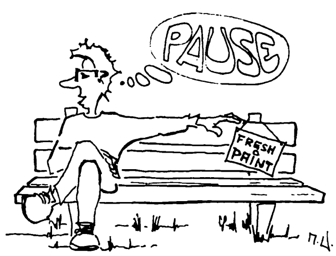
To avoid failure it is important not to attempt to take 'too big a bite'. It is better to study a limited number of pages in the textbook at a time, and to check one's progress at the end of each study period. 69

If you can, it is useful to work with friends because you can help and check each other. Not only is it easier to test someone else, but working with friends also helps you to express yourself orally on the subject matter.

You should also remember that breaks form an integral part of your studies. They should be properly organised and they should not be so long that your work is completely interrupted.

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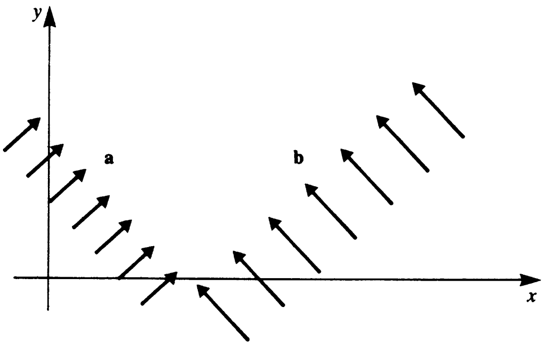
14




Free vectors are considered equal if they have the same magnitude and direction.

Vectors can be shifted:

(a) along their ............................
(b) ............................ to themselves.

--------------------▷ 15

## 1.5 Component Representation in Coordinate Systems

70

**Objective:** Concepts of unit vectors, component of a vector, notation of vectors, position vectors.

**READ: 1.4 Component representation in coordinate systems**
**1.5.1 Position vector**
**1.5.2 Unit vectors**
**1.5.3 Component representation of a vector**
**Textbook pages 9–12**

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(a) direction or line of action
(b) parallel

15

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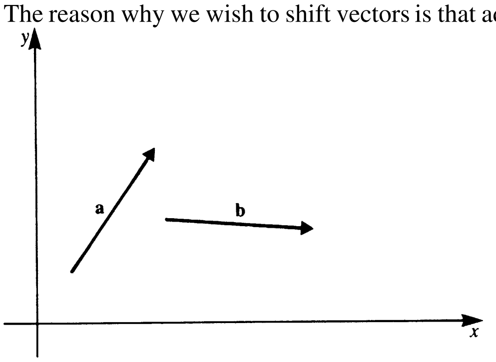
The reason why we wish to shift vectors is that addition and subtraction then become more convenient.




In the diagram, shift vector **b** so that its starting point coincides with that of **a**.

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Vectors whose magnitude is 1 are called ....................................................................

The representation of a vector in the form $\mathbf{a} = (3, \quad 1, \quad 2)$ is called .....................

71

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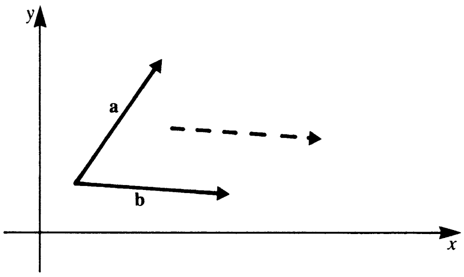


16

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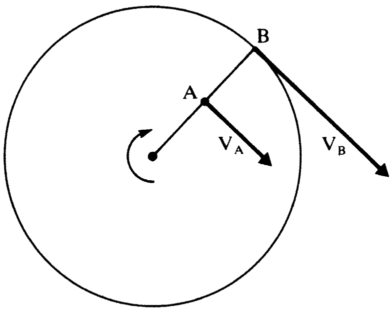


Vectors can be used to indicate the instantaneous velocity of points on a rotating disk.

In this diagram the scale is

$$5\,\text{mm} \mathrel{\widehat{=}} 1\,\text{m/s}.$$

Evaluate from the drawing the magnitude of the velocity of points A and B.

Velocity of A = ............................
Velocity of B = ............................

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unit vectors
component representation

72

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Write down three ways of writing the unit vectors along the $x$-, $y$- and $z$-axes of a Cartesian coordinate system:

....... ....... .......
....... ....... .......
....... ....... .......

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$$V_{\mathrm{A}} = 2.5\,\mathrm{m/s}, \quad V_{\mathrm{B}} = 5\,\mathrm{m/s}$$

17

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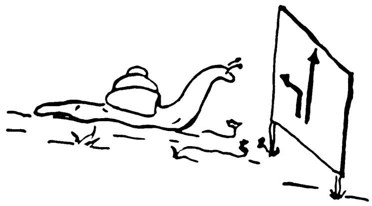
Now decide for yourself how to proceed:



No difficulties; go to --------------------▷ 19

Further example required --------------------▷ 18

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$\mathbf{i},\ \mathbf{j},\ \mathbf{k}$
$\mathbf{e}_x,\ \mathbf{e}_y,\ \mathbf{e}_z$
$\mathbf{e}_1,\ \mathbf{e}_2,\ \mathbf{e}_3$

73

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Let $p_x,\ p_y,\ p_z$ be the components of a position vector $\mathbf{p}$.

Write down the vector $\mathbf{p}$ in terms of its components and the unit vectors.

$$\mathbf{p} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The abbreviated form is

$$\mathbf{p} = (\ldots\ldots\ldots\ldots\ldots) \text{ or } \mathbf{p} = \begin{pmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \end{pmatrix}$$

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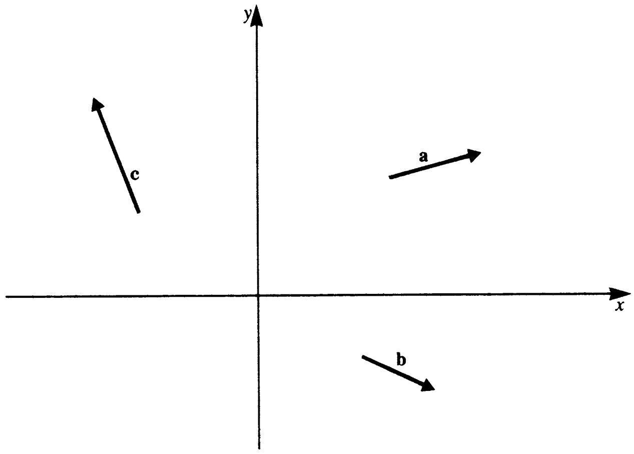
18




Shift **c** and **b** so that all three vectors begin at the starting point of **a**.

------------------▷ 19

74

$$\mathbf{p} = p_x\mathbf{i} + p_y\mathbf{j} + p_z\mathbf{k}$$

$$\mathbf{p} = (p_x,\ p_y,\ p_z) \text{ or } \mathbf{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

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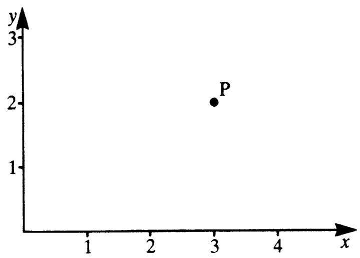
Given the point P = (3, 2)

The position vector **p** has the two ............................ $p_x\mathbf{i}$ and $p_y\mathbf{j}$.

Draw in the components $p_x$ and $p_y$ as well as **p**.

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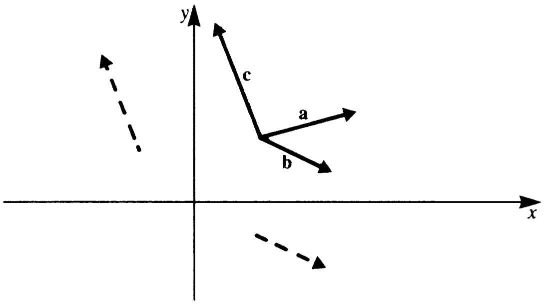


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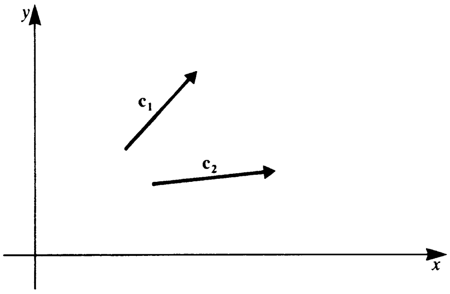


Draw in the lines of action of the vectors $\mathbf{c}_1$ and $\mathbf{c}_2$.

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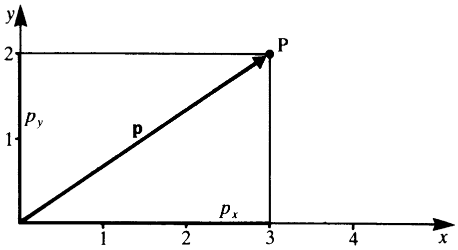
components




75

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What is the component representation of $\mathbf{p}$?

$\mathbf{p}$ = ............................

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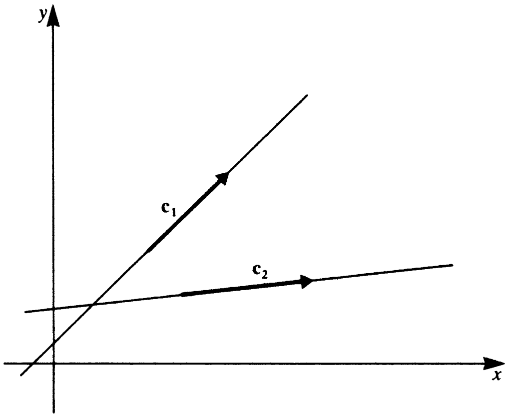
20




21

76

$$\mathbf{p} = (3, \quad 2) \text{ or } \mathbf{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

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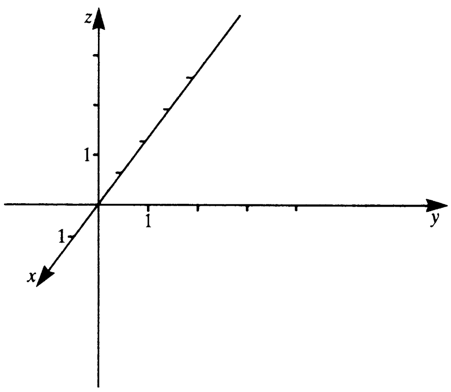
In the three-dimensional coordinate system shown draw the vector **a** by adding the components:
$\mathbf{a} = (-2, \quad 4, \quad 2)$




77

A force is applied at **P**; if in the drawing 1 unit = 1 newton what is the magnitude of the force **F**?

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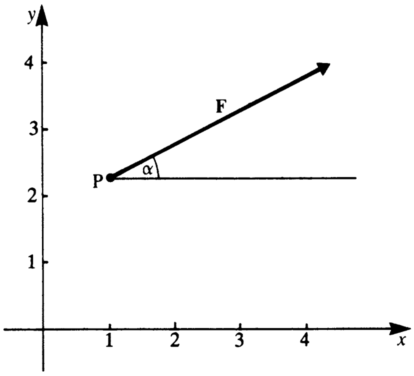


**F** = ....................

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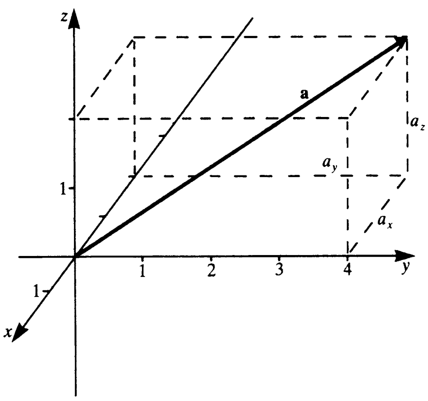
77




Check with the help of the drawing that the order does not matter.

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Having difficulties? -------------------▷ 78

All correct? -------------------▷ 79

$$\mathbf{F} = 3.8\,\mathrm{N}$$

22

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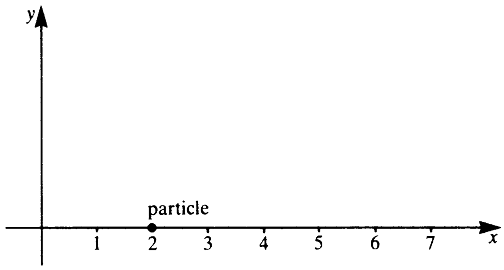
A particle is moving along the positive $x$-axis with a velocity of 4 m/s and it is located at that instant at $x = 2$. If on the drawing 1 unit $= 1\,\mathrm{m/s}$, draw in the velocity vector.

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Go back to the textbook and try to transfer all statements to the two-dimensional case; you should find it easier. 78

Sketch figures analogous to those in the textbook for the two-dimensional case.

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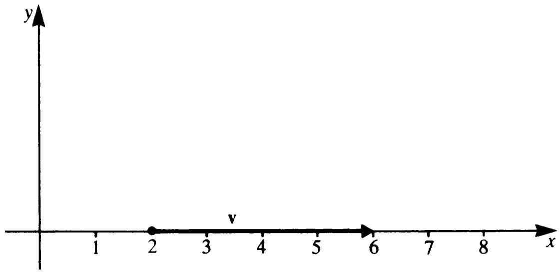
23




24

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## 1.6 Representation of the Sum of Two Vectors in Terms of their Components

79

## 1.7 Subtraction of Vectors in Terms of their Components

**Objective:** Addition and subtraction of vectors in terms of their components.

Many physical quantities can be represented by vectors. So far we have added and subtracted these vectors graphically. The addition and subtraction can also be carried out analytically.

This is achieved by expressing each vector as components along the axes of a coordinate system. Then we can add and subtract components along each axis.

**READ: 1.5.4 Representation of the sum of two vectors in terms of their components**
**1.5.5 Subtraction of vectors in terms of their components**
**Textbook pages 12–14**

80

## 1.2 Addition and Subtraction of Vectors

24

**Objective:** Concepts of geometrical addition, vector sum, resultant vector, negative vector, vector difference.

The principle of geometrical addition and subtraction of vectors is extremely useful.

**READ:** **1.2 Addition of vectors**
**1.3 Subtraction of vectors**
**Textbook pages 4–7**

Then move on to frame 25 in this study guide.

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80

Given two vectors in terms of their components:

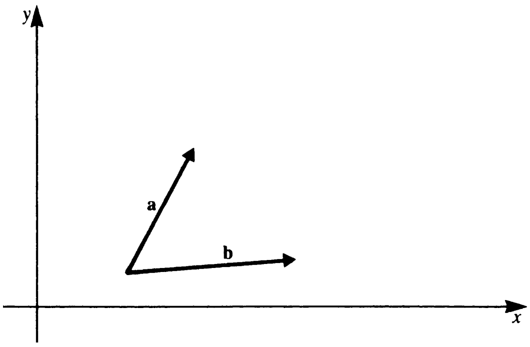
$$\mathbf{a} = (3, \quad 4), \ \mathbf{b} = (2, \quad -2)$$

Find

$$\mathbf{c} = \mathbf{a} + \mathbf{b}$$
$$\mathbf{c} = (\ldots\ldots, \quad \ldots\ldots)$$

Do this analytically, even though it could be solved graphically.

------------------▷ 81




Add the vectors **a** and **b**:

25

$$\mathbf{a} + \mathbf{b} = \mathbf{c}$$

The vector **c** is called the
............................,
............................
or ............................

-------------------▷ 26

---

$$\mathbf{c} = (5, \quad 2)$$

81

---

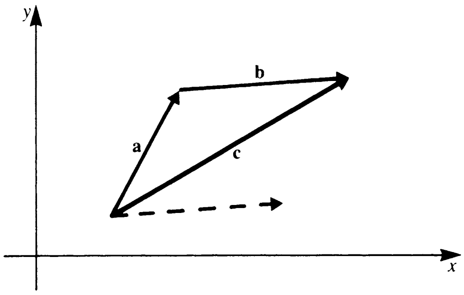
Here are further examples of the same type:

A: $\mathbf{a} = (-2, \quad 1), \quad \mathbf{b} = (1, \quad 3)$
$\mathbf{c} = \mathbf{a} + \mathbf{b} = (........, \quad ........)$

B: $\mathbf{v}_1 = (15\,\mathrm{m/s}, \quad 10\,\mathrm{m/s})$
$\mathbf{v}_2 = (2\,\mathrm{m/s}, \quad -5\,\mathrm{m/s})$

$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 = (.........., \, ..........)$

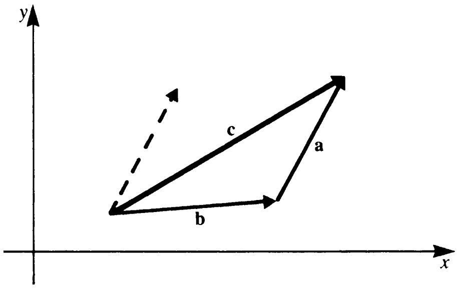
-------------------▷ 82




Resultant vector, vector sum or resultant.

26

---




Interchanging the order of addition does not affect the result.

--------------------▷ 27

---

A: $\mathbf{c} = (-1, \quad 4)$
B: $\mathbf{v} = (17\,\mathrm{m/s}, \quad 5\,\mathrm{m/s})$

82

---

Vectors are added analytically by adding their components. We need to assign units in the case of vectors used to represent physical quantities.

The last example was a case in point because it concerned velocities. Therefore the components represented velocities as well.

Let

$$\mathbf{a} = (4, \quad 2)$$
$$\mathbf{b} = (2, \quad 2)$$

Express the difference vector $\mathbf{d} = \mathbf{a} - \mathbf{b}$ in component form.

$\mathbf{d} = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$

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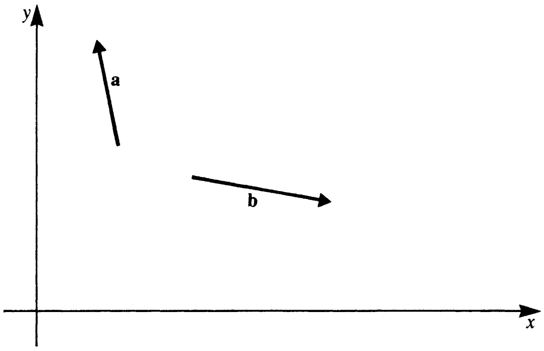



Draw the vector sum
$\mathbf{c} = \mathbf{a} + \mathbf{b}$

27

Other names for the vector sum are:
............................
............................

------------------▷ 28

---

$$\mathbf{d} = (4-2, \quad 2-2) = (2, \quad 0)$$

83

---

Analytically, the subtraction of vectors is carried out by subtracting their components.
Do the following exercises:

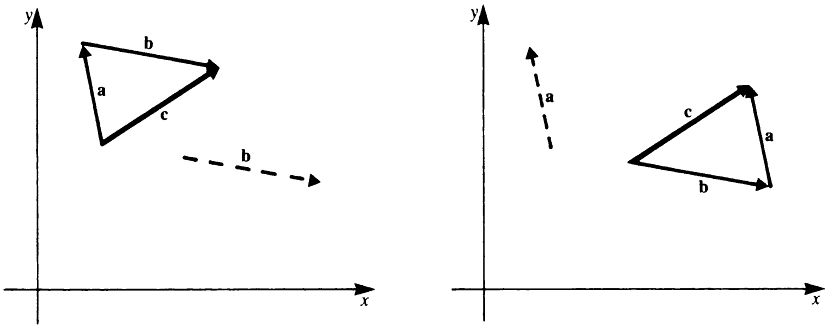
$$\mathbf{v}_1 = (5\,\mathrm{m/s}, \quad 5\,\mathrm{m/s}); \quad \mathbf{v}_2 = (10\,\mathrm{m/s}, \quad 2\,\mathrm{m/s});$$
$$\mathbf{v}_3 = \mathbf{v}_1 - \mathbf{v}_2 = (\ldots\ldots\ldots\ldots\ldots\ldots,\ \ldots\ldots\ldots\ldots\ldots\ldots)$$
$$\mathbf{F}_1 = (2.5\,\mathrm{N}, \quad 0\,\mathrm{N}); \quad \mathbf{F}_2 = (1\,\mathrm{N}, \quad 2\,\mathrm{N});$$
$$\mathbf{F}_1 + \mathbf{F}_2 = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$$
$$\mathbf{F}_1 - \mathbf{F}_2 = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$$

------------------▷ 84

28

resultant vector
resultant

The two solutions shown are equivalent:




29

84

$$\mathbf{v}_3 = (-5\,\mathrm{m/s}, \quad 3\,\mathrm{m/s})$$
$$\mathbf{F}_1 + \mathbf{F}_2 = (3.5\,\mathrm{N}, \quad 2\,\mathrm{N})$$
$$\mathbf{F}_1 - \mathbf{F}_2 = (1.5\,\mathrm{N}, \quad -2\,\mathrm{N})$$

---

In the last example we were dealing with velocities and forces. Velocities are measured in metres/second and forces in newtons.

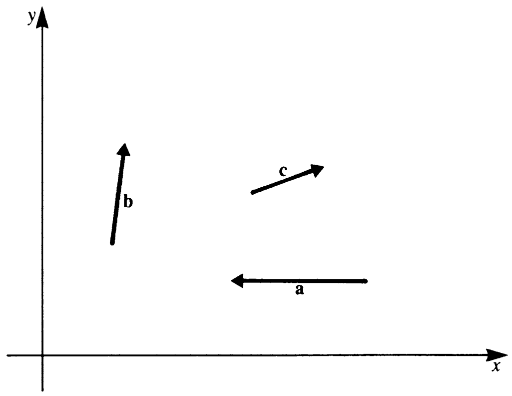
We shall now consider vectors in space; this means that each vector has three components:

$$\mathbf{a} = (1, \quad 2, \quad 1)$$
$$\mathbf{b} = (2, \quad 1, \quad 0)$$
$$\mathbf{c} = \mathbf{a} + \mathbf{b} = (\ldots\ldots, \quad \ldots\ldots, \quad \ldots\ldots)$$
$$\mathbf{d} = \mathbf{b} - \mathbf{a} = (\ldots\ldots, \quad \ldots\ldots, \quad \ldots\ldots)$$

85

Add geometrically the vectors **a**, **b** and **c**: $\mathbf{d} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

29




30

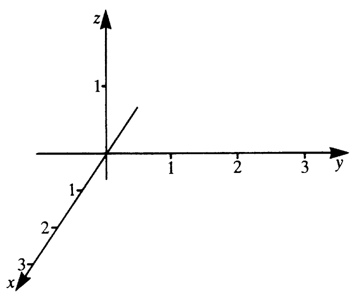
$$\mathbf{c} = \mathbf{a} + \mathbf{b} = (3, \quad 3, \quad 1)$$
$$\mathbf{d} = \mathbf{b} - \mathbf{a} = (1, \quad -1, \quad -1)$$

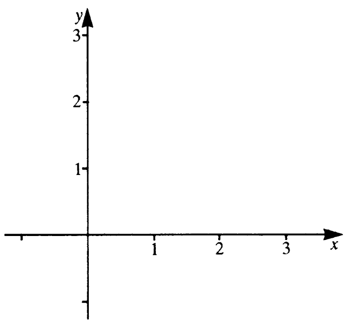
85

---

You can check the analytical result with the help of a drawing.

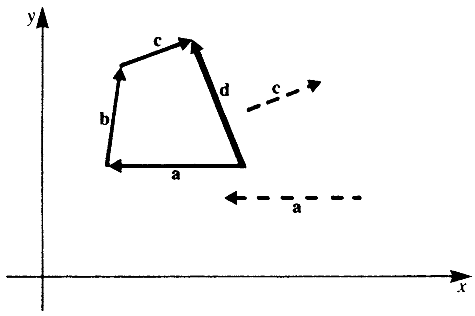
If you find the three-dimensional case too difficult then carry out the graphical method two-dimensionally with $\mathbf{a} = (1, \quad 2)$ and $\mathbf{b} = (2, \quad 1)$.




86

30



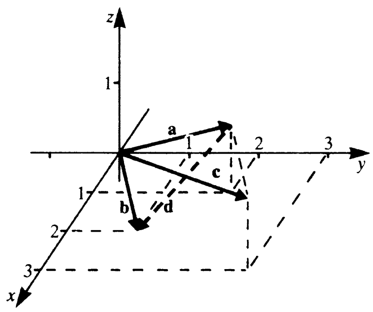

The addition of vectors consists of forming a closed chain, resulting in a polygon. To do this vectors must be shifted. The order is of no importance.

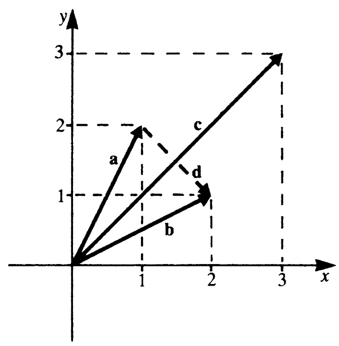
Now decide:

Addition of vectors grasped 33

Further explanation required 31

86




The component method enables us to carry out addition and subtraction analytically and this is obviously very useful. 87

31

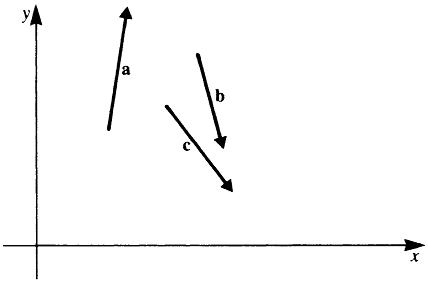
The addition of vectors involves two steps:

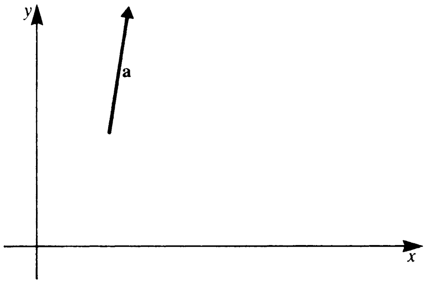
**Step 1:** Formation of a continuous chain:

(a) shift vector **b** so that it starts at the tip of vector **a**.

(b) now shift vector **c** so that it starts at the tip of vector **b**.

Form the chain.

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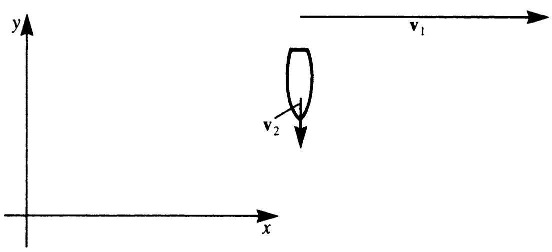
87

A boat is crossing a river. The river has a velocity $\mathbf{v}_1 = (10\,\mathrm{m/s}, \quad 0)$.

The velocity of the boat relative to the water is $\mathbf{v}_2 = (0, \quad -2\,\mathrm{m/s})$.

Then the absolute velocity of the boat, i.e. the velocity of the boat relative to the river bank, is composed of the velocity of the water plus the velocity of the boat relative to the water.

$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2;\ \text{components of } \mathbf{v} = (\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots)$$

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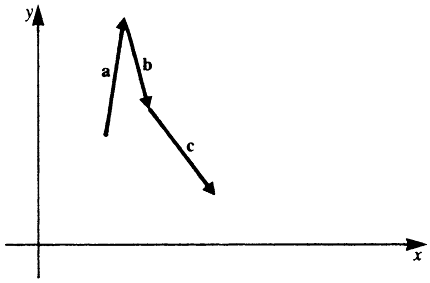
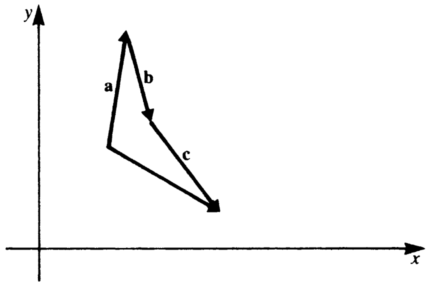
32

**Step 2:** Link up the starting point and end point of the chain.

The vector sum is then the closing vector, i.e. to the tip of vector **c**.

quad The procedure for the graphical addition of vectors is quite simple. We only need to form a chain of the arrows in the sense of each arrow. The last arrow placed on the chain always starts at the tip of the preceding one. The order is immaterial.




-------------------▷ 33

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$$\mathbf{v} = (10\,\mathrm{m/s}, \quad -2\,\mathrm{m/s})$$

88

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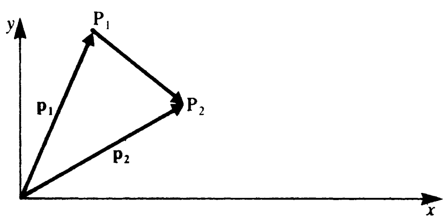



Given two points $P_1$ and $P_2$ with position vectors $\mathbf{p_1}$ and $\mathbf{p_2}$; in component representations: $\mathbf{p_1} = (p_{1x}, \quad p_{1y})$, $\mathbf{p_2} = (p_{2x}, \quad p_{2y})$.

We require the vector which connects $P_1$ and $P_2$.

The connecting vector is the difference vector of the position vectors $\mathbf{p_1}$ and $\mathbf{p_2}$, i.e. $\overrightarrow{P_1P_2}$. It is the difference vector of the two position vectors $\mathbf{p_1}$ and $\mathbf{p_2}$:

$$\overrightarrow{P_1P_2} = \ldots\ldots\ldots\ldots\ldots\ldots$$

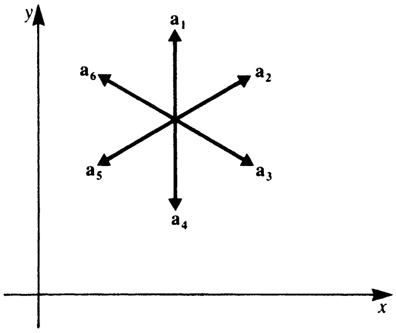
Component representation:

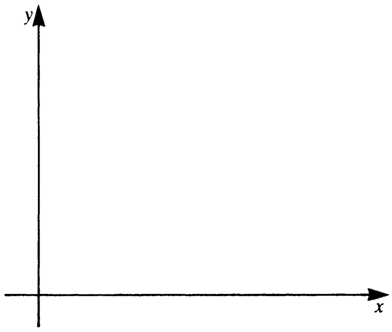
$$\overrightarrow{P_1P_2} = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$$

-------------------▷ 89

The vector sum is independent of the order in which we add the vectors.
Draw the vector polygon in the right-hand diagram. Add all vectors.
What is the resultant vector?

33

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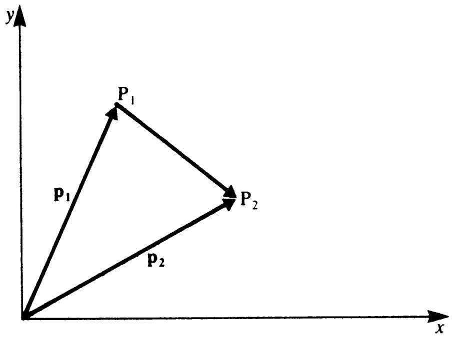



$$\overrightarrow{P_1P_2} = \mathbf{p}_2 - \mathbf{p}_1$$
$$\overrightarrow{P_1P_2} = (p_{2x} - p_{1x}, \quad p_{2y} - p_{1y})$$

89

The equation can easily be checked with the help of the above drawing and with a slight rearrangement:

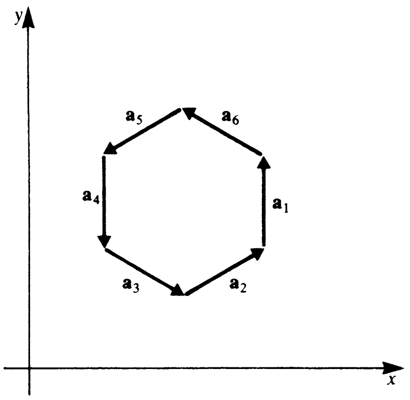
$$\mathbf{p}_1 + \overrightarrow{P_1P_2} = \mathbf{p}_2$$

Given $\mathbf{p_1} = (1, \quad 4)$ and $\mathbf{p_2} = (3, \quad 3)$

$$\overrightarrow{P_1P_2} = (......................)$$

------------------▷ 90

34



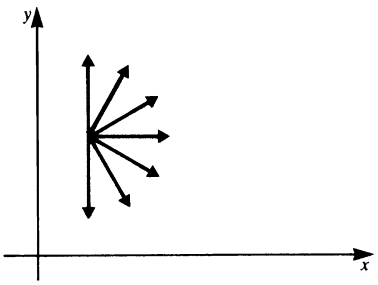

In this case the vector sum, or resultant vector, is zero. Any different order in which we add the vectors yields the same result.
For example, by adding

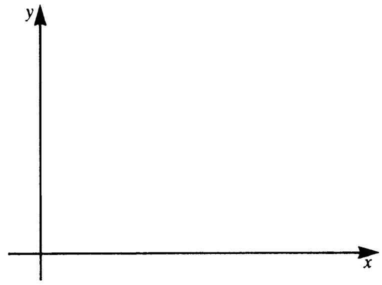
$$(\mathbf{a_1} + \mathbf{a_4}) + (\mathbf{a_2} + \mathbf{a_5}) + (\mathbf{a_3} + \mathbf{a_6})$$

we can see immediately that the resultant vanishes.

---

Now do the same for the following case:




------------------▷ 35

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$$\overrightarrow{P_1P_2} = \mathbf{p_2} - \mathbf{p_1} = (2, \quad -1)$$

90

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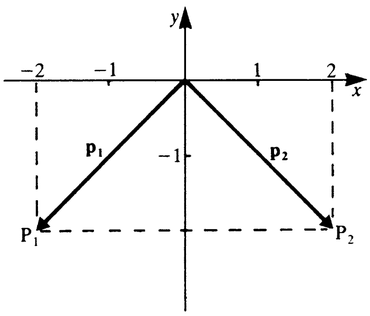
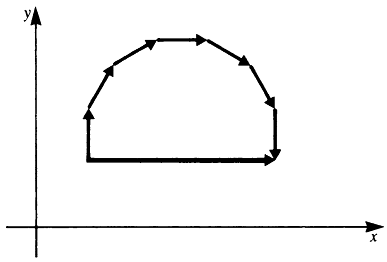



$\mathbf{p_1} = (-2, \quad -2)$
$\mathbf{p_2} = (2, \quad -2)$

(a) Draw the vector $\overrightarrow{P_1P_2}$ which links $P_1$ and $P_2$.
(b) Component representation:

$\overrightarrow{P_1P_2} = (\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots)$

------------------▷ 91




35

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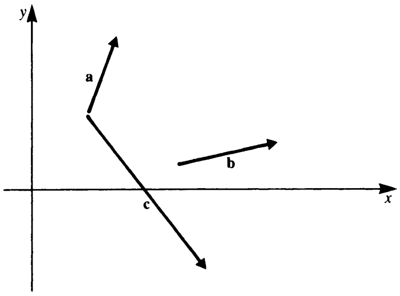


Add geometrically the three vectors **a**, **b** and **c**.

------------------▷ 36

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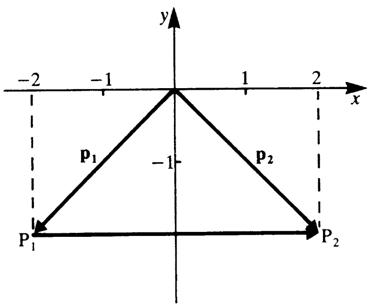
91




$$\overrightarrow{P_1P_2} = (4, \quad 0)$$

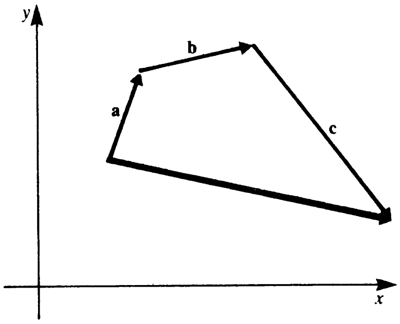
---

All correct ------------------▷ 95

Errors, detailed explanation required ------------------▷ 92

We form a chain with the three vectors as shown.
The vector sum is thus uniquely defined.

36

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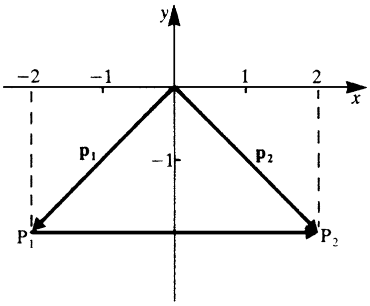
$\overrightarrow{P_1P_2}$ is a vector which starts at $P_1$ and ends at $P_2$ with the arrow head at $P_2$.
Difficulties may be due to the signs.

92




From the diagram we gather

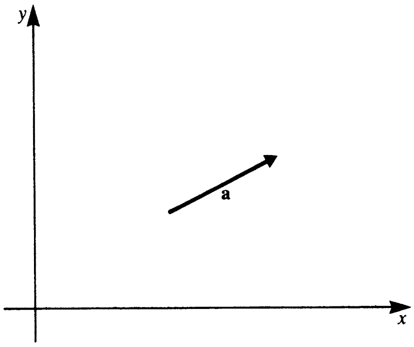
$$\mathbf{p_2} = \mathbf{p_1} + \overrightarrow{P_1P_2}$$

Transforming yields

$$\overrightarrow{P_1P_2} = \mathbf{p_2} - \mathbf{p_1}$$

-------------------▷ 93

Given the vector **a**, draw the vector −**a**. 37




−**a** is called .............................

→ 38

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With 93

$$\mathbf{p_1} = (-2, \quad -2)$$
$$\mathbf{p_2} = (2, \quad -2)$$

then

$$\overrightarrow{P_1P_2} = (2-(-2), \quad -2-(-2)) = (4, \quad 0)$$

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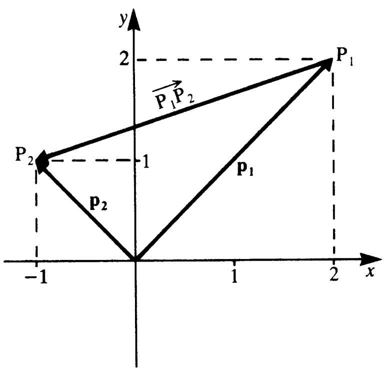
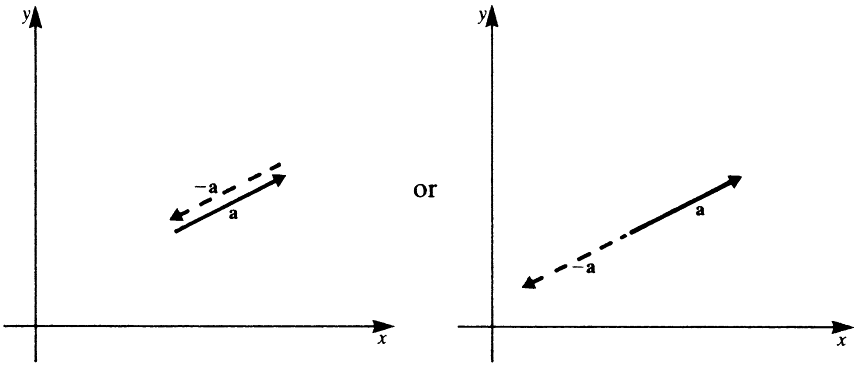



Now try again.

$$\mathbf{p_1} = (2, \quad 2)$$
$$\mathbf{p_2} = (-1, \quad 1)$$
$$\overrightarrow{P_1P_2} = (................, \quad .................)$$

→ 94

$-\mathbf{a}$ is called the negative of the vector $\mathbf{a}$. 38




39

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$$\overrightarrow{P_1P_2} = (-3, \quad -1)$$ 94

---

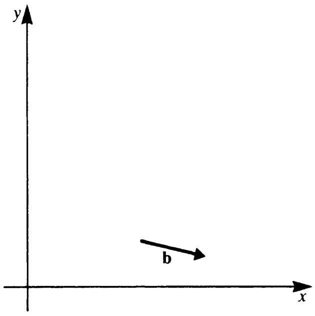
Remember:

The components of the vector from a point $P_1$ to a point $P_2$ are given by the coordinates of the tip of the arrow minus the coordinates of the start of the arrow.

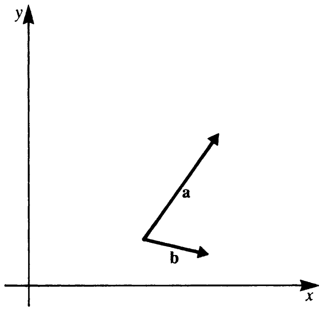
Further exercises are in the textbook.

95

The concept of a negative vector is very useful because it helps us to perform vector subtraction; subtraction is thereby reduced to the addition of vectors. 39




Draw the vector $-\mathbf{b}$




Draw the vector $\mathbf{a} - \mathbf{b}$

------------------▷ 40

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**Some remarks about working in a group and working alone.** 95

Working alone is advisable if facts are to be learnt thoroughly, calculations copied, proofs studied and texts read intensively.

Working in a group is advisable for:

— the solution of exercises and problems with the help of methods just acquired,
— the discussion of results,
— the preparation and identification of problems.

Working in a group should alternate with working alone.

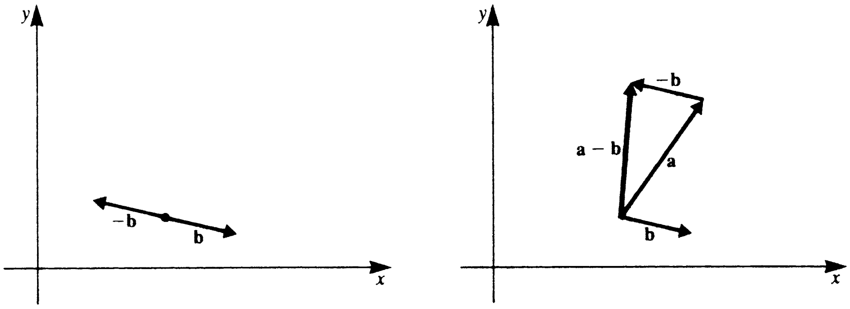
Working in a group cannot replace working alone; similarly, working alone cannot replace those benefits gained by working in a group.

Many students believe that the necessity for expressing facts during group discussions is a good preparation for examinations. They are right, provided of course that in a group ‘nonsense’ is labelled as ‘nonsense’. This means that if someone explains something wrongly he/she must be corrected to ensure that wrong concepts are not passed on.

Time for a break!

------------------▷ 96

40

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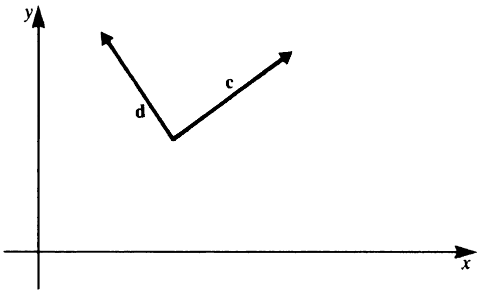
Draw $\mathbf{c} - \mathbf{d} = \mathbf{f}$ in the diagram.

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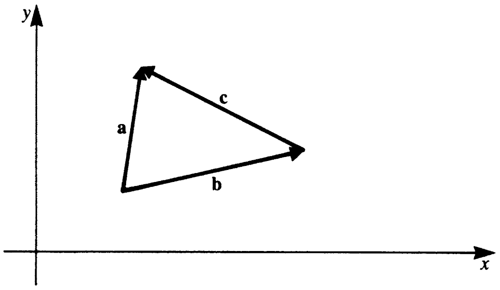
## 1.8 Multiplication of a Vector by a Scalar. Magnitude of a Vector

96

**Objective:** Multiplication of a vector by a scalar, magnitude of a vector in terms of its components, distance between two points.

**READ: 1.6 Multiplication of a vector by a scalar**
**1.7 Magnitude of a vector**
**Textbook pages 14–17**

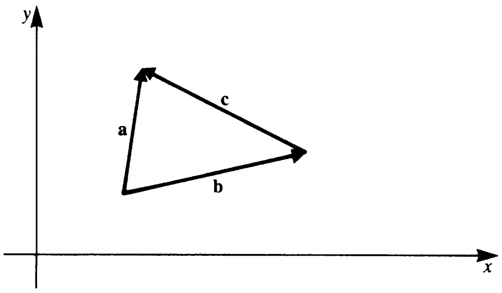
-------------------▷ 97




$$\mathbf{f} = \mathbf{c} - \mathbf{d}$$
$$= \mathbf{c} + (-\mathbf{d})$$

41

---




**c** is a difference vector. Write down the vector equation:

$\mathbf{c} =$ ..............................

--------------------▷ 42

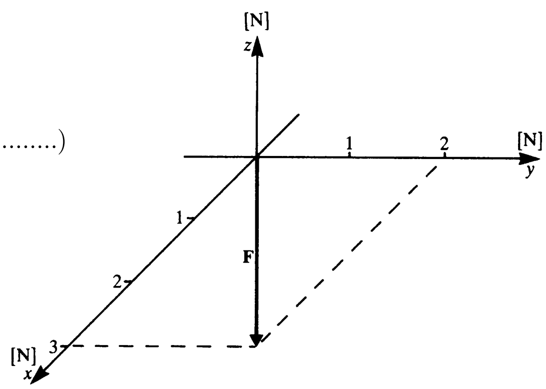
---

97

Let the force

$\mathbf{F} = (3\,\mathrm{N}, \quad 2\,\mathrm{N}, \quad 0\,\mathrm{N})$

be increased by a factor of 2.5;
then $2.5\mathbf{F} = (\ldots\ldots,$ ............, ...............)




--------------------▷ 98

$$\mathbf{c} = \mathbf{a} - \mathbf{b}$$

42

---

You certainly know now that in many cases the decision on how to proceed with this study guide is up to you. From now on you will just be given the options:

No mistakes --------------------▷ 48

Some mistakes, or further explanation required --------------------▷ 43

---

$$2.5\mathbf{F} = (7.5\,\mathrm{N}, \quad 5\,\mathrm{N}, \quad 0\,\mathrm{N})$$

98

---

The vector $0 = (0, \quad 0, \quad 0)$ is called ................................................................

--------------------▷ 99

43

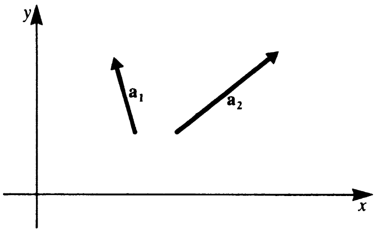
The formation of the difference of two vectors, $\mathbf{a}-\mathbf{b}$, can be reduced to addition with the help of a negative vector,

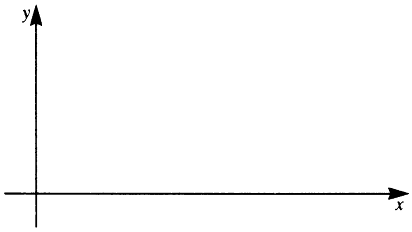
i.e. $$\mathbf{a}-\mathbf{b}=\mathbf{a}+(-\mathbf{b}).$$

This leads to the following procedure to obtain the difference vector $\mathbf{a}-\mathbf{b}$:

**Step 1:** Draw the negative vector $-\mathbf{b}$
**Step 2:** Add the negative vector to $\mathbf{a}$
**Step 3:** Vector $\mathbf{a}$ plus the negative of vector $\mathbf{b}$ is the difference vector $\mathbf{a}-\mathbf{b}$.

Draw the difference vector $\mathbf{a}_3=\mathbf{a}_1-\mathbf{a}_2$




------------------▷ 44

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99

a null vector

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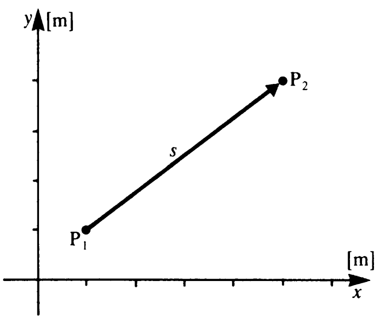
A mechanical digger is crawling along with uniform velocity from a position $\mathbf{P}_1=(1\,\text{m},\quad 1\,\text{m})$ to a position $\mathbf{P}_2=(5\,\text{m},\quad 4\,\text{m})$ in a time of 50 seconds.




The change in position

$\overrightarrow{P_1P_2}=$ ...............................

The magnitude of the distance covered

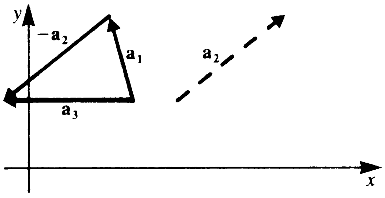
$s=$ ............................

The magnitude of the velocity

$v=$ ............................

The velocity $\mathbf{v}=$ ............................

------------------▷ 100




$\mathbf{a}_3 = \mathbf{a}_1 - \mathbf{a}_2$

44

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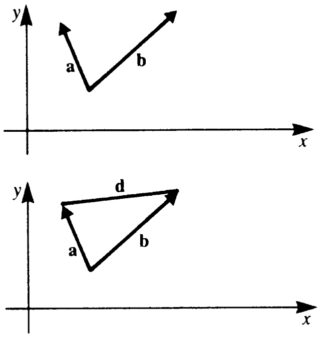
An equivalent method for evaluating the difference vector $\mathbf{a} - \mathbf{b}$ requires the following procedure:




**Step 1:** Draw **a** and **b**.
**Step 2:** Connect the arrowheads of both vectors. The connection is the difference vector.
**Step 3:** Obtain the sense of the difference vector. To do this rearrange the equation:
$\mathbf{d} = \mathbf{a} - \mathbf{b}$ to $\mathbf{b} + \mathbf{d} = \mathbf{a}$
The sense of **d** has to satisfy this equation.

------------------▷ 45

Draw in the sense.

---

$\overrightarrow{P_1P_2} = (4\,\text{m}, \quad 3\,\text{m})$
$s = \sqrt{16\,\text{m}^2 + 9\,\text{m}^2} = 5\,\text{m}$
$v = 5\,\text{m}/50\,\text{s} = 0.1\,\text{m/s}$
$\mathbf{v} = \left(\frac{4}{50}\,\text{m/s}, \quad \frac{3}{50}\,\text{m/s}\right)$

100

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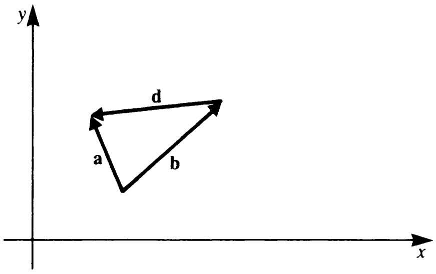
If you made mistakes try once more to solve the problem with the help of the textbook.

------------------▷ 101

45




$\mathbf{a} = \mathbf{b} + \mathbf{d}$

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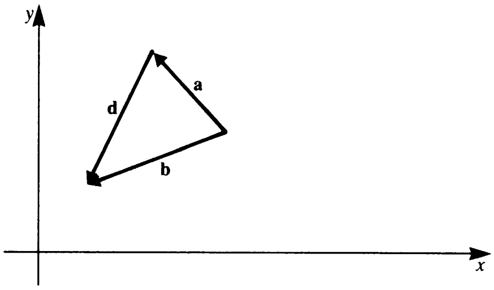
One more exercise. Write down the correct equation for the vectors shown in the figure.




46

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101

Given the vector $\mathbf{b} = (b_x, \quad b_y, \quad b_z)$

its magnitude is $b = \ldots\ldots\ldots\ldots\ldots\ldots$

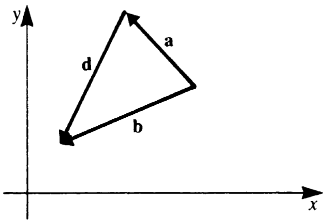
Numerical example

$$\mathbf{b} = (1, \quad 2, \quad 1)$$
$$b = \ldots\ldots\ldots\ldots\ldots$$

102

The correct equation is

$\mathbf{d} = \mathbf{b} - \mathbf{a}$
or $\mathbf{a} + \mathbf{d} = \mathbf{b}$
or $\mathbf{a} + \mathbf{d} - \mathbf{b} = 0$




46

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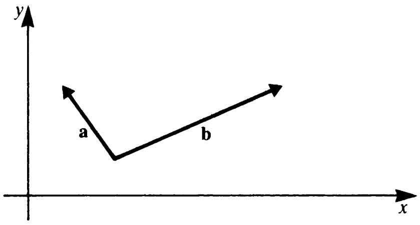
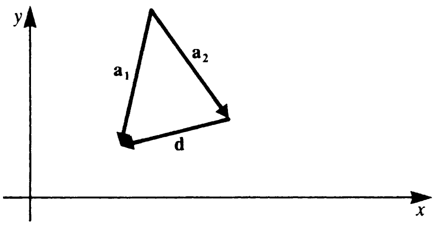



$\mathbf{d} = \mathbf{a} - \mathbf{b}$




Draw in the vector difference,

Write down the equation

$\mathbf{d} =$ .............................

-------------------▷ 47

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$b = \sqrt{\left(b_x^2 + b_y^2 + b_z^2\right)}$
$b = \sqrt{6} = 2.45$ (to two decimal places)

102

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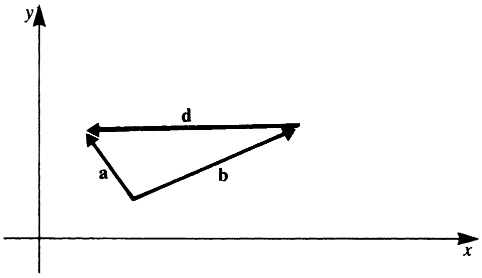
Calculation of unit vectors:
A unit vector has a magnitude equal to .............................

Given the vector $\mathbf{a} = (4, \quad 2, \quad 4)$
then its magnitude

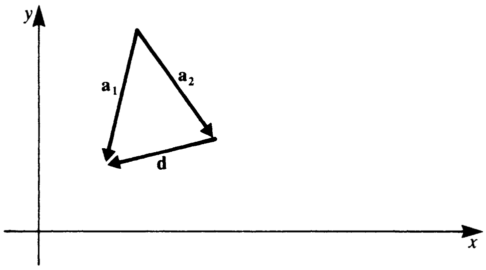
$a =$ .............................

-------------------▷ 103

47




$\mathbf{d} = \mathbf{a} - \mathbf{b}$




$\mathbf{d} = \mathbf{a}_1 - \mathbf{a}_2$

-------------------▷ 48

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1

$a = \sqrt{16+4+16} = \sqrt{36} = 6$

103

---

The unit vector in the direction of $\mathbf{a}$ is denoted by $\mathbf{e}_a$.

The unit vector has a magnitude of 1. The direction of the unit vector in the direction $\mathbf{a}$ is obtained by multiplying $\mathbf{a}$ by the scalar $\frac{1}{a}$.

Obtain the unit vector $\mathbf{e}_a$ for $\mathbf{a} = (4, \quad 2, \quad 4)$.

$\mathbf{e}_a = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

-------------------▷ 104

Further exercises will be found in the textbook. 48

If you have experienced no difficulty, go through those exercises — not now but after two or three days. Then the exercises will be more effective.

→ 49

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$$\mathbf{e}_a = \left(\tfrac{4}{6}, \quad \tfrac{2}{6}, \quad \tfrac{4}{6}\right) = \left(\tfrac{2}{3}, \quad \tfrac{1}{3}, \quad \tfrac{2}{3}\right)$$ 104

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As a check calculate the magnitude of $\mathbf{e}_a$:

$$\mathbf{e}_a = \ldots\ldots\ldots\ldots\ldots\ldots$$

→ 105

## 1.3 Components and Projection of a Vector

49

**Objective:** Concepts of projection, components, projecting one vector on to another.

**READY:** **1.4 Components and projection of a vector**
**Textbook pages 7–9**

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$$\mathbf{e}_a = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 1$$

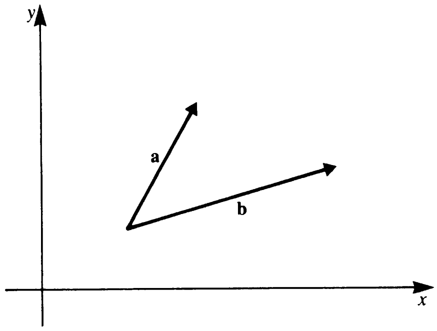
105

---

Calculate the unit vector for $\mathbf{a} = (3, \quad 4)$

$$\mathbf{e}_a = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

--------------------▷ 106




50

Draw the projection of **a** on to **b**.

→ 51

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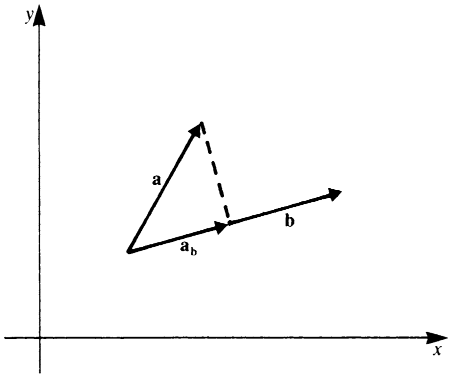
$$\mathbf{e}_a = (0.6, \quad 0.8)$$

106

---

In case of difficulty try to solve the problem with the help of the textbook.

→ 107



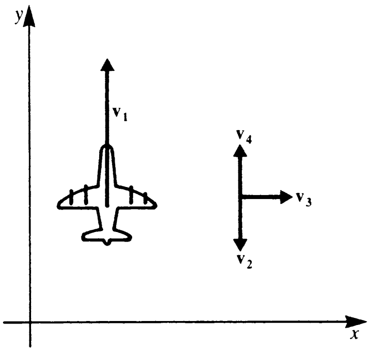

51

Answer correct

-------------------▷ 54

Mistakes, or further explanation required

-------------------▷ 52




An aircraft is flying on a northerly course. Its velocity relative to the air is

107

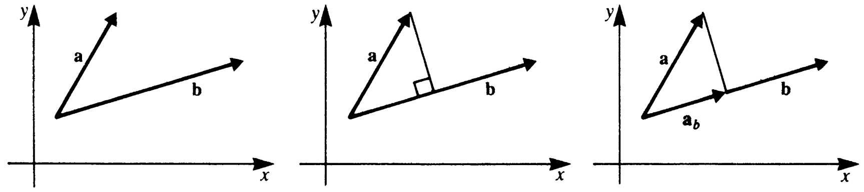
$$\mathbf{v_1} = (0, \quad 200\,\text{km/h}).$$

Calculate the velocity of the aircraft relative to the ground for the following air velocities:

$\mathbf{v_2} = (0, \quad -50\,\text{km/h})$, head wind
$\mathbf{v_3} = (50\,\text{km/h}, \quad 0)$, cross wind
$\mathbf{v_4} = (0, \quad 50\,\text{km/h})$, tail wind

$\mathbf{v_1} + \mathbf{v_2} =$ ............................
$\mathbf{v_1} + \mathbf{v_3} =$ ............................
$\mathbf{v_1} + \mathbf{v_4} =$ ............................

-------------------▷ 108

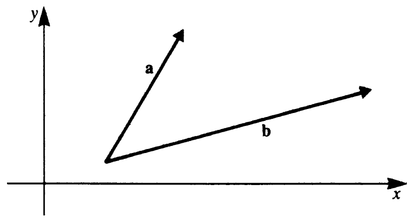



52

The projection of vector **a** on to vector **b** is obtained in two steps.

**Step 1:** Drop a perpendicular from the tip of **a** on to **b**.
**Step 2:** The projection $\mathbf{a_b}$ is the line segment from the starting point to the point of intersection with the perpendicular.

Now draw the projection of **b** on to **a**.

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With a head wind, $\mathbf{v_1} + \mathbf{v_2} = (0, \quad 150\,\text{km/h})$
With a cross wind, $\mathbf{v_1} + \mathbf{v_3} = (50\,\text{km/h}, \quad 200\,\text{km/h})$
With a tail wind, $\mathbf{v_1} + \mathbf{v_4} = (0, \quad 250\,\text{km/h})$

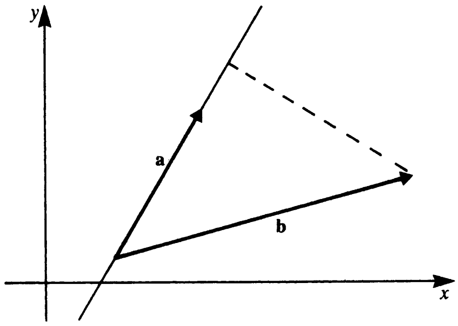
108

---

Calculate the magnitude of the velocity relative to the ground for each of the three cases above:

$|\mathbf{v_1} + \mathbf{v_2}| = \ldots\ldots\ldots\ldots\ldots$ head wind
$|\mathbf{v_1} + \mathbf{v_3}| = \ldots\ldots\ldots\ldots\ldots$ cross wind
$|\mathbf{v_1} + \mathbf{v_4}| = \ldots\ldots\ldots\ldots\ldots\ldots$ tail wind

------------------▷ 109




53

In this case we had to extend the line of action of **a** and then drop the perpendicular from the tip of **b**.

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109

$$|\mathbf{v_1}+\mathbf{v_2}| = \sqrt{(150\,\mathrm{km/h})^2} = 150\,\mathrm{km/h}$$
$$|\mathbf{v_1}+\mathbf{v_3}| = \sqrt{50^2+200^2} = 206.16\,\mathrm{km/h}$$
$$|\mathbf{v_1}+\mathbf{v_4}| = \sqrt{250^2} = 250\,\mathrm{km/h}$$

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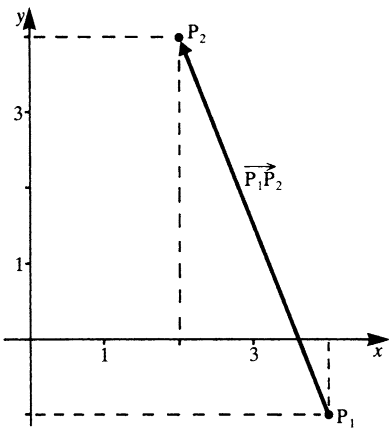


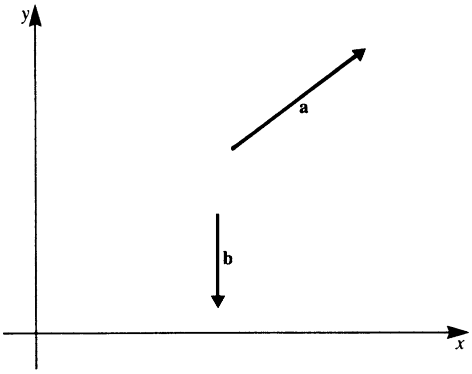

Given the points $P_1 = (4, \quad -1)$ and $P_2 = (2, \quad 4)$:

$\overrightarrow{P_1P_2} = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$

Distance between the points = .........

-------------------▷ 110

54




Draw the projection of **b** on to **a**.

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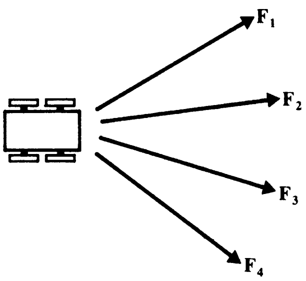
110

$$\overrightarrow{P_1P_2} = (-2, \quad 5)$$
$$\left|\overrightarrow{P_1P_2}\right| = \sqrt{29} \approx 5.39$$

---

A carriage is pulled by four men.

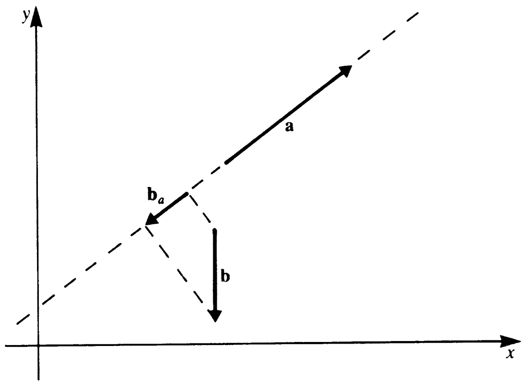



The components of the four forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$ and $\mathbf{F_4}$ are

$\mathbf{F_1} = (200\,\text{N}, \quad 150\,\text{N})$
$\mathbf{F_2} = (180\,\text{N}, \quad 0)$
$\mathbf{F_3} = (250\,\text{N}, \quad -50\,\text{N})$
$\mathbf{F_4} = (270\,\text{N}, \quad -200\,\text{N})$

Resultant force $\mathbf{F}$ = ............................
and $|\mathbf{F}|$ = ............................

------------------▷ 111




55

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All correct --------------------▷ 57

Further explanation required --------------------▷ 56

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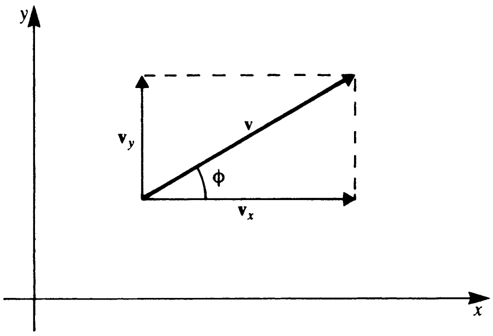
$$\mathbf{F} = (900\,\mathrm{N}, \quad -100\,\mathrm{N})$$
$$|\mathbf{F}| = \sqrt{900^2 + 100^2} = 905.5\,\mathrm{N}$$

111

---

Calculation of the components of a vector, given its magnitude and its angle.



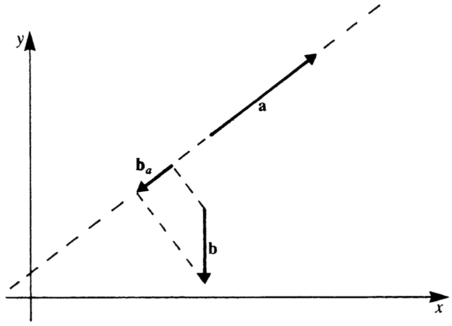

What is **v** in component form?
$\mathbf{v} = (\ldots\ldots\ldots\ldots\ldots\ldots, \quad \ldots\ldots\ldots\ldots\ldots\ldots)$

--------------------▷ 112

56

In this example the vectors **a** and **b** do not have the same starting point.
The projection of **b** on to **a** requires three steps:

**Step 1:** Extend the line of action of **a**.
**Step 2:** Drop perpendiculars from the start and the tip of vector **b** on to the line of action of **a**.
**Step 3:** Draw in the projection as shown.




57

Please continue on page 1
(bottom half)

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$$\mathbf{v} = (v \cos \phi, \quad v \sin \phi)$$

112

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If you have mastered the exercises then it is pointless to do more of the same type; you will not learn more.

But the same exercises may be more difficult if they appear in a different context.

The mathematical methods you have just learnt are frequently used in physics and engineering, although you may find other notations for the same subject matter. Therefore, in the study guide, we sometimes change notation and give exercises related to previous sections of the textbook.