Chapter 17
Vector analysis: Surface integrals, Divergence, Curl, and Potential

-------------------▷ 1

$I_1 = 0.5$ $I_2 = 0$ $I_3 = 0$

60

Hint to improve your study skills.

All textbooks have an index. It should be a habit to use the index. Nobody remembers all he should have learned. Stop your time to find the definition of Bernoulli`s equation in the textbook using the index.

………………………

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17.1 Flow of a vector field through a surface element

1

The prerequisite to study this chapter is a certain familiarity with functions of several variables, partial differentiation, and total differentiation. These topics you have studied in chapter 12. Perhaps a short repetition may help you.

Study in the textbook **17. 1 Flow of a vector field through a surface element**
Textbook pages 461–464

Go to ------------------▷ 2

61

As a rule you will need 20 to 30 seconds to find such a definition. That is not much time.

If when reading a section of a textbook you encounter a concept the meaning of which you have forgotten, it is a normal tendency to overlook this and to hope that this meaning will be explained later or that you will remember it later.
Unfortunately this may cause learning difficulties, sometimes even serious difficulties which will cost you a lot of time. The new subject matter is explained in textbooks using concepts and words the reader is supposed to be familiar with. Unfortunately if a new concept is explained with concepts and words the reader is not familiar with, the probability that he understands the new subject matter is small. To avoid this it is worthwhile to develop two skills which seem quite simple but which are not that simple to apply.
First you should develop a competence in noticing concepts or words you do not understand. Most of us have a strong tendency to overlook them.
Second you should develop a tendency to use the index and read again the meaning of these concepts or words you are not familiar with.
You may think this wastes your time, however, on the whole this saves a lot of time.

------------------▷ 62

2

In the section you studied some new concepts have been introduced and defined.
Write down at least three of them:

1. …………………………………………………..

2. ………………………………………..

3……………………………………….

-------------------▹ 3

62

The tendency to overlook things we do not understand is natural and even necessary to survive. Nobody can understand everything. But if we even do not notice that we do not understand something, this may be dangerous. And may have consequences. Try to develop competence in noticing things you do not understand and develop a tendency to use indexes and encyclopedia. At least a few times per week.

-------------------▹ 63

Flow density $\vec{j}$
Surface element vector $\vec{A}$
Flow of a vector field $\vec{F}$ through a surface $\vec{A}$

3

Try to write down on a separate sheet the definitions and meanings first using your memory but in case of difficulties by consulting your notes and if you do not succeed the textbook.

-------------------▷ 4

17.5 Divergence of a vector field and Gauss's theorem

63

The introduction of the concept of divergence is closely related to the last section on surface integrals.

Study in the textbook **17.5 Divergence of a vector field**
17.6 Gauss's theorem
Pages 475–479

Having concluded go to

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The flow density $\vec{j}$ is the quantity which passes through a unit area per time unit.
The area is assumed to be perpendicular to the flow.

4

The surface element vector is a vector $\vec{A}$ whose direction is perpendicular to the surface and whose magnitude is equal to the area A.

Flow of a vector field through a surface element is given by the dot product of the two vectors $\vec{F} \cdot \vec{A}$.

------------------▷ 5

$\vec{F}$ represents a vector field. Complete the definition

64

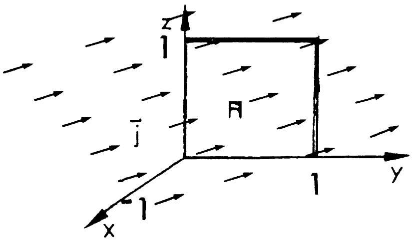
$div \quad \vec{F} =$ ………………………

$div \quad \vec{F}$ is a ………………….

------------------▷ 65

We want the flow of a vector field through an even square surface whose magnitude is A. The surface belongs to the y-z plane. The flow hits the surface at an angle β $\beta \le \frac{\pi}{2}$ 5

The flow vector is constant and given by $\vec{j} = (-j_x, j_y, 0)$



We decompose the task:
1. We determine $\vec{A}$
2. We calculate the flow $\vec{j} \cdot \vec{A}$
The flow is: …………………

Further explanations wanted ------------------▷ 6

Solution found ------------------▷ 11

$$div \quad \vec{F} = \frac{\delta F_x}{\delta x} + \frac{\delta F_y}{\delta y} + \frac{\delta F_z}{\delta z}$$ 65

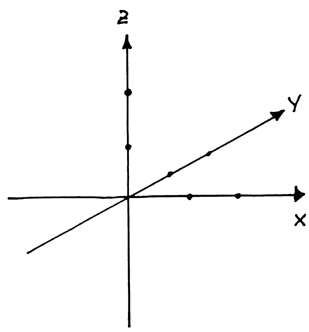
$div \quad \vec{F}$ is a scalar field.

Calculate the divergence for the given vector field:

$\vec{F} = (x, \quad y+b, \quad -z^2)$

$div \quad \vec{F} = \ldots$

Determine the location of sources and sinks.



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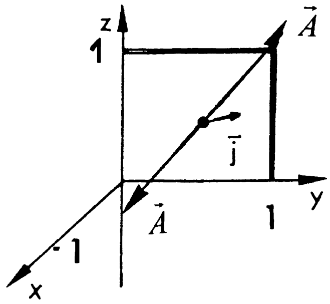
6

We first determine the surface element vector $\vec{A}$. Since the surface element belongs to the y-z plane its direction is parallel to the x-axis. Thus, $\vec{A}$ also has the direction of the x-axis.

$\vec{A} = (+1,0,0)$ or $\vec{A}(-1,0,0)$ Since the flow vector is to hit the surface element at an angle less than $\frac{\pi}{2}$ we obtain $\vec{A} = \ldots\ldots\ldots\ldots\ldots\ldots$



Hint: remember the definition of the given $\vec{j}$.

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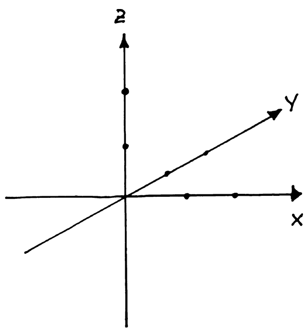
66

$div \quad \vec{F} = (1+1-2z) = (2-2z) = 2(1-z)$



For the plane $z=1$ there are no sources or sinks.
The space above this plane, $z>1$, consists of sinks.
The space beneath this plane, $z \leq 1$, consists of sources

All correct, go ------------------▷ 69

Another exercise wanted ------------------▷ 67

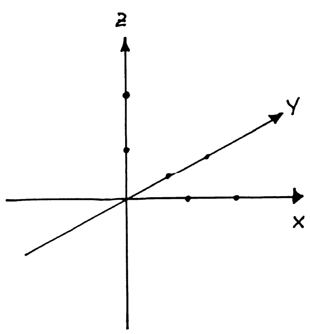
$\vec{A} = A(-1,0,0)$ 7

No more problem --------------------▷ 9

More explication wanted --------------------▷ 8

67

We have no sink or sources if $div \quad \vec{F} = 0$.
With the preceding example this was the case for
$div\vec{F} = 2(1-z) = 0$.
The equation $1 - z = 0$ or $z = 1$ represents a plane which is parallel to the x-y-plane. For it $div \quad \vec{F}$ is negative. This means the space above the plane $z = 1$ consists of sinks.
Beneath the plane $z = 1$ $div\vec{F}$ is positive and thus the space consists of sources.



Calculate the divergence for the following vector field:

$\vec{F} = (x^2 + 1, \quad y, \quad z + 5)$:
$div \quad \vec{F} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

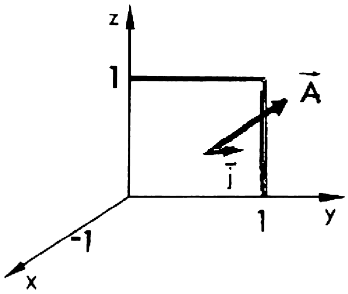
Find the spaces where we have sources or sinks and where the field is free of sources and sinks.

--------------------▷ 68

8

The surface element belongs to the y-z plane. Thus, its surface element vector is directed in the x-direction. But there are two possibilities. It may point in the positive or in the negative x-direction. We have to decide which of these directions is the right one. In our exercise is given the condition that the angle between both given vectors is to be less than 90° or $\frac{\pi}{2}$.

In our exercise the x-component of $\vec{j}$ is negative. Thus, $\vec{A}$ has to point in the negative direction of the x-axis. From this we obtain: $\vec{A} = (-1,0,0,)$

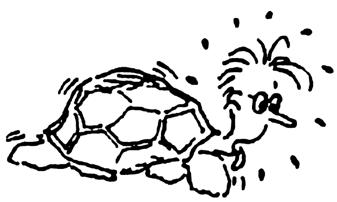


------------------▷ 9

68

$div\vec{F} = 2x + 1 + 1 = 2(x+1)$

For the plane $x = -1$ the field is free of sinks and sources.
The space to the left of this plane, defined by $x < -1$, consists of sinks.
The space to the right of this plane, defined by $x > -1$, consists of sources.



Go on to

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We determined the surface element vector $\vec{A}$ to be $\vec{A} = A(-1,0,0)$ 9

Now we can determine the flow I of $\vec{j}$ through the surface:

$\vec{j} = (-j_x, j_y, 0)$

$I = \vec{j} \cdot \vec{A} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Solution found ------------------▷ 11

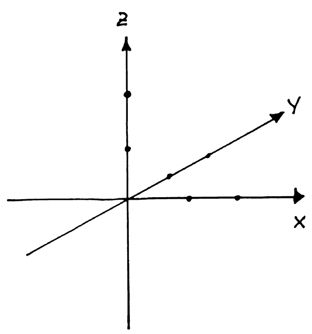
Further help wanted ------------------▷ 10

Calculate the divergence for the vector field 69

$\vec{F} = \left(x^3, \ \ y^3, \ \ -3z\right)$

$div\vec{F} = \ldots\ldots\ldots\ldots\ldots\ldots$

Where do you find sinks and sources?



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10

We remember the dot product of two vectors $\vec{a} = (a_x, a_y,, a_{z)}$ and $\vec{b} = (b_x, b_y, b_z)$ is defined by

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

Now you should be able to calculate using the results obtained before:

$\vec{j} \cdot \vec{A} =$ ……………………………………………………..

------------------▷ 11

70

$div \quad \vec{F} = 3x^2 + 3y^2 - 3 = 3(x^2 + y^2 - 1)$

Sources and sinks vanish for $x^2 + y^2 = 1$.

This is a circle with radius 1 which does not depend on the value of z.

Thus it is a cylinder.

The space inside the cylinder consists of sinks since $div \quad \vec{F}$ is negative.

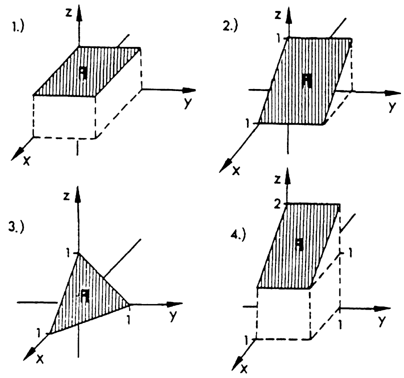
The space outside the cylinder consists of sources since $div \quad \vec{F}$ is positive.

------------------▷ 71

$I = j_x \cdot A$

11

Give the surface element vector for the four given surfaces the magnitude of which is always A



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In the textbook the Nabla operator has been introduced. It is a new notation which seems to be merely formal. But it will prove quite useful because it is a short-hand notation that simplifies notations once you have familiarity with it.

71

$$\vec{\nabla} = \left(\frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z}\right)$$

$\vec{\nabla} =$ ....

-------------------▷ 72

In each case we have two answers. The difference is the sign.
This time a vector field that determines the direction is not given.

12

1. $\vec{A} = A(0,0,1)$. or $\vec{A} = A(0,0-,1)$

2. $\vec{A} = \frac{A}{\sqrt{2}}(1,0,1)$) or $\vec{A} = \frac{A}{\sqrt{2}}(-1,0-,1)$

3. $\vec{A} = \frac{A}{\sqrt{3}}(1,1,1)$ or $\vec{A} = \frac{A}{3}(-1,-1,-1)$

4. $\vec{A} = \frac{A}{\sqrt{2}}(1,0,1)$ or $\vec{A} = \frac{A}{\sqrt{2}}(-1,0,-1)$

All correct ------------------▷ 16

Errors or explanation wanted ------------------▷ 13

$$\vec{\nabla} = \left(\frac{\delta}{\delta x}, \frac{\delta}{\delta y}, \frac{\delta}{\delta z}\right)$$

72

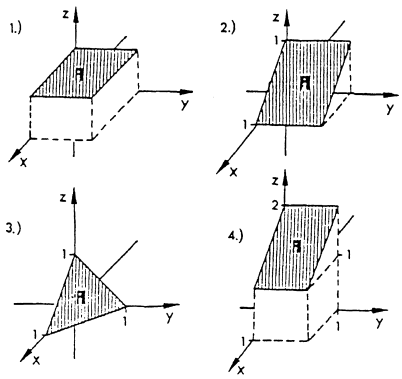
Using the Nabla operator we obtain a short-hand notation to represent
the gradient of a scalar field $f(x,y,z)$ and
the divergence of a vector field $\vec{F}(x,y,z)$.
We start with the calculation of the gradient:

$grad\ f(x,y,z) = \vec{\nabla} \cdot f(x,y,z) =$……………………………..

------------------▷ 73

13

The surface element vector $\vec{A}$ is perpendicular to the surface. Thus, we look at first for a vector perpendicular to the surface, the magnitude of which does not matter for the time being. We give the first two solutions then obtain the next two solutions.
a has not been defined yet.
a will be defined later.



1. $\vec{A} = a(0,0,1)$

2 . $\vec{A} = a(1,0,0)$

3. $\vec{A} = a(.............)$

4, $\vec{A} = a(................)$

------------------▷ 14

73

$$grad f(x,y,z) = \vec{\nabla} \cdot f(x,y,z) = \left(\frac{\delta f}{\delta x} \cdot \vec{e}_x + \frac{\delta f}{\delta y} \cdot \vec{e}_y + \frac{\delta f}{\delta z} \cdot \vec{e}_z \right)$$

$\vec{\nabla}$ is a vector. In this case we compute the product of a vector with a scalar since the function $f(x,y,z)$ is a scalar.
The result is a vector since the product of a vector with a scalar is a vector.

Now we calculate the divergence of a vector field

$$div \quad \vec{F} = \vec{\nabla} \cdot \vec{F}(x,y,z) = \frac{\delta F_x}{\delta x} \vec{e}_x + \frac{\delta F_y}{\delta y} \vec{e}_y + \frac{\delta F_z}{\delta z} \cdot \vec{e}_z$$

Again $\vec{\nabla}$ is a vector. But this time we compute
the dot product of the vector Nabla with the vector $\vec{F}$. The result is a scalar. We remember well from vector algebra the dot product of two vectors results in a scalar.
Calculate $\vec{\nabla} \cdot \varphi(x,y,z)$ for $\varphi = (x^2 + y^2 + z^2)$:
$grad \quad \varphi = \vec{\nabla} \cdot \varphi = ...$

Calculate $\vec{\nabla} \cdot \vec{F}$ for $\vec{F} = (x^2, y^2, z^2)$:
$div \quad \vec{F} = \vec{\nabla} \cdot \vec{F} = ...$

------------------▷ 74

14

1. $\vec{A} = a(0,0,1)$
2. $\vec{A} = a(1,0,1)$
3. $\vec{A} = a(1,1,1)$
4. $\vec{A} = a(1,0,1)$

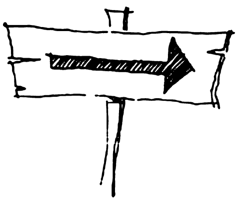
Now we will determine a. $\vec{A}$ must have the magnitude A. To obtain this we have to choose a.
For the first surface it is evident that:
$\vec{A} = A(0,0,1)$ In this case $a = A$
For the second exercise we have
$\vec{A} = \frac{A}{\sqrt{2}}(1,0,1)$ We may prove it: $A^2 = \frac{A^2}{2}(1+1)$
Now you may try to calculate a for the third exercise $\vec{A} = ...(1,1,1)$

-------------------▷ 15

74

$grad \quad \varphi = \vec{\nabla} \cdot \varphi = (2x, 2y, 2z) = (2x\vec{e}_x + 2y\vec{e}_y + 2z\vec{e}_z)$
(The vector $grad \quad \varphi$ may be written down in short-hand or extensively)

$div \quad \vec{F} = \vec{\nabla} \cdot \vec{F} = (2x + 2y + 2z)$



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. $\vec{A} = \frac{A}{\sqrt{3}}(1,1,1)$ Prove: $(\vec{A})^2 = \frac{A^2}{3}(1+1+1) = A^2$ 15

We give the systematic solution. Given: $\vec{A} = a(a_x, a_y, a_z)$.

If $\vec{A}$ is to be of magnitude A this results in $(\vec{A})^2 = A^2$ this results in

$$A^2 = a^2(a_x^2 + a_y^2 + a_z^2)$$

Thus, finally we obtain

$$a = \frac{A}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

In our case no direction of the vector field is given.
Thus, the sign of the surface element vector may be inverted.

-------------------▷ 16

In the textbook page 470 the electrical field of a sphere is discussed. 75
Given a sphere with homogeneous charge distribution ρ, the total charge Q,
and radius *R*. Then outside the surface of the sphere the electrical field is given by

$$\vec{E}(x,y,z) = \frac{Q}{4\pi\varepsilon_0} \frac{(x,y,z)}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

Now calculate the divergence $\vec{E}$ outside the surface of the sphere.

$div \quad \vec{E}$ =………………………………………………….

-------------------▷ 76

17.2 Surface integral

16

Study in the textbook 17.2 Surface integral

Textbook pages 464–466

This done go to ------------------▷ 17

$div \quad \vec{E} = 0$

76

In case of difficulties consult the textbook again.

……………………………………………………………………………………..

Inside the sphere the electrical field is given by

$$\vec{E}(x,y,z) = \frac{Q}{4\pi\varepsilon_0 R^3}(x,y,z)$$

Calculate the divergence inside the sphere:

:

$div \quad \vec{E} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 77

The following integral is named ………. 17

$I = \oint \vec{F} \cdot d\vec{A}$

The circle in the integral $I = \oint \vec{F} \cdot d\vec{A}$

significates the integral has to be taken over a ……………………….surface.

------------------▷ 18

$div \quad \vec{E} = \frac{3Q}{4\pi\varepsilon_0 R^3} = \frac{\rho}{\varepsilon_0}$ 77

In case of difficulties remember $\frac{3}{4}\pi R^3 = V$ and $\frac{Q}{V} = \rho$.

…………………………………………………………………………………………

Write down from memory the Gauss theorem. If possible do not consult the textbook. You should memorize this theorem.

………………………………………………..
………………………………………………..
………………………………………………..

------------------▷ 78

Surface integral

Closed surface

18

Give at least three examples for a closed surface ….

1.: ………………………………………………………..

2: ………………………………………………

3:……………………………………………..……………………………………………...……
……………………………………..

-------------------▷ 19

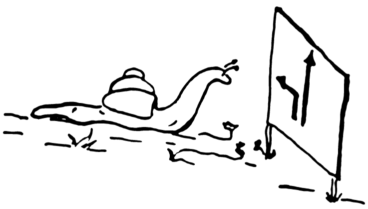
$$\int_V div \ \ \vec{F} \cdot dV = \oint_{A(V)} \vec{F} \cdot \overrightarrow{dA}$$

78

………………………………………………………………………………………………
In the following we will use Gauss's theorem to calculate the given electrical field inside and outside of the surface of the sphere.

First calculate the electrical field outside the surface of the sphere:

$\vec{E} =$…………………………………..................



Solution found -------------------▷ 82

Help and explanations wanted -------------------▷ 79

Control your examples using the definitions given in the textbook 19

This control is necessary if there are no right answers given.
In your further study and in practice this is the normal case.

-------------------▷ 20

In the textbook page 470 the electric field of a sphere is given with radius R and charge density ρ. Outside of the sphere the electrical field is: 79

$$\vec{E}(x,y,z) = \frac{Q}{4\pi\varepsilon_0} \frac{(x,y,z)}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

We can obtain this result using Gauss's theorem. The center of the sphere is to coincide with the origin of the coordinate system.

First we calculate the total charge Q of the sphere. Given radius R and charge density inside the sphere ρ:

$Q = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

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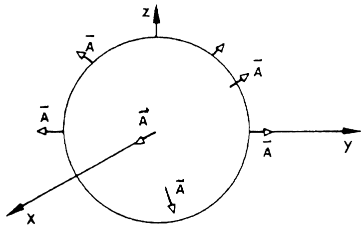


The direction of the surface element vectors for closed surfaces is defined

20

unambiguous ------------------▷ 21

ambiguous ------------------▷ 22

$$Q = \int_{V_{Kugel}} \rho \quad dv = \frac{4\pi}{3}\rho R^3$$

80

Secondly we calculate the electrical field outside the sphere using Gauss's theorem.

$$\int_V div\vec{F} \cdot dV = \oint_{A(V)} \vec{F} \cdot \overrightarrow{dA}$$

From electrodynamics we know that the flow of the electrical field through a closed surface is proportional to the enclosed total charge Q.

$$\oint_{Surfase} \vec{E} \cdot \overrightarrow{dA} = \frac{Q}{\varepsilon_0}$$

Calculate the flow through the surfaces outside the sphere with the given radius $R_{outside}$:

$$\oint \vec{E}\overrightarrow{dA} = ... = \frac{Q}{\varepsilon_0}$$

------------------▷ 81

21

You are right. With closed surfaces the sign is defined unambiguously.

With closed surfaces the surface element vector always points to …………

………………………………………………………….……………………

------------------▷ 23

81

$$\int \vec{E}\cdot\overrightarrow{dA} = |E|\cdot 4\pi R^2_{außen} = \frac{Q}{\varepsilon_0}$$

From this result we obtain:

$$|E| = \frac{Q}{\varepsilon_0 \cdot 4\pi R^2_{außen}}$$

The field has spherical symmetry. The field vector points to the outside and its direction is given by the unit vector:

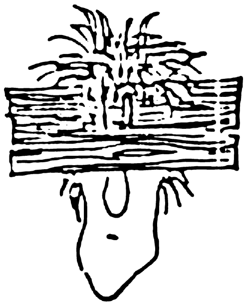
$$\frac{\vec{r}}{|r|} = \frac{(x,y,z)}{\sqrt{x^2+y^2+z^2}}$$

Thus we obtain: $|r| = R = \sqrt{x^2+y^2+z^2}$

$$\vec{E} = \frac{Q}{\varepsilon_0 4\pi R^2}\cdot\frac{\vec{r}}{|r|} = \text{……………………………......}$$

------------------▷ 82

Unfortunately you are wrong. 22



Look again into the textbook and read again the definition or better the convention of the direction of surface elements.

The surface element vectors are
a) perpendicular to the surface
b) they point with closed surfaces always to…………………………..

------------------▷ 23

$$\vec{E}(x,y,z) = \frac{Q}{4\pi\varepsilon_0}\frac{(x,y,z)}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$ 82

Now we are to calculate the electrical field inside the sphere.
Let us regard a surface of a sphere inside with radius R_{inside}.
We are to calculate the enclosed charge. Use Gauss's theorem again and use the total charge Q of the original sphere.

$\vec{E}_{innen}$=…………………………………………

Further help and detailed calculation ------------------▷ 83

Solution happily found ------------------▷ 86

23

The surface element vector for closed surfaces always point to the outer space.

This is a convention, but it is worth memorizing.

-------------------▷ 24

83

The surface of an inner sphere encloses a part of the total charge given by:

Q_{innen} =………………

-------------------▷ 84

17.3 Special cases of surface integrals

24

Study in the textbook **17.3.1 Flow of a homogeneous vector field through a cuboid**

Pages 466–468

-------------------▷ 25

$$Q_{innen} = \int \rho dV = \frac{4\pi}{3} \cdot R_{innen}^3 \cdot \rho$$

84

Now we apply Gauss's theorem. Given: $div \quad \vec{E} = \frac{\rho}{\varepsilon_0}$.

$$\oint_{surface} \vec{E} \cdot \overrightarrow{dA} = \int_{volume} div \quad \vec{E} \quad dV$$

We calculate both integrals:

a) $\oint_{Surface} \vec{E} \cdot \overrightarrow{dA}$ =………………

b) $\int_{Volume} div \quad \vec{E} dV$ =……………….

Inserting into Gauss's theorem we obtain:……………=………………….

-------------------▷ 85

Decide whether the following vector fields are homogeneous or not 25

	Vector field is homogeneous	
	yes	no
1. $\vec{F} = \frac{(x,y,z)}{x^2+y^2+z^2}$	☐	☐
2. $\vec{F} = (1,0,x)$	☐	☐
3. $\vec{F} = (y,x,z)$	☐	☐
4. $\vec{F} = (6,3,5)$	☐	☐
5. $\vec{F} = (2,0,0)$	☐	☐

------------------▷ 26

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a) $\int \vec{E} \cdot \overrightarrow{dA} = E \cdot 4\pi R^2_{innen}$ 85

b) $\int div\vec{E}dV = \frac{\rho}{\varepsilon_0} \cdot \frac{4\pi}{3} \cdot R^3_{innen}$

Inserting into Gauss's theorem we obtain

$$\left|\vec{E}\right| \cdot 4\pi R^2_{innen} = \frac{\rho}{\varepsilon_0} \cdot \frac{4\pi}{3} \cdot R^3_{innen}$$

$$\left|\vec{E}\right| = \frac{\rho}{\varepsilon_0} \frac{R_{innen}}{3}$$

To obtain $\vec{E}$ we have to regard the direction of $\vec{E}$:

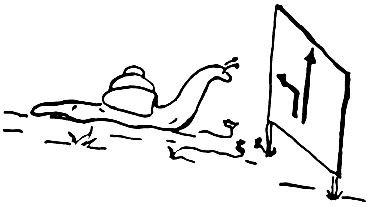
$$\vec{E} = |E| \cdot \frac{\vec{r}}{|r|} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Remember: $|\vec{r}| = R_{inside}$

------------------▷ 86

Homogeneous vector fields are 4. 5.

26



All correct

-------------------▷ 29

Not all answers correct

-------------------▷ 27

$$\vec{E}_{innen} = \frac{\rho}{\varepsilon_0} \cdot \frac{(x, y, z)}{3}$$

86

Finally, we substitute ρ by the total charge Q of the original sphere using the known equation

$$Q = \rho \cdot \frac{4\pi}{3} R^3$$

$\vec{E}_{innen} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots..$

-------------------▷ 87

In this case we suggest you read again the section you just studied.
Theses questions did not imply calculations. Thus, you have problems with
the understanding of the definition of a homogeneous vector field.
Read again to try and find the correct answers.

27

Vector field	homogeneous	not homogeneous
1. $\vec{F} = \frac{(1,2,3)}{x^2+y^2+z^2}$	□	□
2. $\vec{F} = (\frac{1}{x}, \frac{1}{y}; \frac{1}{z})$	□	□
3. $\vec{F} = (1;0;0)$	□	□
4. $\vec{F} = (x;0;0)$	□	□
5. $\vec{F} = (21;1;)$	□	□

------------------▷ 28

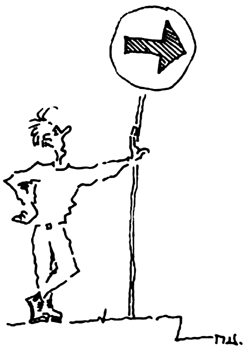
$$\vec{E}_{innen} = \frac{Q}{4\pi\varepsilon_0 \cdot R^3} \cdot (x, y, z)$$

87

In case of remaining difficulties repeat this section of the study guide and consult the textbook again.

------------------▷ 88

Go on to



1. $\vec{F} = \frac{(1,2,3)}{x^2 + y^2 + z^2}$ not homogeneous 28

2. $\vec{F} = (\frac{1}{x}, \frac{1}{y}; \frac{1}{z})$ not homogeneous

3. $\vec{F} = (1;0;0)$ homogeneous

4. $\vec{F} = (x;0;0)$ not homogeneous

5. $\vec{F} = (21;1;)$ homogeneous

------------------▷ 29

17.6 Curl of a vector field and Stoke's theorem

88

The concept of curl will be introduced. After completing section 17.7 it is suggested you have a break and a cup of coffee or tea. But do not forget to take notes of new definitions on a separate sheet.

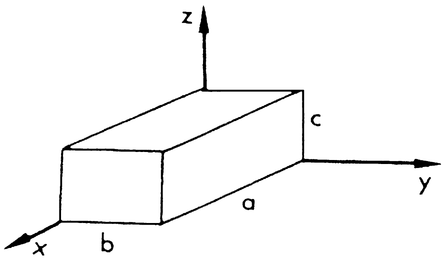
Study **17.7 Curl of a vector field.**
17.8 Stoke's theorem
Textbook pages 480–485

Having studied ------------------▷ 89

Determine the flow of the field $\vec{F}(x, y, z) = (1,4,3)$ through a cube whose faces are parallel to the axis of the coordinate system. 29

The flow I of the vector field $\vec{F}$ is

$I =$………………………..



Solution found -------------------▷ 31

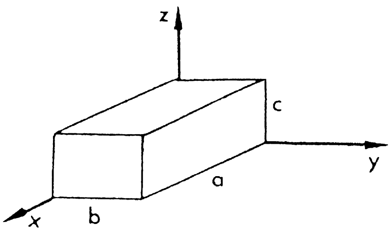
One more hint wanted -------------------▷ 30

If a vector field $\vec{F}_1$ is curl free we have: …………=………..
If a vector field $\vec{F}_2$ has curl we have gilt……...…..=……….. 89

-------------------▷ 90

Hint: $\vec{F} = (1,4,3)$ is a homogeneous vector field.
Thus, you can apply rule 17.7 given in the textbook.

30



Give the flow of the vector field $\vec{F} = (1;4;3)$ through a cube whose faces are not parallel to the axes of the coordinate system.

$I =$.....................................

------------------▷ 31

Field is curl free $\oint \vec{F}_1 \cdot \overrightarrow{ds} = 0$

Field has curl $\oint \vec{F}_2 \cdot \overrightarrow{ds} \neq 0$

90

It is not that easy, but it is worthwhile to memorize the definition of curl or to reconstruct it using the formula $rot \quad \vec{F} = \vec{\nabla} \times \vec{F}$.

$rot \quad \vec{F} =$ ....

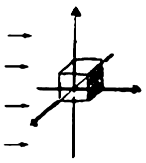
------------------▷ 91

$I = 0$

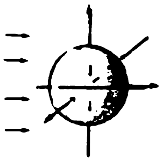
31

For which of the given surfaces below does the flow of the homogeneous field $\vec{F} = (0,2,0)$ not vanish? The surfaces are closed.

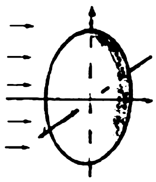
cuboid ☐



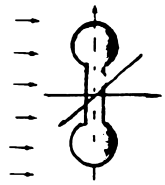
sphere ☐



ellipsoid ☐



dumbbell-shaped ☐



------------------▷ 32

91

$$rot \quad \vec{F} = \begin{pmatrix} \dfrac{\delta F_z}{\delta y} - \dfrac{\delta F_y}{\delta z} \\ \dfrac{\delta F_x}{\delta z} - \dfrac{\delta F_z}{\delta x} \\ \dfrac{\delta F_y}{\delta x} - \dfrac{\delta F_x}{\delta y} \end{pmatrix}$$

Write down as a determinant:

$rot \quad \vec{F} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 92

None of these. All are closed. A homogeneous vector field vanishes for all closed surfaces. 32

Given the vector field $\vec{F} = \frac{(x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$. Does its flow through a sphere vanish if the sphere's center coincides with the origin of the coordinate system?

Answer found ------------------▷ 35

Help or more explanation wanted ------------------▷ 33

92

$$rot\vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ F_x & F_x & F_z \end{vmatrix}$$

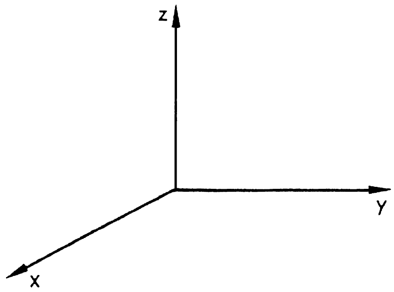
Expand the determinant and give:

$rot\vec{F} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 93

The surface of the sphere is closed. 33

Given the vector field $\vec{F} = \frac{(x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$. This vector field is not homogeneous.



Sketch some vectors of the field along the axes of the coordinate system

-------------------▷ 34

93

$$rot \quad \vec{F} = \begin{pmatrix} \frac{\delta F_z}{\delta y} - \frac{\delta F_y}{\delta z} \\ \frac{\delta F_x}{\delta z} - \frac{\delta F_z}{\delta x} \\ \frac{\delta F_y}{\delta x} - \frac{\delta F_x}{\delta y} \end{pmatrix}$$

Regard a flow of water. The field of velocities may be given by:

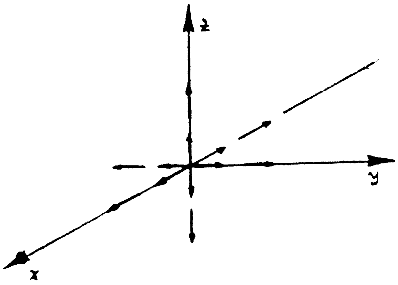
$\vec{v} = (1, \quad \ln z, \quad 0)$

Calculate:

$div \quad \vec{v} =$ ....

$rot \quad \vec{v} =$ ....

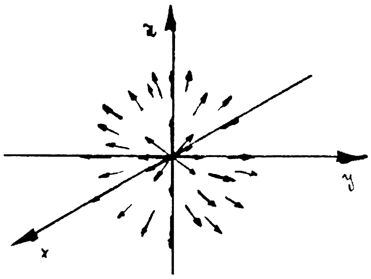
-------------------▷ 94



34

If vectors in all directions are sketched, the field is represented as shown to the left.
Imagine a sphere with its center at the origin of the coordinate system. The field passes through the surface always from the inner side to the outer side.

Does the flow through the surface vanish?



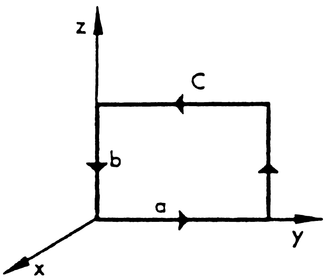
------------------▷ 35

94

$div\vec{v} = 0$

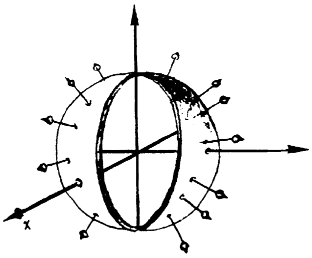
$rot\vec{v} = (-\frac{1}{z}, 0, 0)$

Calculate the line integral for the rectangle with sides a and b for a complete circulation.
Given $\vec{F}$ to be $\vec{F}(x,y,z) = (5,0,z^2)$



$\oint \vec{F}\vec{ds} =$ ....

------------------▷ 95



No. The flow of $\vec{F} = \frac{(x, y, z)}{\sqrt{x^2 + y^2 + z^2}}$ does not vanish. 35

The field passes at all points of the surface from the inner space to the outer space.

A vector field $\vec{F}$ has radial symmetry if

1……………………………………………………

2………………………………………….

In case of doubt look again at textbook chapter 13 (section "coordinate systems").

------------------▷ 36

The field of velocities is free of curl. Thus: 95

$\oint \vec{F}\,\overrightarrow{ds} = 0$

------------------▷ 96

A vector field has radial symmetry if
1. All vectors point in a radial direction and;
2. Its amount depends only on the radius r.

36

Decide whether the following field possesses radial symmetry:

2. Its amount depends only on r. $\vec{F} = \frac{(x, y, z)}{(\sqrt{x^2 + y^2 + z^2})^3} = \frac{\vec{r}}{r^3}$

Solution found -------------------▷ 39

Further explanation or help wanted -------------------▷ 37

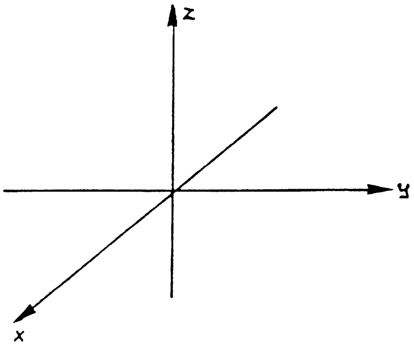
17.7 Potential of a vector field

96

Study **17.9 Potential of a vector field**
Textbook pages 485–487

-------------------▷ 97

Firstly sketch some of the vectors of the field $\vec{F} = \frac{\vec{r}}{r^3}$ 37

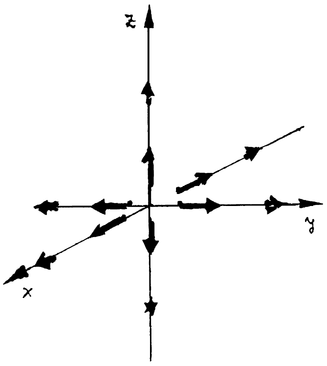


------------------▷ 38

In the textbook we treated the example of the gravitational field of a sphere, the mass of which is *M*. The case most familiar to us is the earth. At the surface of the earth, however, we simplify the situation by calculating with a homogeneous field of gravitation. In this simplification the x-y plane is parallel to the earth's surface. The z-axis points upwards and the origin of the coordinate system coincides with the earth's surface. This approximation holds for a lot of calculations. In this case the gravitational force acting on a mass *m* is given: 97

$\vec{F} = $ ………………………………

------------------▷ 98



Your sketch may be similar to the sketch to the left. 38
The vectors point outwards. Because of r^3
in the denominator the amount of the vectors decreases with the distance from the origin.
The vectors at other positions point in a radial direction as well.
Since $|(x,y,z)| = \mathrm{r}$ we obtain:

$$F = \frac{(x,y,z)}{(\sqrt{x^2+y^2+z^2})^3} = \frac{r}{r^3} = \frac{1}{r^2}$$

F only depends on r.
Thus, this vector field has ……………………………………

------------------▷ 39

$\vec{F} = (0, \ 0, \ -m \cdot g)$ 98

In the case of a gravitational field the force $\vec{F}$ is the product of the mass m of the body in question and the gravitational field vector $\vec{F}_g$ of the gravitational field. This field vector is in this case

$\vec{F}_g = -g \cdot \vec{e}_z = (0, 0, -g)$

In the following we discuss fields and their field vectors. This holds as well for electrical fields and field vectors. In the case of a static electrical field the force acting on a charge Q is the product of Q with the electrical field vector $\vec{E}$:

$\vec{F} = Q \cdot \vec{E}$

The field is completely represented by the electrical field vector $\vec{E}$.

Check gravitational field $\vec{F}_g$.
Is it free of curl?

$rot \ \vec{F}_g =$…………………

$\vec{F}_g$ is……………………..

------------------▷ 99

The vector field $\vec{F}$ has radial symmetry because 39

$\vec{F} = \frac{(x,y,z)}{(\sqrt{x^2+y^2+z^2})^3}$ points in a radial direction and its amount depends only on *r*.

------------------▷ 40

$rot(0,0,-g) = 0$ 99

$\vec{F}_g$ is curl-free, it is a conservative field.

Calculate the potential of $\vec{F} = m \cdot \vec{F}_g$.

Remember: The following convention holds in physics. For a given force field $\vec{F}_g$ the potential φ is the work done against the force field

$\varphi(x, y, z) = \ldots\ldots\ldots\ldots\ldots\ldots\ldots..$

------------------▷ 100

17.3.1 Flow of a spherically symmetrical field through a sphere

40

In this section we show the calculation of surface integrals through a surface of a sphere.
In physics this is an important special case.

Read in the textbook **17.3.2 Flow of a spherically symmetrical field through a sphere**
Textbook pages 468–469

------------------▷ 41

$\varphi(x, y, z) = m \cdot g \cdot z + C$

100

The potential is thus determined except the integration constant C:
Determine the potential given above for three different situations:

1) $\varphi_1 = 0$ for the ground floor with $z_0 = 0$

   $\varphi_1 = \dots\dots\dots\dots\dots\dots$

2) $\varphi_2 = 0$ for the underground of a high building with $z_0 = -10$

   $\varphi_2 = \dots\dots\dots\dots\dots\dots$

3) $\varphi_3 = 0$ for the roof of a high building with $z_0 = 90$

   $\varphi_3 = \dots\dots\dots\dots\dots\dots$

------------------▷ 101

Calculate the surface integral $\oint \vec{F} \cdot d\vec{A}$ of the field $\vec{F} = \frac{\vec{e}_r}{r^2}$ 41

with $\vec{e}_r = \frac{\vec{r}}{r}$ for a surface of a sphere with radius *R*.

Wanted is the flow of $\vec{F}$ through the surface of the sphere.

$\oint \vec{F} \cdot d\vec{A} =$ ……………………………………..

------------------▷ 42

$\varphi_1 = (0,0,mgz)$ 101

$\varphi_2 = (0,0,mg(z+10))$

$\varphi_3 = (0,0,mg(z-90))$

Give the gravitational field $\vec{F}_g$ for these three cases:

$\vec{F}_{g1} =$ …………………

$\vec{F}_{g2} =$ …………………

$\vec{F}_{g3} =$ …………………

------------------▷ 102

$$\oint \vec{F} \cdot d\vec{A} = \oint \frac{dA}{r^2} = 4\pi R^2 \cdot \frac{1}{R^2} = 4\pi$$

42

Given a field of a force with radial symmetry: $\vec{F}(r) = \frac{a}{r^3}\vec{e}_r$ with $\vec{e}_r \frac{\vec{r}}{r}$

Compute the flow of the field of the force $\vec{F}(r)$ through the surface of a sphere which has a distance R from the origin of the field. The center of the field is defined by $r = 0$.

Solution found -------------------▷ 44

Explanation or help wanted -------------------▷ 43

$$\varphi_1 = \varphi_2 = \varphi_3 = (0,0,mgz)$$

102

In the textbook you studied the gravitational field of a mass M. This mass is assumed to be distributed homogeneously in the inner space of a sphere with radius R. Outside of the sphere the gravitational field is:

$$\vec{F}_g(x,y,z) = -\gamma \cdot M \frac{x,y,z}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

Simplify using $\vec{r} = (x,y,z)$

$\vec{F}_g(x,y,z) =$ …………………..

-------------------▷ 103

The field of the force is given by $\vec{F}(r) = \frac{a}{r^3}\vec{e}_r$ with $\vec{e}_r \frac{\vec{r}}{r}$ 43

Wanted: the flow given by $\oint \vec{F} \cdot d\vec{A}$

First hint: The field has radial symmetry of the following form: $\overline{F} = f(r) \cdot \vec{e}_r$
Second hint: $\oint \vec{F} \cdot d\vec{A}$ is calculated for the general case in the section of the textbook you just studied.
Repeat your study and try again:

$\oint \vec{F} \cdot d\vec{A}$ = ……………………………………………………………………..

------------------▷ 44

$\vec{F}_g = -\gamma M \cdot \frac{\vec{r}}{r^3} = -\gamma M \frac{1}{r^2} \cdot \frac{\vec{r}}{r}$ 103

Now we calculate the potential $\varphi = -\int \vec{F}\overrightarrow{dr}$ for a path of integration in the direction of a radius.

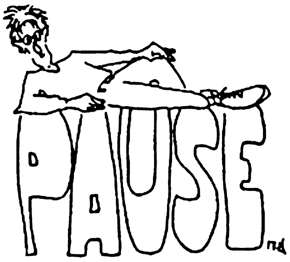
Then we have: $\frac{\vec{r}}{r}\overrightarrow{dr} = dr$.

$\varphi = \gamma M \int_{r_0}^{r} \frac{1}{r_2} \cdot \frac{r}{r} dr =$……………………………

------------------▷ 104

$$\oint \frac{a}{r^3}\vec{e}_r d\vec{A} = \oint f(r)\vec{e}_r d\vec{A} = 4\pi R^2 \cdot f(R) = 4\pi R^2 \cdot \frac{a}{R^3} = \frac{4\pi \cdot a}{R}$$ 44

Have a short break



------------------▷ 45

$$\varphi(r) = \gamma M \cdot \int_{r_0}^{r} \frac{dr}{r^2} = -\gamma M \left[\frac{1}{r}\right]_{r_0}^{r}$$ 104

$$\varphi(r) = \gamma M \left[\frac{1}{r_0} - \frac{1}{r}\right]$$

We can scale the potential of the gravitational field of the mass M in a way that it vanishes for $r \to \infty$, but the potential may be scaled as well to vanish for the surface for which holds $r = r_{surface}$

In the textbook we explained the first case.
In the study guide we calculated the second case.
Calculate the potential for the first case in order to vanish for infinity.

$\varphi_1 =$ …………………..

------------------▷ 105

17.3.2 Application: The electrical field of a point charge.

45

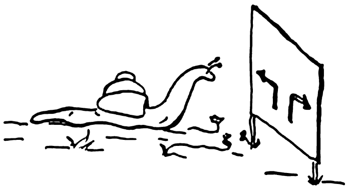
Here we apply the new knowledge

Study in the textbook **17.3.3 Application: The electrical field of a point charge**
Textbook page 470

------------------▷ 46

$$\varphi_1 = -\gamma M \frac{1}{r}$$

105



All correct ------------------▷ 108

Further explanation wanted ------------------▷ 106

46

The calculations in the foregoing section are obvious. In this case using physical units facilitates the calculations.

Thus, we can proceed to the next section without further exercises.

Go to -------------------▷ 47

106

The potential of the gravitational field was represented by:

$$\varphi(r) = \gamma M \int_{r_0}^{r} \frac{dr}{r^2} = \gamma M \left[\frac{1}{r_0} - \frac{1}{r} \right]$$

The condition was $\varphi(r = \infty) = 0$

In this case the bracket must be zero. This is obtained by letting $\frac{1}{r_0} = 0$ or $r_0 = \infty$.

A difficulty in understanding this may arise because r_0 is the lower limit of integration and the lower limit can not be ∞ while the upper limit of integration is finite. But this problem is solved; we invert the direction of integration. We integrate from r to r_0.

Thus we obtain $\varphi(r) = \gamma M \int_{r_0}^{r} \frac{dr}{r^2} = -\gamma M \int_{r}^{r_0} \frac{dr}{r^2} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

-------------------▷ 107

17.4 General case of computing surface integrals

47

In the following section we compute the surface integral for a general case. This section is slightly formal and more difficult. It is worth studying this section if you are not in a hurry and did not have difficulty with the preceding subject matter. Decide according to your preferences.

I prefer to skip section 17.4 for the time being and want to proceed

------------------▷ 55

I prefer to study section 17.4.

Then study in the textbook **17.4 General case of computing surface integrals Textbook pages 470–474**

------------------▷ 48

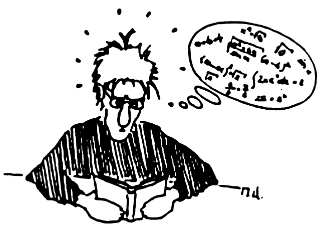
107

$$\varphi(r) = \gamma M\left[\frac{1}{r_0} - \frac{1}{r}\right]$$

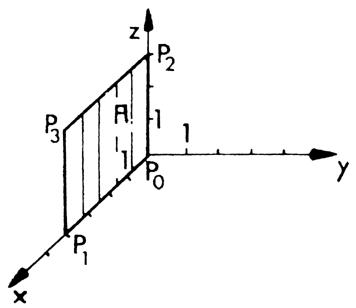
Now we let r_0 grow to infinity to obtain the result of the foregoing frame.

$$\varphi_1 = -\gamma M\,\frac{1}{r}$$

Further problems may arise if we change signs. Do not underestimate the problems related to signs. It is always advisable to calculate meticulously.



------------------▷ 108



A rectangular plane A is defined by the points $P_0 = (0,0,0)$, $P_1 = (4,0,0)$, and $P_3 = (4,0,0)$. Thus it lies in the x-z plane.

48

An inhomogeneous vector field is given by $\vec{F} = (0,2x,0)$.

Compute $\int \vec{F} d\vec{A} =$ ……..

Solution found -------------------▷ 54

Explanation or hints wanted -------------------▷ 49

Now we will treat the second case and let the potential of the gravitational field vanish for the surface of the earth.

108

We calculated before: $\varphi(r) = \gamma M\left[\frac{1}{r_0} - \frac{1}{r}\right]$

The height z above the surface is given by $r = r_0 + z$

Thus: $\varphi(z)$

$\varphi(z)=$……………………….

-------------------▷ 109

49

In this case we have an inhomogeneous field. The field is quite simple, because it has only one component in the y-direction. Remember the field was given by $\vec{F} = (0, 2x, 0)$

To obtain $I = \int_A \vec{F} \cdot d\vec{A}$ we determine $\vec{F}$ and $d\vec{A}$

$\overline{F} = \ldots\ldots\ldots\ldots$

$d\vec{A} = \ldots\ldots\ldots$

$I = \ldots\ldots\ldots\ldots\ldots$

Solutions found ------------------▷ 52

Further help and explanations ------------------▷ 50

109

$$\varphi(r) = \gamma M\left[\frac{1}{r_0} - \frac{1}{r_0 + z}\right]$$

For places near the surface we state $z << r_0$. Applying this approximation we write:

$$\frac{1}{r_0 + z} = \frac{1}{r_0\left(1 + \frac{z}{r_0}\right)} \approx \frac{1}{r_0}(\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots)$$

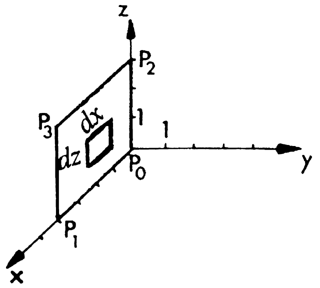
We insert this into the equation above and obtain:

$$\varphi(z) = \gamma M[\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots]$$

------------------▷ 110

We have to determine $\vec{F}$ and $d\vec{A}$. 50

Difficulties may arise in determining $d\vec{A}$. Plane A lies in the x-z plane, as was pointed out in frame 48. Thus, the surface element vector points in the y-direction.



The amount of a differential surface element for this plane is given by dA $= dx \cdot dz$.
The surface element vector in the y-direction with amount dxdz is given by

$d\vec{A} = (.............)$

Remember: $\vec{F}$ is given to be $\vec{F} = (....................)$

------------------▷ 51

Approximation: $\dfrac{1}{r_0\left(1+\dfrac{z}{r_0}\right)} \approx \dfrac{1}{r_0}\left[1-\dfrac{z}{r_0}\right]$ 110

$\varphi(z) = \gamma M \dfrac{z}{r_0^2}$

For the case of the earth with mass M and radius r_E this corresponds to the potential we used in previous frames. There we used the expression

$\varphi(z) = (0{,}0 \quad g \cdot z) = g \cdot z$

Both expressions are identical if we define g appropriately. Calculate and define:

$g = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 111

$d\vec{A} = (0, dx \cdot dz, 0)$
$\vec{F} = (0, 2x, 0)$

51

We want to determine the flow $I = \int_A \vec{F} \cdot d\vec{A}$ a

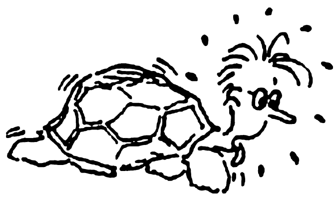
Now we can calculate the dot product in the integral using the results obtained:

$I = \int_A \vec{F} \cdot d\vec{A} = \int_A (0, 2x, 0) \cdot (0, dxdz, 0) = \int_A \ldots$ ………………..….

-------------------▷ 52

$g = \frac{\gamma M}{r_0^2}$

111



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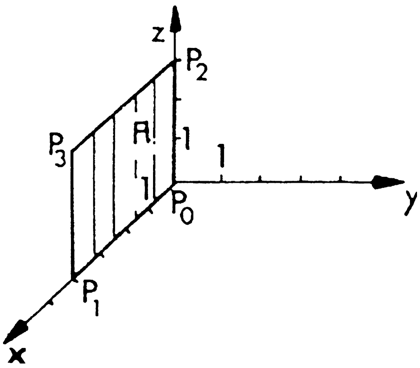
$$\int_A \vec{F} d\vec{A} = \int_A 2x dx dz$$

52

This integral must be calculated for plane A.
Written correctly this is a double integral. Write this integral as a double integral and insert the limits given by our plane A:

$$A\int 2x \cdot dx \cdot dz = \int_{x=......}^{..........} \int_{z=.....}^{.......} 2x \cdot dx \cdot dz$$



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At the end of this chapter we recapitulate:

112

Definition of divergence of a vector field $\vec{F}$:

$div \quad \vec{F} =$ ……………………=………

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$$I = \int_A 2x dx \cdot dz = \int_{x=0}^{4} \int_{z=0}^{3} 2x dx \cdot dz$$ 53

You can calculate the double integral. It was explained in chapter 13 “Multiple integrals.”
In case of difficulties you should repeat that chapter, at least section 13.2 “multiple integrals with constant limits.”

$$I = \int_A 2x dx \cdot dz = \int_{x=0}^{4} \int_{z=0}^{3} 2x dx \cdot dz = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

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$$div \quad \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \vec{\nabla} \cdot \vec{F}$$ 113

Calculate the divergence of the given vector field $\vec{F}$:

$$\vec{F} = \left(\frac{x^3}{3}, \frac{y^3}{3}, -\frac{z^2}{2} \right)$$

$div \quad \vec{F} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Distribution of sources and sinks:

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

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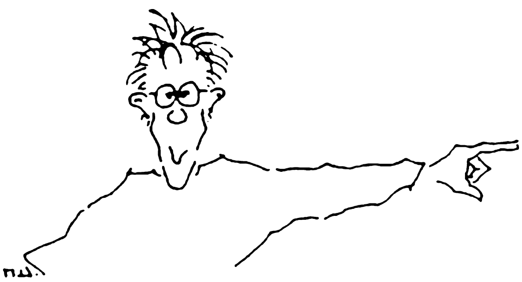
$$I = \int_A \vec{F} \cdot d\vec{A} = 48$$

54

Congratulations. You have mastered some difficult reasoning.



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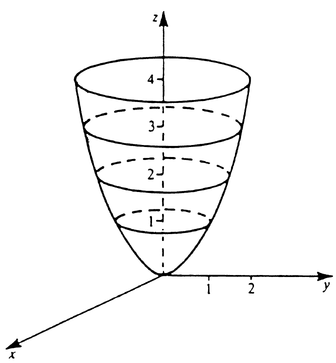
$div \quad \vec{F} = x^2 + y^2 - z$

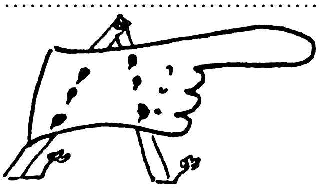
114



No sinks or sources for:
$div \quad \vec{F} = 0$:
Thus, we obtain for this condition $0 = x^2 + y^2 - z$
or $z = x^2 + y^2$. This represents a paraboloid
of revolution around the z-axis.
The inner space consists of sinks. The outer space consists of sources.

Give Gauss's theorem:

……………………………………………..=………………………………



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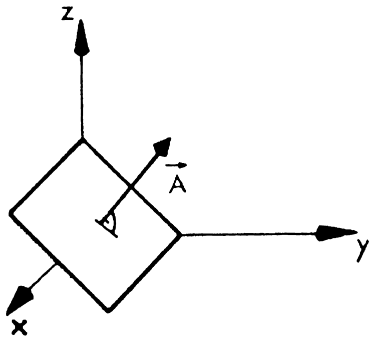
Before we finish this section we should recapitulate. 55
In a vector field $\vec{F}$ we have a quadratic plane with an area of 2.
The position is sketched below.

Give the surface element vector $\vec{A}$. $\vec{A}=$.........



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$$\int_V div \quad \vec{F}dV \qquad\qquad = \int \vec{F} \cdot d\vec{A}$$ 115

Volume integral Surface integral. l

Give the definition of the curl of a vector field $\vec{F}$:

$rot \quad \vec{F} = \vec{\nabla} \times \vec{F} =$................................

------------------▷ 116

$\vec{A} = (0, \sqrt{2}, \sqrt{2})$ You may have factorized the root: $\vec{A} = \sqrt{2}(0,1,1)$

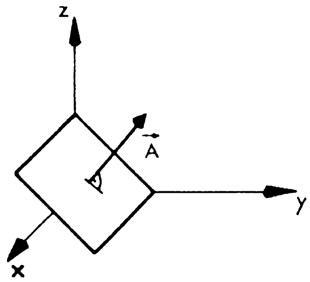
56

Calculate for this plane A the flow of three vector fields:

$F_1 = (0,6,0)$ $I_1 =$ ....

$F_2 = (0,2,1)$ $I_2 =$ ....

$F_3 = (6,0,0)$ $I_3 =$ ....



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116

$$rot \quad \vec{F} = \begin{pmatrix} \frac{\delta F_z}{\delta y} - \frac{\delta F_y}{\delta z} \\ \frac{\delta F_x}{\delta z} - \frac{\delta F_z}{\delta x} \\ \frac{\delta F_y}{\delta x} - \frac{\delta F_x}{\delta y} \end{pmatrix}$$

Given a flow of water

$\vec{V} = \left(z^2, \ 0, \ 0\right)$

$rot \quad \vec{V} =$ ………………..

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$I_1 = 6 \cdot \sqrt{2}$ $I_2 = 3 \cdot \sqrt{2}$ $I_3 = 0$

57

Given a vector field with radial symmetry $\vec{j}$. The origin coincides with the origin of the coordinate system.

The amount of $\vec{j}$ is constant.

Calculate the flow I of $\vec{j}$ through a sphere with radius R. The center of the sphere lies in the origin of the coordinate system.

I =…………….

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$rot(z^2, 0, 0) = (0, 2z, 0)$

117

Give Stoke's theorem:

………………………..=……………………….

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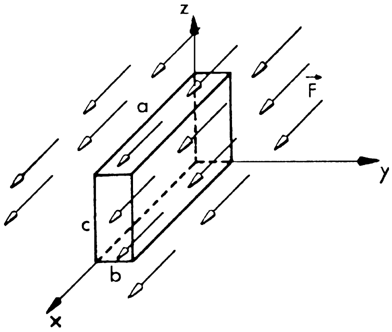
$I = \int \vec{j} \cdot d\vec{A} = 4\pi R^2 \left| \vec{j} \right|$

58



Calculate the flow Φ of a vector field

$\vec{F} = (1,0,0)$

Through the sketched cuboid with the edges

a = 6, b = 1, c = 3

$\Phi =$……………………………………

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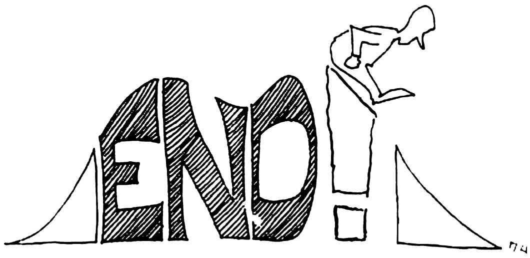
$\int\limits_V div\vec{F}dV =$ $\oint \vec{F} \cdot d\vec{A}$ $\int\limits_A rot\vec{F} \cdot \overrightarrow{dA} =$ $\oint\limits_{C(A)} \vec{F} \cdot \overrightarrow{ds}$

118

Volume integral Surface integral Surface integral Line integral

The theorems of Gauss and Stokes are worth understanding and memorizing in order to avoid difficulties when applying them in physics.

You have reached the end of chapter 17.
You have now made considerable progress.



$\Phi = 0$

59

Given $\vec{F} = (0.5, 0, 0.5)$

Calculate the flow for three planes:

$\vec{A}_1 = (1, 1, 0)$ $I_1 =$...........

$\overline{A}_2 = (1, 0, 1)$ $I_2 =$..................

$\vec{A}_3 = (1, 1, 1,)$ $I_3 =$.................

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Please continue on page 1
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