Chapter 3
Functions

3.1 The mathematical concept of functions and its meaning in physics and engineering

For many readers, most parts of chapter 3 will be a mere recapitulation of well-known facts.
If, however, you are not sufficiently familiar with its content, we suggest that you carefully take notes of all new concepts and notations.

Now start to study the textbook

READ 3.1 The mathematical concept of functions and its meaning in physics and engineering
Textbook page 39-42

When you have finished return to this study guide frame -------------------▷ 2

84

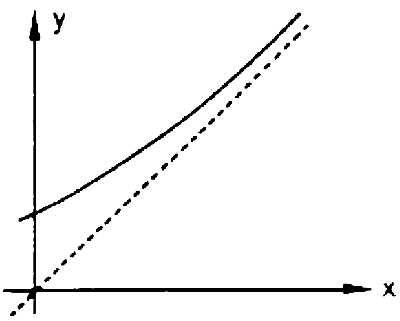
The graph of the inverse function is obtained by reflecting the line *f*(*x*) in the line $y = x$ which bisects the first and third quadrant.

Or any different wording with the same meaning

Obtain the inverse functions

$y_1 = \dfrac{1}{x+1}$ $\qquad$ $y_1^{-1}(x) = \ldots\ldots$

$y_2 = 5x + 1$ $\qquad$ $y_{-12}(x) = \ldots\ldots$



Sketch the graph of the inverse function for the plotted function.

------------------▷ 85

After having read the textbook, answer the following questions in order to ensure that you have grasped the main ideas. **2**
Very often we understand facts but we fail to memorize them.

The expression $y = f(x)$ is called …………..
The particular parts are named as follows

x: ……………

y: ……………

$f(x)$ …………..

Check your answers by going to frame 3

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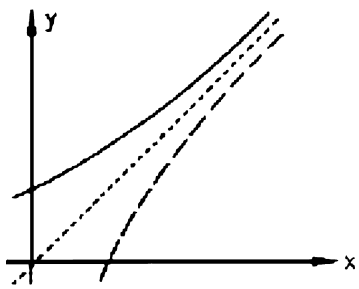
$y_1^{-1}(x) = \frac{1}{x} - 1$ $\quad$ $y_2^{-1}(x) = \frac{x-1}{5}$ **85**



You certainly know by now that in case of difficulties you should always consult the textbook first.

Successful students eliminate with this technique misunderstandings.

------------------▷ **86**

functional expression
x is the independent variable
y is the dependent variable
$f(x)$ is the defining expression

3

The set of all x values for which the function is defined is called ………..

The set of all corresponding y-values is called …………..

-------------------▹ 4

3.6 Trigonometric or Circular Functions

86

An important prerequisite to understanding trigonometric functions is that you know to express angles in radians.

READ 3.6.1 Unit circle
Textbook page 52

Then go to

-------------------▹ 87

Domain of definition, or simply domain

range or codomain

4

If you are still unsure how to use these concepts and notations consult your notes or the textbook.
Click your answer

With a function we assign to a given x-value:

☐ one and only one y-------------------value --------------------▷ 5

☐ one or more y-------------------values --------------------▷ 6

Complete the table

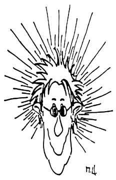
87

Degrees		radians
180^0	=	…….rad
…..	=	2π rad
57^o	=	….rad
……	=	2 rad

--------------------▷ 88

You are right. Functions assign to a given x-value one and only one y-value

5



Which are functions?

$y = x^2 + 2$ ☐

$y = \pm\sqrt{x^2 + 2}$ ☐

$y = \frac{1}{x}$ ☐

$y = \frac{1}{x} \pm \sqrt{x}$ ☐

$y = \frac{1}{x^2 + 1}$ ☐

Now go to frame ------------------▷ 6

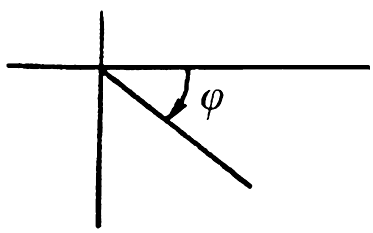
88

$180^0 = \pi$ rad
$360^0 = 2\pi$ rad
$57^0 = 1$ rad
$115^0 = 2$ rad

Angles in clockwise direction are

☐ positive

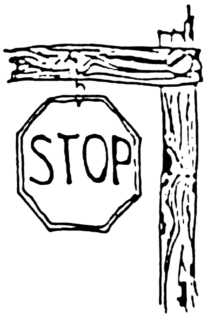
☐ negative



Convert in radians

$1^0 \approx ..$ rad
$45^0 \approx ..$ rad

------------------▷ 89



6

Sorry your answer is not right. Functions assign to a given *x*-value one and only one *y*-value. Functions are unambiguous. But unfortunately during the last decades the term function changed a bit. Thus, in engineering literature you may find the terms “two-valued” or “many valued function.” In modern terminology these are “relationships.”

An ambiguous relation is $y = \sqrt{x+3}$

For $x = 1$ this results in $y = \pm 2$

Is the following term unambiguous?

$$y = \left(4 \pm \sqrt{\frac{1}{x}}\right)^2$$

□ yes

□ no

--------------------▷ 7

Negative

89

$1^0 \approx 0.017$ rad
$45^0 \approx 0.78$ rad

Hint: the notation of angles may vary slightly in different books.

I am not familiar with these conversions and want more exercises --------------------▷ 90

I am familiar with the conversion of angles --------------------▷ 95

No: $y = \left(4 \pm \sqrt{\frac{1}{x}}\right)^2$ is ambiguous

7

Select and mark the functions.

- ☐ $y = x^2 + 2$
- ☐ $y = \pm\sqrt{x^2 + 2}$
- ☐ $y = \frac{1}{x}$
- ☐ $y = \frac{1}{x} \pm \sqrt{x}$
- ☐ $y = \frac{1}{x^2 + 1}$

-------------------▷ 8

In practice you should regard carefully how angles are denoted. It is very useful indeed to be familiar with the conversion of degrees into radians and vice versa.
Convert:

90

$1^0 \approx \ldots$ rad
$90^0 = \ldots$ rad
$180^0 = \ldots.$ rad
$360^0 = \ldots.$ rad

-------------------▷ 91

Only the following are functions

8

$y = x^2 + 2 \qquad y = \frac{1}{x} \qquad y = \frac{1}{x^2 + 1}$

The symbol $\pm$ means that there are two values. But often this symbol is omitted because everyone is supposed to know that a square root has two solutions. You can transform the unambiguous term $y = \sqrt{x^2 + 2}$ to a function if you limit yourself to using only the positive or the negative result of the root. In this case you must denote this.

Transform the expression $y = \frac{1}{\sqrt{x}}$ into functions.

$y_1 = \ldots\ldots\ldots \qquad y_2 = \ldots\ldots\ldots$

-------------------▷ 9

$1^0 \approx 0.017$ rad

91

$90^0 = \frac{\pi}{2}$ rad ≈ 1.57 rad

$180^0 = \pi$ rad ≈ 3.14 rad

$360^0 = 2\pi$ rad ≈ 6.28 rad

It is useful to always remember the following relationship:

The angle around four quadrants of a circle is 2π radians or 360^0.

In other words:
An angle of 360^0 has in radians the value of the circumference of the circle namely 2π.

A positive angle is

□ clockwise
□ anticlockwise

In case of remaining difficulties and errors go back to the textbook and solve the exercises once again

-------------------▷ 92

$y_1 = +\frac{1}{\sqrt{x}}$ $y_2 = -\frac{1}{\sqrt{x}}$

9

The equation $y = x^2$ is a function.

Its argument is ……………..
Its dependent variable is ..…………….
Its domain of definition is ……………
Its codomain is ……………

------------------▷ 10

Anticlockwise.
Hint: The definition of the sign of the angle is a convention.

92

Convert the angles. Calculate the values approximately. It matters only that you gain a certain familiarity with these conversions. Later on you will use your calculator

radians		degrees
3.14	=	……….
1	≈	……….
0.1	≈	……….
1.79	≈	……….

------------------▷ 93

10

x
y
$-\infty \leq x \leq +\infty$
$o \leq y \leq +\infty$

Control of your knowledge using simple questions is important to eliminate errors or misunderstanding from the beginning. Lectures cannot help you with this task.
If your answer was wrong this is not important but you must do something to eliminate the reason for your error.

------------------▷ 11

93

radians		degrees
3.14	≈	180^0
1	≈	57^0
0.1	≈	$5,7^0$
1.79	≈	102^0

If we denote the angles in degrees α and radians in φ then you can write down the conversion formulae:

$\alpha = ...\varphi$
$\varphi = ...\alpha$

Hint: always remember $2\pi rad \approx 360^0$

------------------▷ 94

3.2 Graphical representation of functions

11

READ **3.2. Graphical representation of functions**
3.2.1 Coordinate system, position vector
3.2.2 The linear function; The straight line
Textbook page 42–44

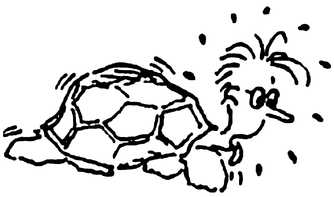
After your study return to this study guide

------------------▷ 12

$$\alpha = \frac{360^0}{2\pi}\varphi \qquad \varphi = \frac{2\pi}{360}\alpha$$

94

We repeated many times: You should be familiar with these relationships because conversions are often used and it helps you if you can do it from the beginning.



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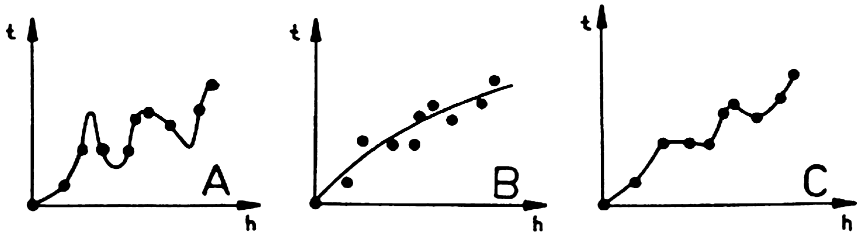
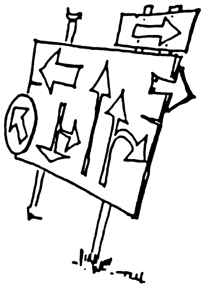
A short introductory and entertaining problem will be presented first. 12

A body falls from a certain height *h*. The body hits the ground after a time *t*. Both values are measured.

The three graphs show the measured values and a curve to represent the result.
Which is the best curve?





- A ------------------▷ 13
- B ------------------▷ 14
- C ------------------▷ 15

3.6.1 Sine function

95

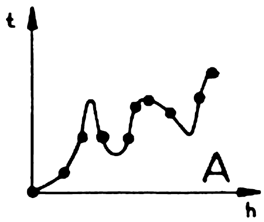
The following section is more extensive. Probably you will know some content from school. Please take notes of all new definitions and rules. If you are tired during your study you may have short breaks.

READ **3.6.2 Sine function**
Pages 53–58

------------------▷ 96

No, no, no 13

Perhaps you are curious to know what will be said here.
You chose this curve



It is highly improbable that this curve with its irregular pattern represents the relationship between height of fall and time of fall.

Go back and try again

--------------------▷ 14

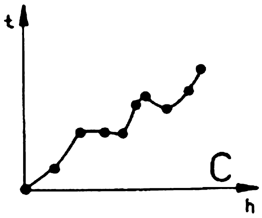
We assume you remember the geometrical definition of the sine. 96
But the definition of the sine – function regarding the unit circle may be new to you.
The sine function is called a …… function. It is defined for angles or in other words its domain is

☐ $0 \le \varphi \le 2\pi$
☐ $0 \le \varphi \le \infty$
☐ $-\infty \le \varphi \le +\infty$

--------------------▷ 97

You chose this curve

14



This curve represents all measured values, but it would not be chosen by physicists or engineers. We know that the accuracy of measurements depends on the used instruments and we always have errors in measurements. In our case we assume that the time of fall increases with the height of fall. Thus, we prefer curve B as the best fitting curve and regard the deviations from the fitting curve as errors of measurement. In chapter 21 "Theory of errors" we will discuss this topic again.

------------------▷ 15

Trigonometric function
Domain of the sine function
$-\infty \leq \varphi \leq +\infty$

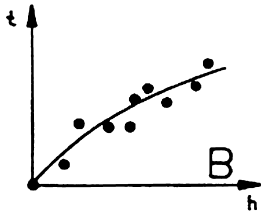
97

The value of $y = \sin x$ never surpasses the value $y_{\max} = \ldots$

The value of $y = \sin x$ never is less the $y_{\min} = \ldots$

The codomain of $\sin x$ is given by $\ldots\ldots\ldots\ldots \leq \sin x \leq \ldots\ldots.$

------------------▷ 98



15

Very good. This was the right choice.
We know that all measurements are subject to measurement errors.
To construct fitting curves involves knowledge of the used instruments and insight into the physical relationships.
In chapter 21 “Theory of errors” we will treat methods to deal with errors of measurement.

------------------▷ 16

98

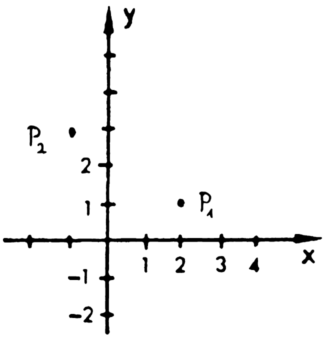
$y_{max} = +1$
$y_{min} = -1$
$-1 \leq \sin x \leq +1$

We can denote this result
$|\sin x| \leq 1$
Give some zeros of $y = \sin x$

$x_{zero} = \ldots = \ldots = \ldots. = \ldots$

------------------▷ 99

Give the coordinates of point P_1 and P_2 16



$P_1 =$ ………………..

$P_2 =$ ……………….

The x-coordinate is named ……………

The y-coordinate is named ……………

------------------▷ 17

$x_{zeros} = 0, \pm\pi, \pm 2\pi, \pm 3\pi$ and so on. 99

Give two different notations for the argument

$y = \sin ....$

$y = \sin ....$

The sine function has the period ……

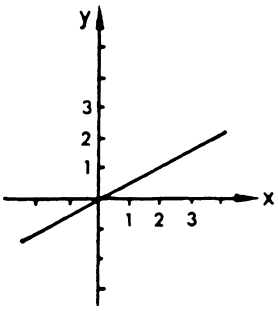
------------------▷ 100

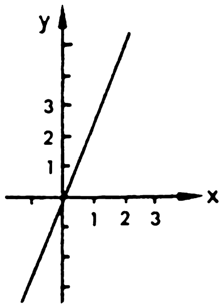
$P_1 = (2, 1)$ $P_2 = (-1, 3)$ 17

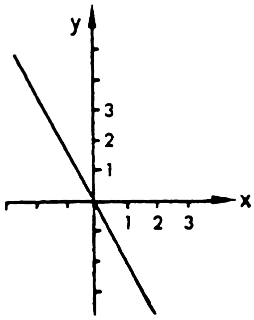
17

abscissae
ordinate

Give the equations of the three straight lines







$y_1 = ...$ $y_2 = ...$ $y_3 = ...$

If this question is too simple for you skip it and go to -------------------▷ 23

Answers and more exercises -------------------▷ 18

$y = \sin\varphi$, $y = \sin x$ or similar notations
The period is 2π

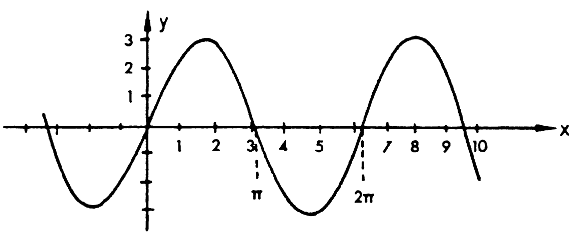
100

Given the function

$y = A\sin x$

A is the …………

The plot represents $y =$ ……………..

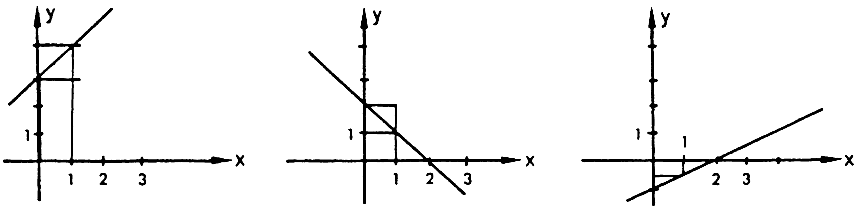


-------------------▷ 101

$y_1 = 0.5x$ $y_2 = 2.5x$ $y_3 = -2x$

18

It is more difficult to give the equation if the straight line does not cross the origin of the coordinate system.
In this case we determine first the constant b and afterwards the slope *a*.
Give the equations



$y_1 =$ .... $y_2 =$ .... $y_3 =$ ....

------------------▷ 19

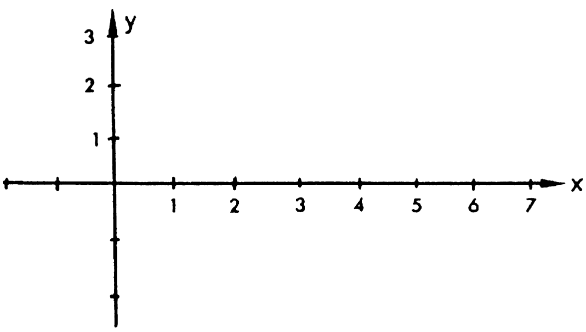
Amplitude
$y = 3\sin x$

101

Sketch by free hand the two functions

$y_1 = 2\sin x$
$y_2 = 0.5\sin x$

Accuracy does not matter. The plots must be fundamentally correct.

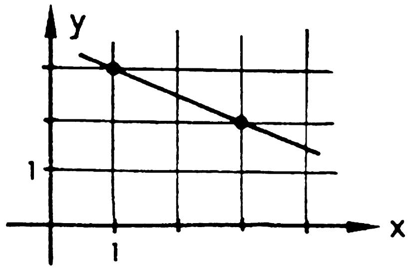


------------------▷ 102

$y_1 = x + 3$ $y_2 = x + 2$ $y_3 = \frac{1}{2}x - 1$ 19

In case of difficulties go back to the textbook and solve this task according to the text. Having done this solve the following exercises in which you can calculate the slope by dividing the increase or decrease of y for a given section of the abscissae. To do this you have to choose appropriate sections of the abscissae.

Determine the slope

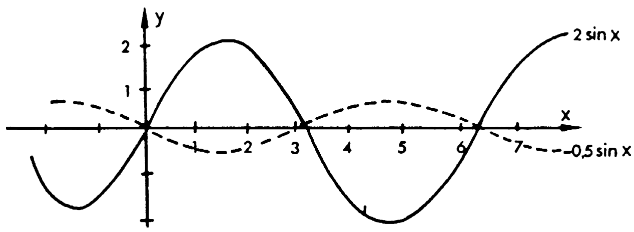


$y = ax + b$

$a = \ldots\ldots\ldots\ldots$

-------------------▷ 20

102



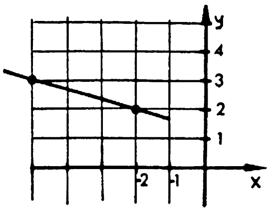
All correct -------------------▷ 105

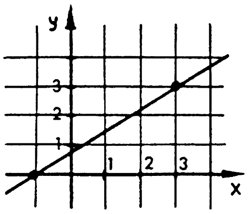
In case of errors -------------------▷ 103

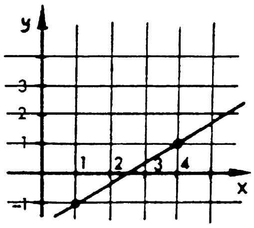
$a = -\dfrac{1}{2}$

20

Determine the slope a for the given straight lines.





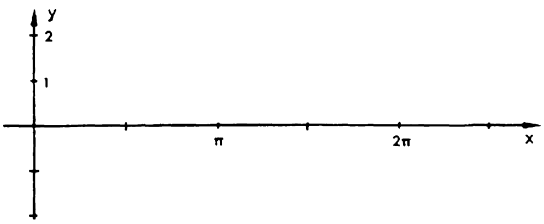


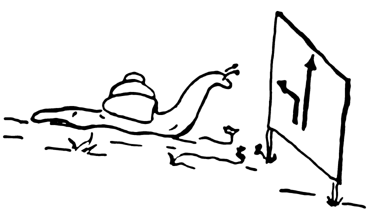
$a_1 = \ldots.$ $a_2 = \ldots.$ $a_3 = \ldots.$

-------------------▷ 21

103

Read again in the textbook the section “amplitude.” The amplitude can be negative.
Sketch by free hand
$y = -2\sin x$





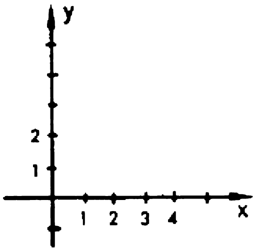
-------------------▷ 104

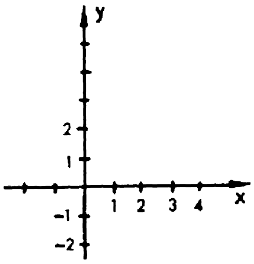
$a_1 = -\frac{1}{3}$ $\quad a_2 = \frac{3}{4}$ $\quad a_3 = \frac{2}{3}$

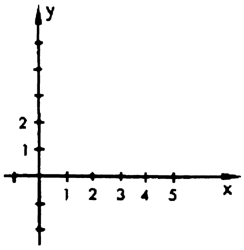
21

Give the graphical representation

$y_1 = 0{,}1x + 2$ $\quad y_2 = -2x - 2$ $\quad y_3 = \frac{x+1}{2}$







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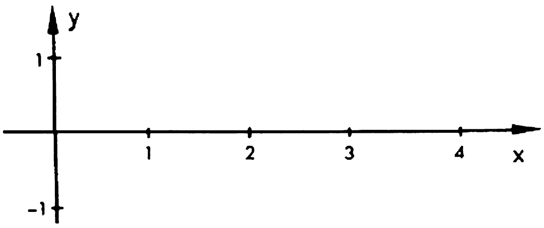
104

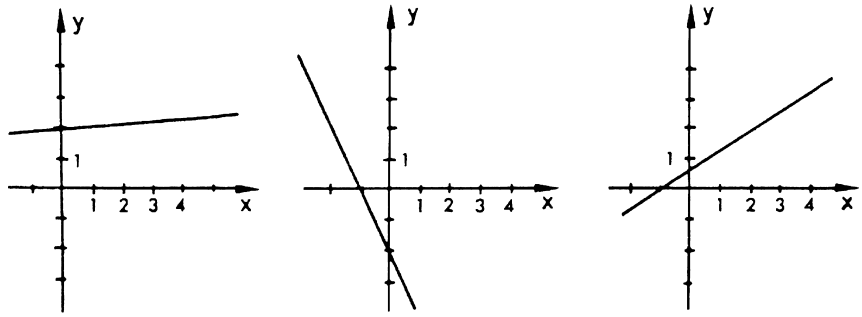


In case of remaining difficulties you may do the following exercise.
Sketch the function $y = \sin x$ on a separate sheet of paper.
Then construct the plot of
$y = (-2) \cdot \sin x$
All values must be multiplied by the factor –2.

Thus, you will obtain the solution shown above.

------------------▷ 105

22



In case of difficulties study again section 3.3.1 in the textbook.

-------------------▷ 23

105

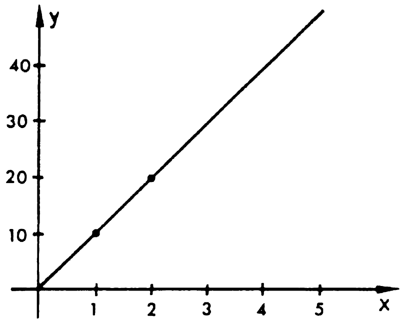
The sine function is a periodic function with a period of 2π .
If we add to the argument the value 2π we obtain the same value.
Thus, $\sin x = \sin(x + 2\pi)$
Given $y = \sin(bx)$. Which value must you add to the argument x to obtain the same value of the function again?

$\sin(b[\ x + x_{period}\ ]) = \sin(bx)$

$x_{period} = \ldots\ldots\ldots\ldots\ldots\ldots$

-------------------▷ 106

In practice the calibration of the coordinate axis has to be chosen according to the problem. Give the function that represents the graph. 23



$y = \ldots\ldots\ldots\ldots$

--------------------▷ 24

$\frac{2\pi}{b}$ 106

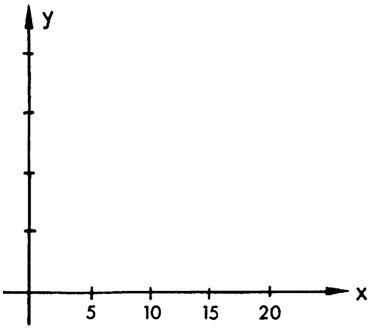
The function $y = \sin bx$ has the period $\frac{2\pi}{b}$

Correct answer --------------------▷ 109

You need help or further explanation --------------------▷ 107

$y = 10x$

24



Given $y = 50x + 1000$
Domain of x: $0 \le x \le 20$
Give an appropriate calibration for the ordinate and sketch the straight line

------------------▷ 25

$y = \sin x$ has the following zeros: $x = 0. \pm \pi. \pm 2\pi.......$

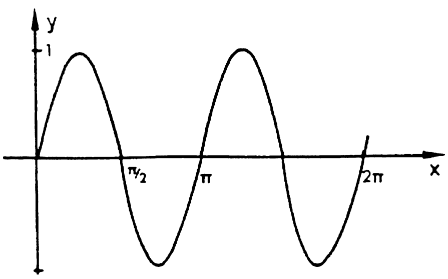
107

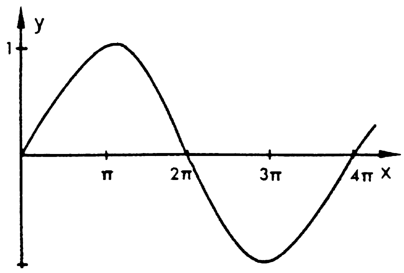
The period equals twice the distance between zeros.

The function $y = \sin bx$ has zeros at $bx = 0. \pm \pi. \pm 2\pi..........$

The distance between zeros is $\frac{\pi}{b}$

Thus, the period is $\frac{2\pi}{b}$



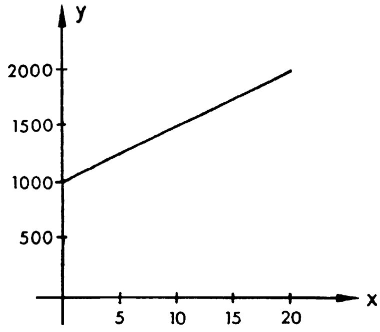


Analyze the graphs and give the function and the period:

$y_1 = ...$ $y_2 = ...$

$period : y_{period} = ...$ period: $y_{2\,period} = ...$

------------------▷ 108



25

Everything correct --------------------▷ 30

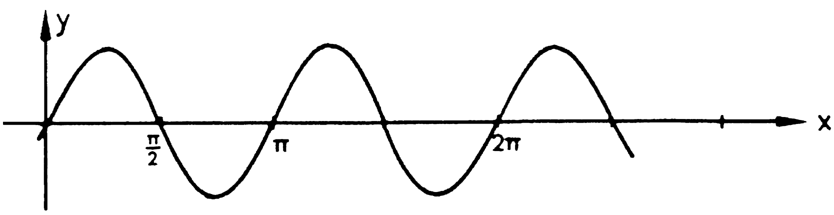
Explanation wanted --------------------▷ 26

108

$y_1 = \sin(2x)$ $y_2 = \sin\frac{x}{2}$

$period_1 : \pi$ $period_2 : 4\pi$

Here is the detailed solution for the first exercise.



The general formula of the sine functions is $y = (b \cdot x)$

The period of the plotted function is $x_{period} = \pi$

Thus $b \cdot x_{period} = 2\pi$

We insert x_{period} and obtain: $b \cdot \pi = 2\pi$

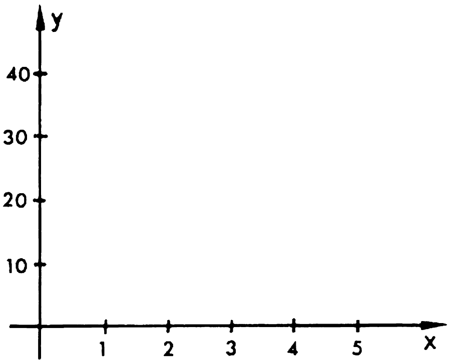
Thus: b = 2

Finally, the solution is: $y_1 = \sin 2x$

--------------------▷ 109

The calibration of the coordinate axis can be chosen deliberately. As a rule the axis is calibrated to aid observation of the relevant features of a curve.
Sketch the straight lines for 26

$y_1 = x$ $y_2 = 10x$ $y_3 = 20x$



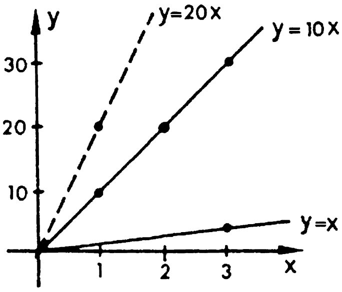
27

Calculate the periods of the following sine functions 109

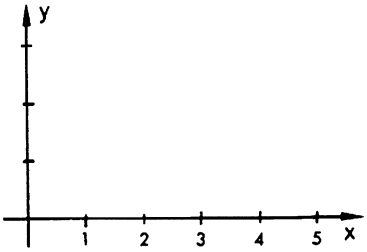
$y_1 = 5\sin(2x)$ $x_{period} = ...$
$y_2 = 0.5\sin(2x)$ $x_{period} = ...$
$y_3 = 0.5\sin(2\pi x)$ $x_{period} = ...$

110

27



Good observables are the functions $y_2 = 10x$ and $y_3 = 20x$



Calibrate the ordinate to obtain a good representation of the function $y = 0.01x$
Domain: $0 \le x \le 5$

------------------▷ 28

110

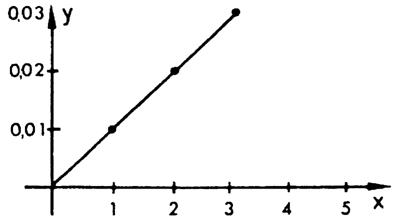
$x_{1\,period} = \pi$
$x_{2\,period} = \pi$
$x_{3\,period} = 1$

Try to reconstruct the period of the function $y = A \cdot \sin(bx)$

$x_{period} = ...$

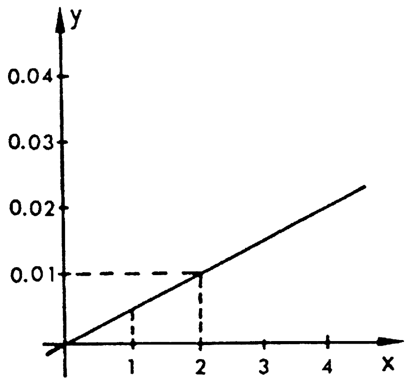
Hint: A period is completed if the argument of the sine function increases by 2π . In case of doubt go back to the textbook.

------------------▷ 111



The rule is simple. The domain must have space on the x-axis and the codomain must have enough space on the y-axis

28



If we have to determine the function for a given straight line we first determine the slope using the expression

$$a = \frac{y_s - y_1}{x_2 - x_1}$$

It makes sense to use the origin and an appropriate point of the line.

$a = \ldots\ldots.$ $y = \ldots\ldots..$

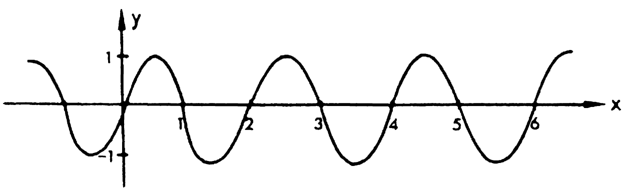
-------------------▷ 29

$$x_{period} = \frac{2\pi}{b}$$

111

This results from $b \cdot x_{period} = 2\pi$

Given the plotted function



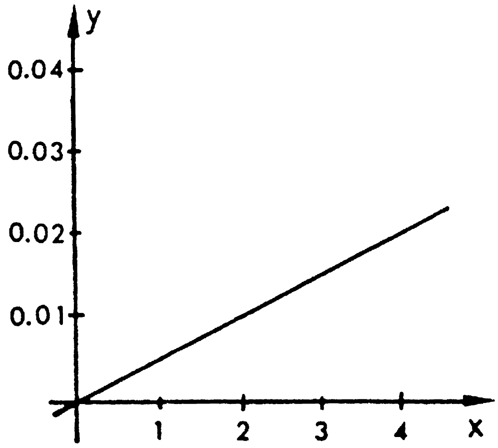
Period = ……

Function: $y = \ldots\ldots$

-------------------▷ 112

$a = \frac{0.01}{2} = 0.005$ $y = 0.005x$

29



Verify for yourself that the slope does not depend on the chosen interval. Repeat the calculation of a by choosing $x_1 = 0$ and $x_2 = 1$ and then $x_1 = 2$, $x_2 = 3$, and then $x_1 = 0$, $x_2 = 3$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$a_1 = \ldots\ldots.$

$a_2 = \ldots\ldots.$

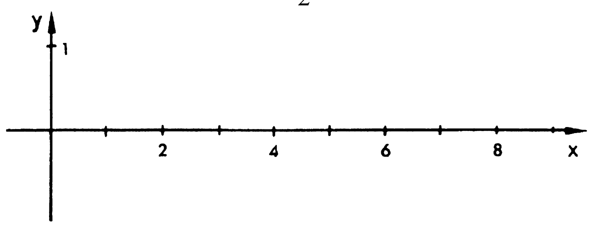
$a_3 = \ldots\ldots.$

-------------------▷ 30

Period = 2
Function: $y = \sin(\pi x)$

112

Sketch free-handed $y = \sin(\frac{1}{2}x)$

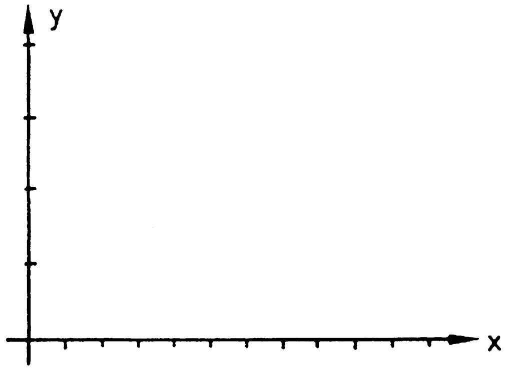


-------------------▷ 113

All slopes a are the same: $a = 0.005$

30

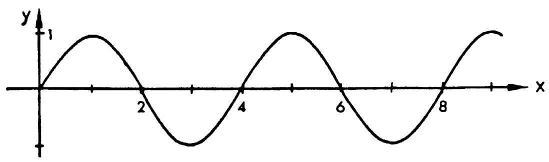
Calibrate the axis to represent $y = 0.02x$ for the domain $0 \le x \le 1000$



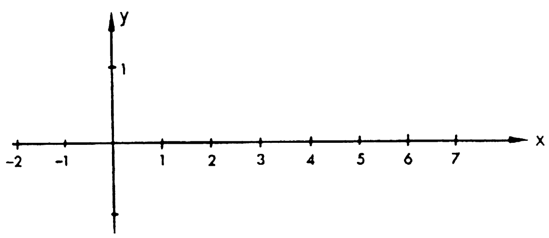
------------------▷ 31

Sketch of $y = \sin\left(\frac{1}{2}\pi x\right)$

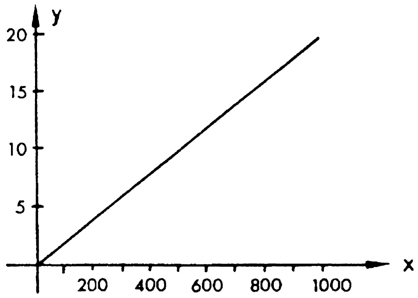
113



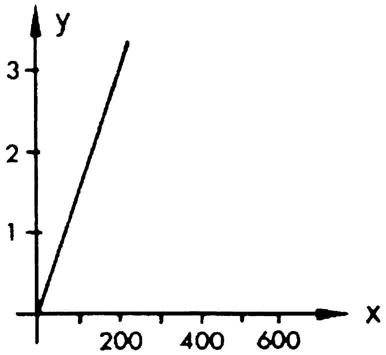
Plot the function $y = \sin(x + \pi)$



------------------▷ 114



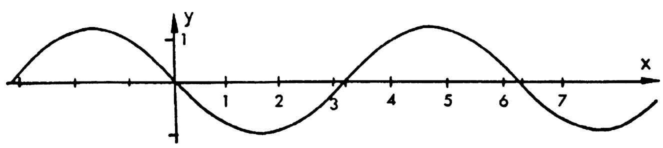
31



Give the equation of the given graph

$y = \dots.$

------------------▷ 32



114

Hint: With $y = \sin(x+\pi)$ the argument of the sine has its first zero if $x = -\pi$.
At this point starts the normal sine function.
The curve is shifted by the constant π to the left.

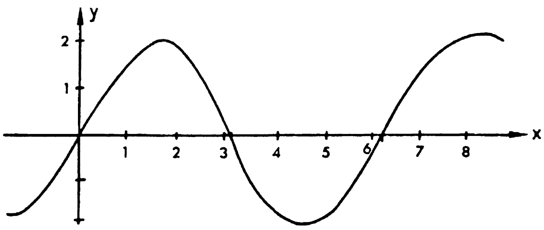
------------------▷ 115

$y = \frac{3}{200}x = 0.01x$ 32

All correct ----------> 35

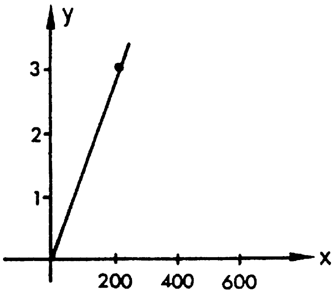
Further explanation wanted ----------> 33

Sketch the function $y = \sin(\pi + x)$ 115



----------> 116

33

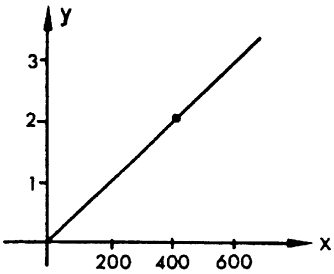


We choose in the graph two points

$x_1 = 0$ $\quad y_1 = 0$

$x_2 = 200$ $\quad y_2 = 3$

According to these points we calculate the slope of $y = ax$

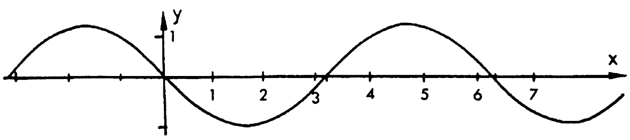


$$a = \frac{y_2 - y_1}{x_2 - x_1} \qquad a = \frac{3}{200}$$

Give the function of the graph to the left

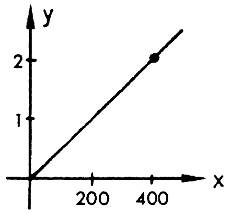
-------------------▷ 34

116



Hint: given: $y = \sin(\pi + x)$. The term in brackets has its zero for $x = -\pi$. At this point starts loosely speaking the curve if we start with sin(o). The sine-curve is shifted by the distance π to the left.

-------------------▷ 117



$$y = \frac{2}{400} = 0.005x$$

34

To determine the function of a given graph of a straight line you must select two points.
Then you must calculate the quotient of

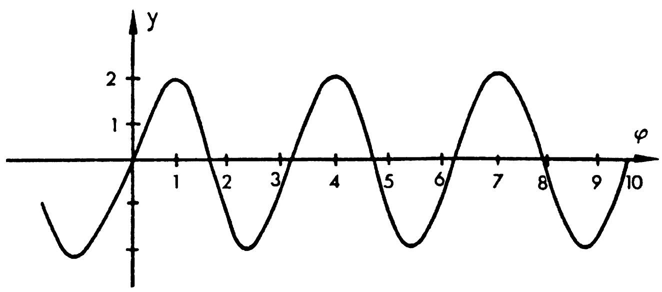
- the difference of the y-values
- the difference of the x-values

Of course you have to observe the calibration.

------------------▷ 35

You should measure your competence and your vocabulary by answering some test questions.
Give the function which is plotted beneath

117

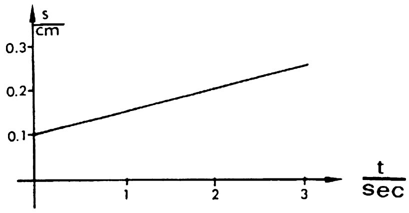


y = ...

------------------▷ 118

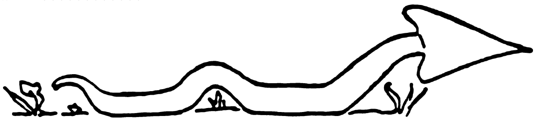
In practice the notations x and y are substituted by variables with different dimensions. Determine the function of the given graph

35



The graph represents the movement of a snail

$s = \ldots\ldots\ldots\ldots$



--------------------▷ 36

$y = 2\sin x$

118

Given $y = A\sin\varphi$. Write down the names

φ is the …….
A is the …….

--------------------▷ 119

$s = 0.05\frac{cm}{s} \cdot ts + 0.1cm$

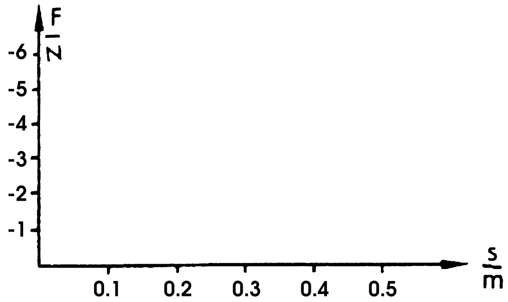
36

A spring is fixed at one end and stretched at the other end. This results in a force which opposes the displacement.
The paired values of force and displacement are tabled beneath.

displacement force

m	N
0	0
0.1	–1.2
0.2	–2.4
0.3	–3.6
0.4	–4.8
0.5	–6.0

Sketch the graph and give the function



$F =$ ……………………….

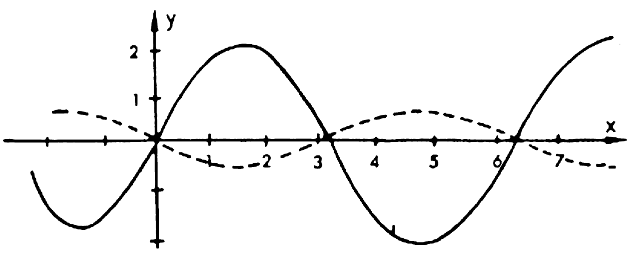
$a =$ ……………………….

-------------------▷ 37

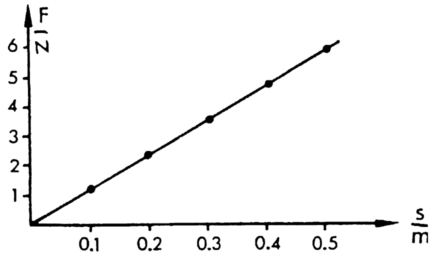
φ is the argument or independent variable
A is the amplitude

119

The functions of the curve beneath are $y_1 =$ .... $y_2 =$ ....



-------------------▷ 120



$$F = a \cdot s = -12\frac{sN}{m}$$
$$a = -12\frac{N}{m}$$

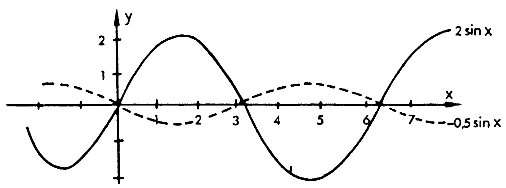
37

With practice you will gain experience in choosing appropriate calibrations.

-------------------▷ 38

$y_1 = 2\sin x$ $\qquad$ $y_2 = -\frac{1}{2}\sin x$

120



The function $y_1 = 2\sin\rho$ has the period $\rho_{period} =$ ....

The function $y_2 = A\sin bx$ has the period $x_{period} =$ ....

-------------------▷ 121

3.2.1 Graph plotting

38

Again you may find contents which you are familiar with from school. Whether you study this section quickly or carefully depends on your level of knowledge.

READ **3.2.3 Graph plotting**
Pages 44–46

Afterwards return to this study guide

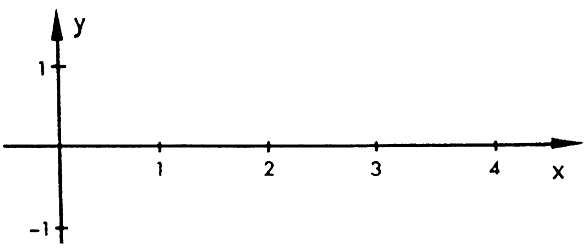
-------------------▷ 39

121

$$\varphi_{period} = \pi$$

$$y_2 = x_{period} = \frac{2\pi}{b}$$

Sketch free-handed the function $y = \sin(2\pi x)$. Your plot must not be perfect. It only matters that it is basically correct.



-------------------▷ 122

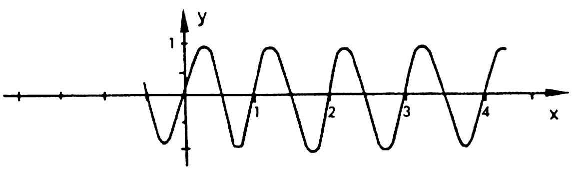
Find the zeros of the function

$y = x^2 - 4$

Zeros …………

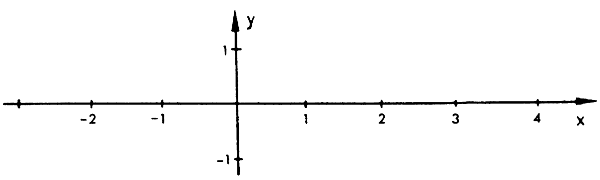
39

-------------------▷ 40



122

Sketch free-handed the function $y = \sin\left[\pi x + \frac{\pi}{2}\right]$



-------------------▷ 123

$x_1 = +2$
$x_2 = -2$

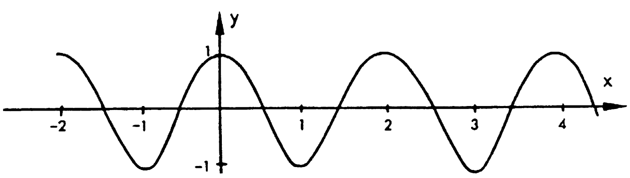
40

Find the pole for $y = \frac{1}{x+1} - 1$

Pole: ……..

------------------▷ 41

123



It is important to be familiar with the effect of b and c in the function
$y = A\sin(bx + c)$
b determines the period. Positive c shifts the graph to the left.
In case of remaining difficulties we suggest you repeat the section “sine function” in the textbook and solve the exercises in this study guide once more.

------------------▷ 124

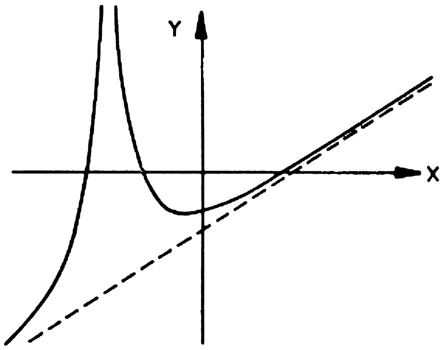
$x = -1$

41

How many zeros has the sketched function?

………….

The dashed line is named ………………



-------------------▷ 42

3.6.2 Cosine Function

124

Relationships between the sine and the cosine function.
Divide your study and have breaks.

READ **3.6.3 Cosine Function**
3.6.4 Relationships between the sine and cosine function
Textbook pages 58–60

Having done

-------------------▷ 125

3 zeros
asymptote

42

Given the function

$$y = \frac{1}{x^2 - 4}$$

This function has

………. zero(s)

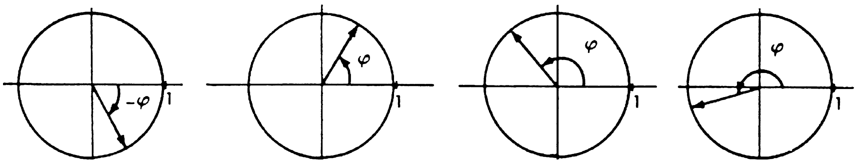
………. pole(s)

………. asymptote(s)

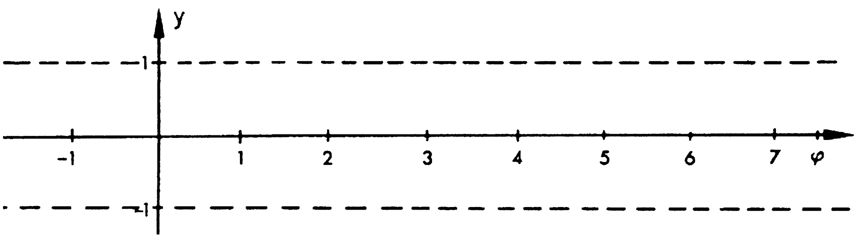
-------------------▷ 43

Sketch the cosine of the angle φ into the plots:

125



Sketch the function $y = \cos\varphi$



-------------------▷ 126

43

No zeros
2 poles
1 asymptote

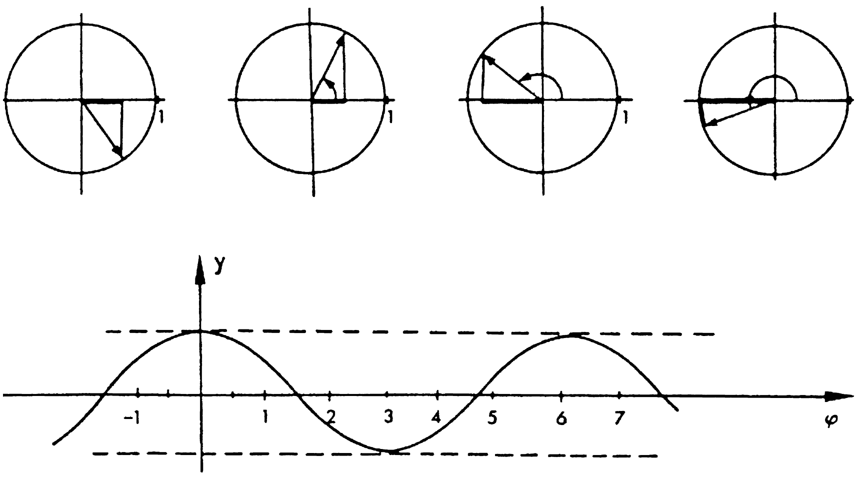
Write down in your own words:

To calculate poles ………………………………….

………………………………….

------------------▷ 44

126



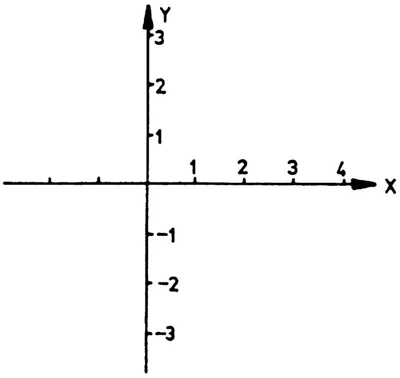
Go ahead to

------------------▷ 127

Poles are found by calculating the zeros in the denominator of a fraction. The numerator of which is not zero.

44

Sketch the function $y = \frac{2}{1}$



The function has

…. zero(s)

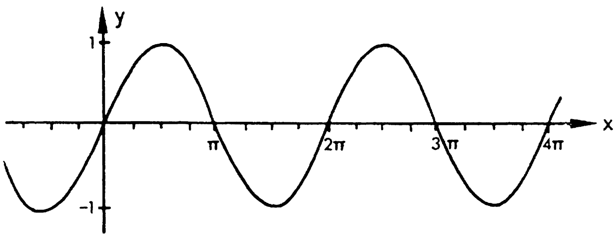
…. pole(s)

…. asymptote(s)

------------------▷ 45

The sketch beneath shows $y = \sin x$

127

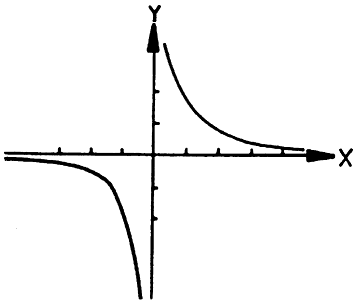


Plot in the given sketch the function $y = \sin(x + \frac{\pi}{4})$

------------------▷ 128

No zeros a pole no asymptotes

45



$y = \frac{2}{x}$ is a hyperbola

It has two separate parts

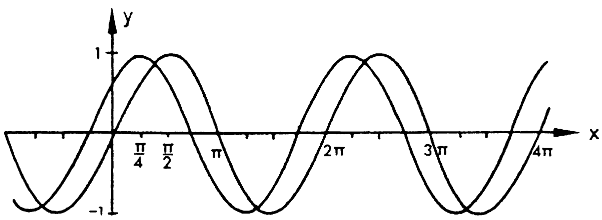
Is $y = \frac{a}{x} + b$ a hyperbola as well?

☐ yes

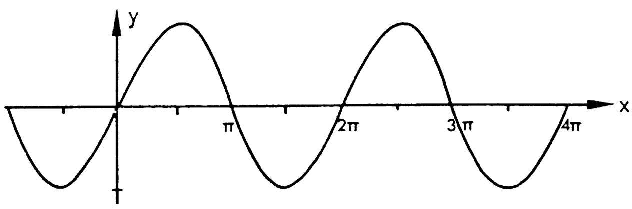
☐ no

------------------▷ 46

128



The sketch below shows $y = \sin x$.



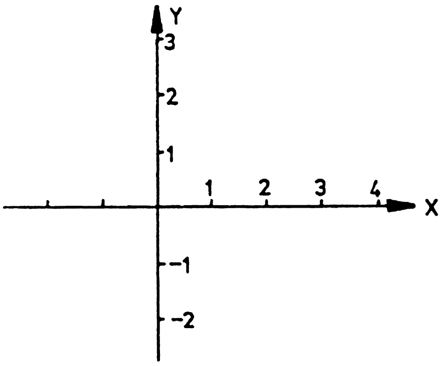
Plot in the given sketch the function $y = \sin(x + \frac{\pi}{2})$

You know the function you plotted this time. It is the ………. function

------------------▷ 129

Yes, it is shifted in the y-direction

46

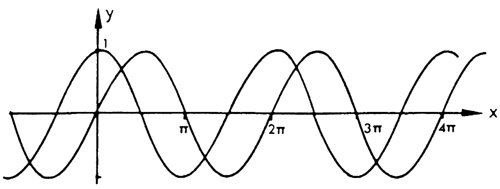


Sketch a representation of $y = \frac{1}{x} + 2$

-------------------▷ 47

cosine function

129



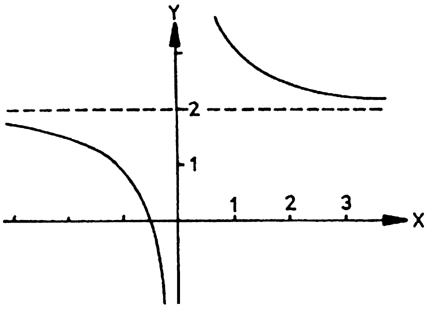
We consider the transition from the sine function to the cosine function.
To obtain the function $y = \sin x$ from $y = \cos x$ you shift the cosine function to the right.
Written down this looks like this

$\cos(x \ldots) = \sin x$

To obtain $y = \cos x$ from $y = \sin x$ you shift the sine function to the left.

$\sin(x \ldots) = \cos x$

-------------------▷ 130



Hopefully you sketched both parts of the hyperbola, which has

47

…………zero(s)

…………asymptote(s)

…………..pole(s)

------------------▷ 48

$\cos(x-\frac{\pi}{2})=\sin x$

130

$\left(x+\frac{\pi}{2}=\cos x\right)$

The sine function and the cosine function are to some extent similar functions.
If they are shifted by the phase $\frac{\pi}{2}$ they happen to coincide. Thus, both functions may be used to represent mechanical or electrical oscillations.

------------------▷ 131

48

1 zero
1 asymptote
1 pole

You may sketch more functions and determine zeros, poles, and asymptotes. This is up to you.
If you still feel unsure we suggest you continue with the exercises.

It is quite an annoying fact that if we have enough competence we like to do the exercises. We even enjoy it. But in this case we do not need more training. In the following you will find more exercises.

------------------▷ 49

131

There is no error or contradiction if in one textbook you read:
The oscillation of a pendulum is represented by $s = s_0 \cos(\omega t)$
And in another textbook you read the oscillation of a pendulum is represented by
$A = A_0 \sin(\omega t)$
The representations only differ in two points:

- the amplitude is denoted differently. This is of no importance.
- The position of the pendulum for t = o. For t = o the function s has its maximum while A has its zero.

It is obvious that this does not change the basic characteristics of the oscillation.

------------------▷ 132

Given some functions: $y = x^2 + x + 1$ 49

$$y = \frac{1}{x^2 + x + 1}$$

$$y = \frac{1}{x^2}$$

Sketch the graphs!

Solutions and further exercises ------------------▷ 50

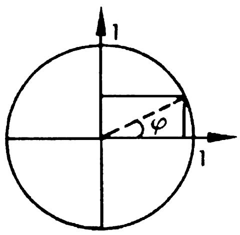
If you solved the exercises and characterized them as quite easy ------------------▷ 56

The relationship between sine function and cosine function results from an analysis using the unit circle and the theorem of Pythagoras 132

$\sin^2\varphi + \cos^2\varphi = \ldots\ldots\ldots$

$\sin^2\varphi = \ldots\ldots\ldots$

$\cos^2\varphi = \ldots\ldots\ldots$

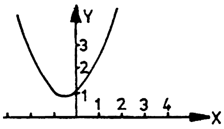


------------------▷ 133

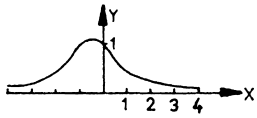
Given three functions

50

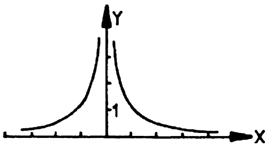
$y_1 = x^2 + x + 1$



$y_2 = \frac{1}{x^2 + x + 1}$



$y_3 = \frac{1}{x^2}$



Sketch the graphs

$y_1 = \frac{1}{x} + x \qquad y_2 = -\frac{1}{x} \qquad y_3 = \frac{3}{x} - 2$

Solutions and further exercises --------------------▷ **51**

If you solved the exercises and characterized them as quite easy --------------------▷ **56**

133

$\sin^2\varphi + \cos^2\varphi = 1$
$\sin^2\varphi = \cos^2\varphi$
$\cos^2\varphi = \sin^2\varphi$

It is easy to memorize: $\sin^2\varphi + \cos^2\varphi = 1$
Often you will find the following notation

$\sin\varphi = \sqrt{1 - \cos^2\varphi}$
$\cos\varphi = \sqrt{1 - \sin^2\varphi}$

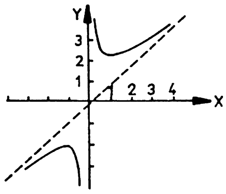
In this case the sign of the root must not be forgotten
$\cos\ \varphi$ is positive in the ….. and ….. quadrant.
Thus, in this case you must write $\cos^2\varphi = ...\sqrt{1 - \sin^2\varphi}$

$\cos^2\varphi$ is negative in the …. and …. quadrant.
Thus, in this case you must write $\sin\varphi = ...\sqrt{1 - \cos^2\varphi}$

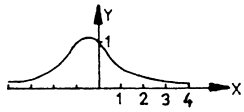
--------------------▷ **134**

51

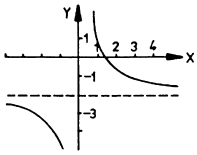
$y_1 = \frac{1}{x} + x$



$y_2 = -\frac{1}{x}$



$y_3 = \frac{3}{x} - 2$



More exercises --------------------▷ 52

If these exercises are simple go to --------------------▷ 56

134

$\cos\varphi$ is positive in the first and fourth quadrant. In this case you must write.
$\cos\varphi = +\sqrt{1-\sin^2\varphi}$

$\cos\varphi$ is negative in the second and third quadrant. In this case you must write
$\cos\varphi = -\sqrt{1-\sin^2\varphi}$

--------------------▷ 135

Do you remember the correct names? 52

The x-axis is named ………

The y-axis is named ……….

Calculate the zeros

a) $y = x - 2$
zero(s): ………….

b) $y = x^2 - 4$
zero(s):

------------------▷ 53

………....

3.6.3 Tangent and Cotangent 135

3.6.4 Addition formulae

With this section two new functions are presented. They are combinations of the trigonometric functions which you are now familiar with.

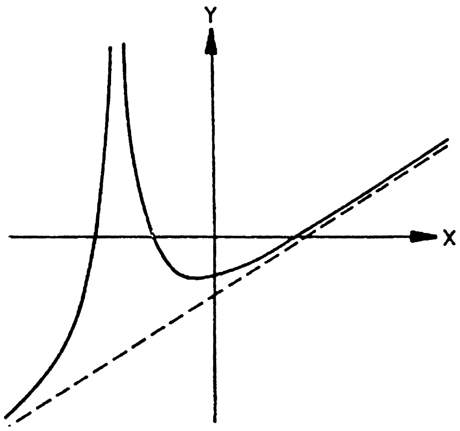
READ **3.6.5 Tangent and Cotangent**
3.6.6 Addition formulae
Textbook pages 61–64

Go to

------------------▷ 136

abscissae ordinate 53

Zeros: a) $x = 2$ b) $x_1 = +2$
$x_2 = -2$



The plotted function has
………zeros and ………… poles
The dotted line is named
…………..

------------------▷ 54

The quotient $\frac{\sin\varphi}{\cos\varphi}$ is named ……… 136

Give the definition:

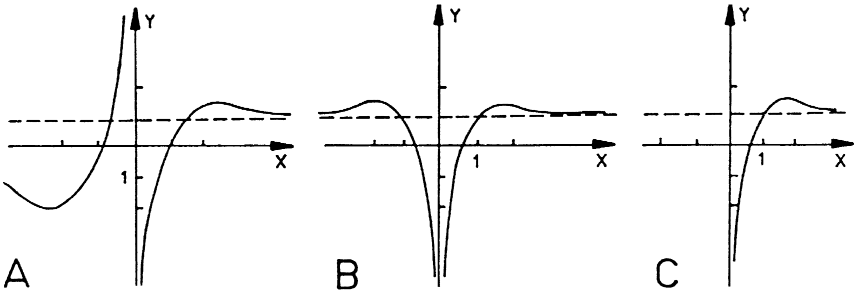
$\cot\varphi =$ ..... ……

------------------▷ 137

3 zeros 1 pole asymptote

54

Which is the plot of $y = \frac{x^2 - 1}{x^4} + 1$



The function has …. zeros …. poles ….. asymptote

-------------------▷ 55

$\tan\varphi$

$\cot\varphi = \frac{\cos\varphi}{\sin\varphi}$

137

You can obtain the important features of $\tan\varphi = \frac{\sin\varphi}{\cos\varphi}$ by analyzing nominator and denominator of the fraction.

Regarding the domain of $\varphi \quad 0 \le \varphi \le 2\pi$

The tangent function has

zeros for $\varphi =$ ..... …….

poles for $\varphi =$ ..... ……..

If $\tan\varphi = 1 \quad \varphi =$ ....

Regarding the domain $0 \le \varphi \le \infty$

The tangent function has

zeros for $\varphi =$ ..... …. …..

poles for $\varphi =$ ..... ….. ….

If $\tan\varphi = 1 \qquad \varphi =$ .....

-------------------▷ 138

Plot B 55

2 zeros 1 pole 1 asymptote

If you solved all problems correctly congratulations.

-------------------▷ 56

Domain of φ: 138

$0 \le \varphi \le 2\pi$
zeros $\varphi = o, \pi, 2\pi$
poles $\varphi = \frac{\pi}{2}, \frac{3\pi}{2}$
$\tan\varphi = 1$ for $\varphi = \frac{\pi}{4}, \frac{5}{4}\pi$

Domain of $\varphi: 0 \le \varphi \le \infty$
zeros $\varphi = 0, \pi, 2\pi, 3\pi, 4\pi.....$
poles $\varphi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, ...$
tan1 for $\varphi = \frac{\pi}{4}, \frac{5}{4}\pi, \frac{9}{4}\pi, \frac{13\pi}{4}....$

In case of difficulties consult the textbook and analyze the drawings. Let φ start with $\varphi = 0$ and let it increase up to 2π

-------------------▷ 139

3.3 Quadratic Equations

56

Quadratic equations should and may be known from school. But since this topic is often used a rehearsal may be worthwhile.

Read **3.3 Quadratic Equations**
Textbook page 47–48

-------------------▷ 57

Express the sine by the cosine and vice versa. There are some solutions. Find at least two.

139

$\sin\varphi = \ldots\ldots\ldots$

$\sin\varphi = \ldots\ldots\ldots$

$\cos\varphi = \ldots\ldots\ldots$

$\cos\varphi = \ldots\ldots\ldots$

-------------------▷ 140

57

Given
$ax^2+bx+c=o$
Calculate the roots.

$x_1 =$ ………..

$x_2 =$ ………..

In case of difficulties consult the textbook.

-------------------▷ 58

140

$$\sin\varphi = \cos(\varphi-\frac{\pi}{2}) = -\cos(\varphi+\frac{\pi}{2})$$
$$\sin\varphi = \sqrt{1-\cos^2\varphi}$$
$$\cos\varphi = \sin(\varphi+\frac{\pi}{2}) = -\sin(\varphi-\frac{\pi}{2})$$
$$\cos\varphi = \sqrt{1-\sin\ \ \varphi}$$

Simplify using the table in the textbook

a) $\dfrac{\sin(\omega_1+\omega_2)+\sin(\omega_1-\omega_2)}{\cos(\omega_1+\omega_2)+\cos(\omega_1-\omega_2)}$

b) $\cos(45^0+\delta)+\cos(45^0-\delta)$

c) $\dfrac{\cos^2\varphi}{\sin 2\varphi}$

-------------------▷ 141

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$ 58

$$x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Give the roots for the equation

$x^2 + px + q = o$

$x_1 =$ ....

$x_2 =$ ....

-------------------▷ 59

---

a) $\dfrac{2\sin\omega_1 \cdot \cos\omega_2}{2\cos\omega_1 \cos\omega_2} = \tan\omega_1$ 141

b) $2\cos 45^0 \cos\delta = \sqrt{2}\cdot\cos\delta$

c) $\dfrac{\cos^2\varphi}{\sin 2\varphi \cos\varphi} = \dfrac{1}{2}\cot\varphi$

Hint: Having solved the exercises you may solve them as well with a computer program if you are familiar with it.

-------------------▷ 142

$$x_1 = \frac{-p + \sqrt{p^2 - 4q}}{2}$$ 59

$$x_2 = \frac{-p - \sqrt{p^2 - 4q}}{2}$$

Given $2x_2 + 2x - 4 = o$

$x_1 = \ldots\ldots\ldots\ldots..$

$x_2 = \ldots\ldots\ldots\ldots\ldots.$

Solution -------------------▷ 63

Help and hints wanted -------------------▷ 60

Substitution of variables often helps to understand equations. 142

In physics we use different variables

t = time	v = velocity
ρ = density	g = gravitational acceleration
h = height	p = pressure
E = electrical field	a = acceleration

The equation for pressure as a function of depth reads $p = \rho \cdot g \cdot h$

If we substitute pressure p by y, depth h by x and the product of constants $\rho \cdot g$ by a we obtain a well-known equation

$y = \ldots.$

-------------------▷ 143

Given $2x^2 + 2x - 4 = 0$

60

You may use the formula

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2q}$$

Write down

$a = \ldots\ldots$ $b = \ldots.$ $c = \ldots..$

Inserting gives

$x_1 = \ldots\ldots\ldots$

$x_2 = \ldots\ldots\ldots$

-------------------▷ 61

$y = a \cdot x$

143

This equation is easily understood because we are familiar with it.
To understand relationships in physics or engineering we must understand the mathematical relationship.
Our understanding is often more easily reached if we substitute unfamiliar notations with familiar ones like x, y, z, or substitute complex terms with simpler ones.
This procedure is performed in three steps.
Step 1: Substitution of unfamiliar symbols with familiar ones.
Step 2: Work, calculate and discuss the relationship
Step 3: Resubstitute the variables

-------------------▷ 144

$a = 2$ $b = 2$ $c = -4$ 61

$x_1 = 1$
$x_2 = -2$

You may try the other formula as well.
Given $2x^2 + 2x - 4 = 0$
Dividing by 2 gives

……………. = 0

Thus p = …………. q = …………….

-------------------▷ 62

3.7 Inverse trigonometric functions

144

Here we use the concept of inverse functions to obtain a new type of function

READ **3.7 Inverse trigonometric functions**
Textbook pages 64–66

-------------------▷ 145

$x^2 + x - 2 = 0$

62

$p = 1$ $q = -2$

Inserting in the given formula you obtain

$x_1 =$ …………..

$x_2 =$ ……………

------------------▷ 63

The expression $arc \sin 1 = y$ means
y is the …. whose …. has the value …

145

Try to solve for y: …….
y = arc cos1
y = …..

------------------▷ 146

$x_1 = 1$ $x_2 = -2$ 63

Given

$3x^2 - 9x - 30 = 0$

Try to use both formulae to obtain x_1 and x_2

Hints welcome --------------------▷ 64

Solution --------------------▷ 65

---

y is the angle whose value is 1 146
$y = o$ or $y = 0^0$

No difficulties go to --------------------▷ 148

For more explanation --------------------▷ 147

64

Given $3x \;\; -9x-30=0$

To apply the first formula you write down

$a = \ldots.$ $b = \ldots.$ $c = \ldots.$

Then insert into the given formula to obtain

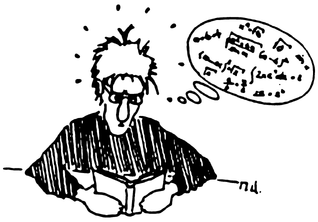
$x_1 = \ldots\ldots.$

$x_2 = \ldots\ldots.$

To try the other formula as well divide the equation above by 3 to get q and p and to obtain.

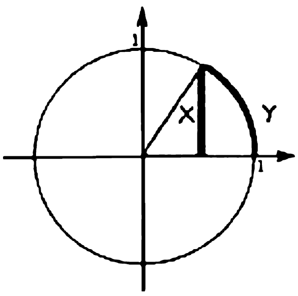
$x_1 = \ldots\ldots.$

$x_2 = \ldots\ldots.$



-------------------▷ 65

147



In the unit circle is marked the angle y in radians and its sine.
In this case we may say y is the angle in radians whose sine has the value x.
This is the meaning of the expression
$y = \text{arc}\sin x$
Difficulties may arise if you do not discern between $\sin(x)$ and $\sin x$ in the equation above. $\sin(x)$ is the sine of the angle x.
In the equation above $\sin x$ means x is the value of the sine whose angle is required.

-------------------▷ 148

$x_1 = 5$
$x_2 = -2$ 65

------------------▷ 66

Let us reiterate. 148

In the equation
y = arc sin x the term "sin x" means.
The sine has the value x

In the equation
y = arc cos x the term "cos x" means
The …. has the value ….

------------------▷ 149

3.4 Parametric changes of functions and its graphs

66

READ **3.4 Parametric changes of functions and its graphs**
Textbook page 49–50

Having studied

------------------▷ 67

149

The cosine has the value *x*

This notation is new, unfamiliar, and more difficult than the underlying mathematics.
Try to formulate the question in your mind before solving the exercises

$\varphi = arc\cos 0.5 = ...$
$y = arc \quad \sin 1 = ...$
$\alpha = arc\sin 0.5 = ...$

φ	α	$\cos\alpha$ $\cos\varphi$	$\sin\alpha$ $\sin\varphi$
0 = 0,00	0°	1	0
$\frac{\pi}{6}$ = 0,52	30°	0,87	0,5
$\frac{\pi}{4}$ = 0,78	45°	0,71	0,71
$\frac{\pi}{3}$ = 1,05	60°	0,50	0,87
$\frac{\pi}{2}$ = 1,56	90°	0	1

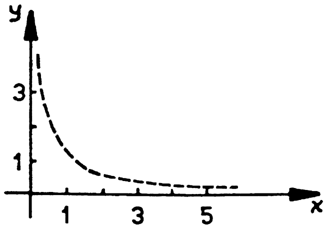
------------------▷ 150

67

In the following we regard the function

$y_1 = f(x) = \frac{1}{x}$

We will limit our considerations to one part of the hyperbola



Multiplication of the function with a constant, e.g. 3.
Sketch the graph
$y_2 = 3 \cdot f(x)$

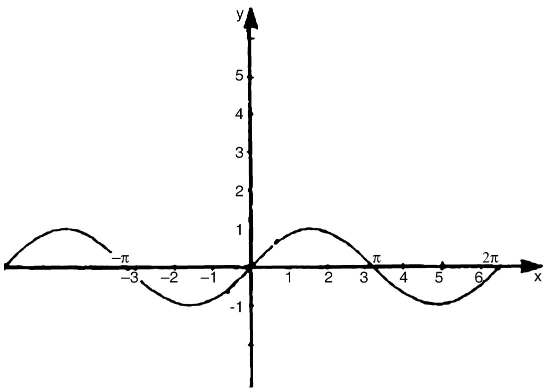
-------------------▷ 68

150

$\varphi = \frac{2}{3}\pi rad$ or $\varphi = 60^0$

$y = \frac{\pi}{2} rad$ or $y = 90^0$

$\alpha = \frac{\pi}{6} rad$ or $y = 30^0$

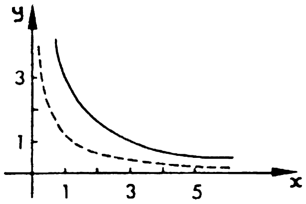


Plotted are two periods of the sine function sketch the inverse.

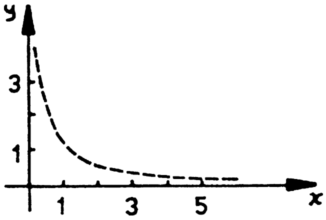
-------------------▷ 151

$y_2 = 3f(x) = \frac{3}{x}$

68



Addition of a constant to a function:

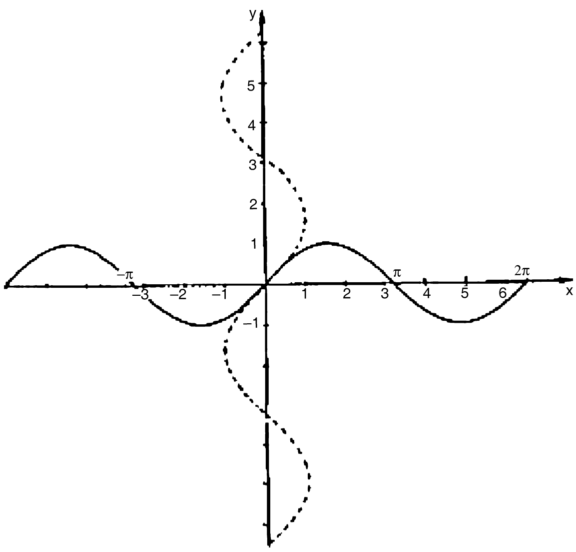


Given $y_1 = f(x) = \frac{1}{x}$

Sketch

$y_2 = f(x) + 3 = ...$

------------------▷ 69



151

The sketched inverse is a

☐ function

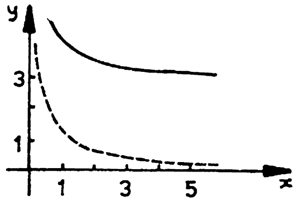
☐ relation

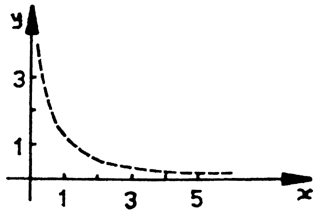
Mark the values of the inverse for $x = 0.5$

------------------▷ 152

69

$y_2 = f(x) + 3 = \frac{1}{x} + 3$





Multiplication of the argument by a constant.

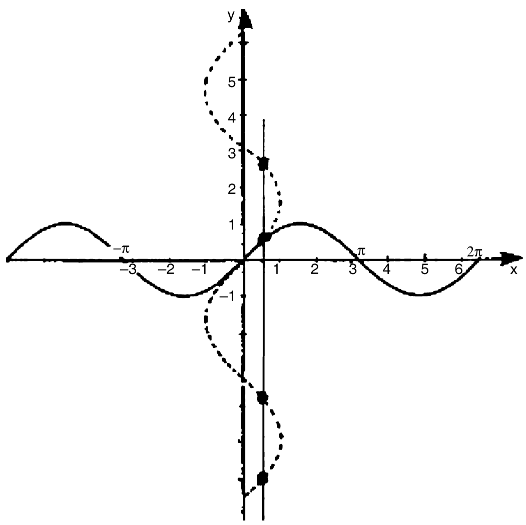
Given $y_1 = f(x) = \frac{1}{x}$

Multiplying the argument by 3 gives $y_2 = \ldots \ldots\ldots$

Sketch the modified function.

------------------▷ 70

152

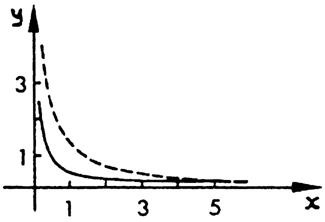


Relation
Hint: For $x = 0.5$ we have four values sketched but more exist.
Now mark the main values for which the inverse may be regarded as a function.

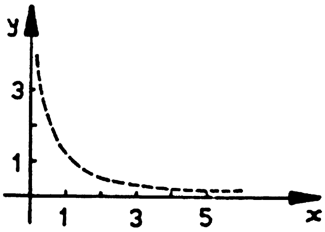
------------------▷ 153

$y_2 = \frac{1}{3x}$

70



Addition of a constant to the argument



Given $y_1 = f(x) = \frac{1}{x}$

Adding the constant 3 to the argument results in

$y_2 = f(x+3) = ...$

Hint: We have to replace the argument by $(x + 3)$

Sketch the function y_2

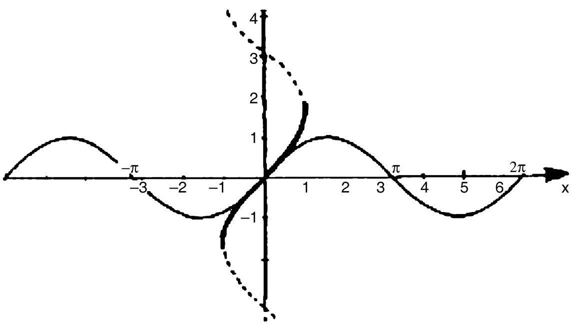
Modification of the graph:

☐ shift to the right

☐ shift to the left

------------------▷ 71

153



a) $y_1 = arc\cos 0{,}71 \quad y_1 = ...$

b) $y_2 arc\sin 0.87 \quad y = ...$

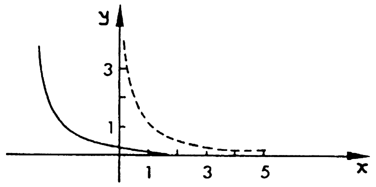
φ	α	cos α cos φ	sin α sin φ
0 = 0,00	0°	1	0
$\frac{\pi}{6}$ = 0,52	30°	0,87	0,5
$\frac{\pi}{4}$ = 0,78	45°	0,71	0,71
$\frac{\pi}{3}$ = 1,05	60°	0,50	0,87
$\frac{\pi}{2}$ = 1,56	90°	0	1

------------------▷ 154

71

$y_2 = \frac{1}{x+3}$

Shift to the left



In the following exercises we will modify the function by $c = -3$
Decide for yourself

I do not need the exercise --------------------▷ 73

I want the exercise --------------------▷ 72

154

$y_1 = \frac{\pi}{4} rad$ or $y_1 = 45^0$

$y_2 = -\frac{\pi}{3} rad$ or $y_2 = -60^0$

If we denote angles by φ or α, you may solve the same questions

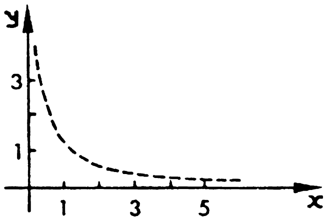
$y_1 = arc\sin 0.5 \qquad \varphi_1 = ...$

$\varphi_2 = arc\sin -0.5 \qquad \varphi_2 = ...$

$\alpha = arc\cos -0.71 \qquad \alpha = ...$

--------------------▷ 155

Multiplication of the argument by a constant $c = -3$ 72



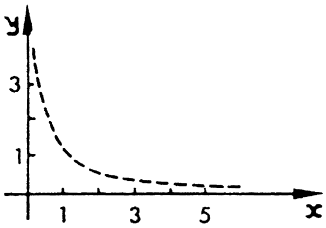
Given $y_1 = f(x) = \frac{1}{x}$

Multiply the argument by $c = -3$

$y_2 = f(x \cdot (-3)) = \ldots\ldots\ldots$

Sketch the function

Addition of a constant to the function: c – 3



Given $y_1 = \frac{1}{x}$

$y_2 = f(x) - 3 = \ldots\ldots$

Sketch y_2

-------------------▷ 73

155

$\varphi_1 = \frac{\pi}{6} rad$ or $\varphi_1 = 30^0$

$\varphi = -\frac{\pi}{6} rad$ or $\varphi = -30^0$

$\alpha = \frac{3\pi}{4}$ or $\alpha = 135^0$

At last a question regarding the arc tan relation

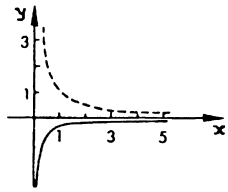
$y = arc \tan 1$ $\qquad y = \ldots.$

$\varphi = arc \tan o$ $\qquad \varphi = \ldots.$

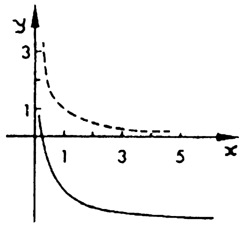
$\alpha = arc \tan -1$ $\qquad \alpha = \ldots..$

-------------------▷ 156

$y_2 = \frac{1}{-3x}$



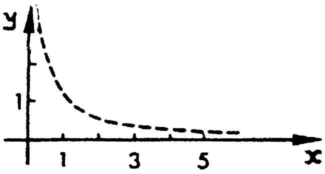
$y_2 = \frac{1}{x} - 3$



73

Addition of a constant c to the argument $c = -3$

Given $y_1 = f(x) = \frac{1}{x}$



$y_2 = f(x-3) = \ldots\ldots\ldots$

------------------▷ 74

156

$\varphi_1 = \frac{\pi}{4} rad$ or $y = 45^0$

$\varphi = o\,rad$ or $\varphi = 0^0$

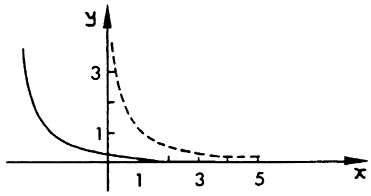
$\alpha = \frac{\pi}{4} rad$ or $\alpha = -45^0$

Hint: $\tan\frac{\pi}{4} = \tan 45^0 = 1$

We use arc relations if we know certain values of sin, cos, or tan and want to know the related angles.

------------------▷ 157

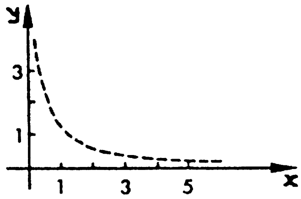
$y_2 = \dfrac{1}{x-3}$ 74



Multiplication of the function by a constant.

$e \cdot g: \quad c = -3$

$y_1 = \dfrac{1}{x}$ $\qquad y_2 = c \cdot y_1 = \ldots\ldots\ldots$



Sketch the new function y_2

------------------▷ 75

3.8 Functions of a Function (Composition) 157

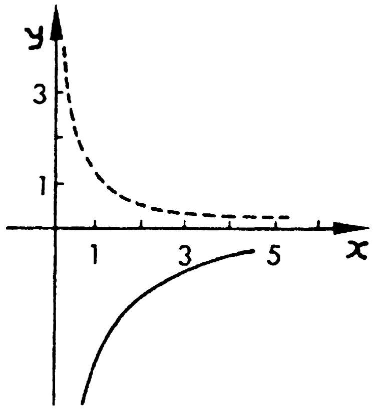
Often the concept of composition explained in the textbook helps to simplify equations

READ **3.8. Function of a Function (Composition)**
Textbook pages 66–67

Having completed your study go to

------------------▷ 158

$y_2 = -\frac{3}{x}$ 75



------------------▷ 76

A function of a function or in other words a composition is a special form of substitution. 158
Given two equations:
$y = f(u)$
$u = g(x)$

In this case u substitutes $g(x)$

Then $y = f(u) = f[g(x)]$

$f(u)$ is called ………

$g(x)$ is called ……..

------------------▷ 159

3.5 Inverse Functions

76

The concept of the inverse function is often used and it helps in obtaining new functions

READ **3.5. Inverse functions**
Textbook page 50–51

-------------------▷ 77

$f(u)$ is called outer function
$g(x)$ is called inner function

159

Let us solve a function of a function

Given
$y = u^2 - 1$
$u = x^2 + 1$

$y = \ldots\ldots$

-------------------▷ 160

To obtain an inverse function we proceed in two steps: 77

1. …………………...
2. …………………..

------------------▷ 78

$y = x^4 + 2x^2$ 160

Given:

$y = \sin(u + \pi)$

$u = \frac{\pi}{2} x$

$x = 1$

Calculate $y = f[u(x)]$ for x = 1

$y(x = 1) =$…..

------------------▷ 161

1. Interchange x and y to obtain a new function
2. Solve the new function for y

78

Obtain the inverse function y^{-1} for

$$y = 1 - \frac{1}{x}$$

Choose the correct solution

□ $x^{-1}(y) = \frac{1}{1-y}$ --------------------▷ 79

□ $y^{-1}(x) = \frac{1}{1-x}$ --------------------▷ 81

$y = -1$

161

Solution correct --------------------▷ 165

Further explanations wanted --------------------▷ 162

Sorry, this is the wrong choice. 79

We suggest you study again section 3.4 “inverse functions” in the textbook. Consulting the textbook obtain the inverse functions of

$y_1 = \frac{1}{x+1}$ $\qquad y^{-1}(x) = \ldots\ldots\ldots\ldots\ldots$

$y_2 = e^{2x}$ $\qquad y^{-1}(x) f = \ldots\ldots\ldots\ldots\ldots$

------------------▷ 80

Given: 162

$y = \sin(u+\pi)$

$u = \frac{\pi}{2} x$

$x = 1$

For $x = 1$ we obtain $u = \frac{\pi}{2}$

Inserting u into the given function results in

$y = \sin(\frac{\pi}{2} + \pi) = \sin(\frac{3}{2}\pi) = \ldots\ldots$

------------------▷ 163

$y^{-1}1(x) = \frac{1}{x} - 1$ 80

$y_2^{-1}(x) = \frac{1}{2}\ln x$

Try again to get the inverse function of

$y = 1 - \frac{1}{x}$

$y^{-1} =$ ....

------------------▷ 81

$y = -1$ 163

Now try to solve $y = u + \sqrt{u}$

$u = \frac{x^4}{6}$

$y =$ ……

------------------▷ 164

$y^{-1}(x) = \frac{1}{1-x}$ is the correct solution

81

Obtain the inverse function of

$y = 27x^3$

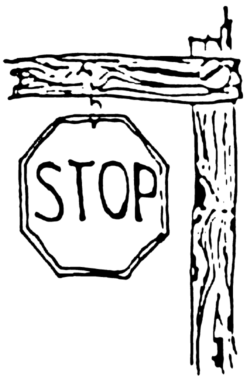
$y^{-1}(x) = ...$……………..

--------------------▷ 82

$y = 2$

164

If you are unsure go back to frame 165 and try again



--------------------▷ 165

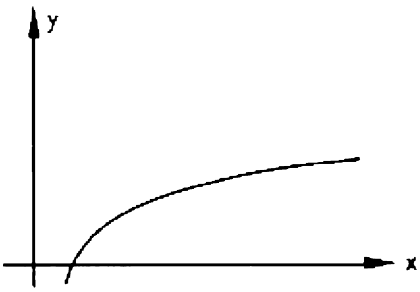
$y^{-1}(x)=\frac{3\sqrt{x}}{3}$

82

The inverse function is a new function. You obtain the inverse function in two steps

Step 1: ………………….

Step 2: …………………



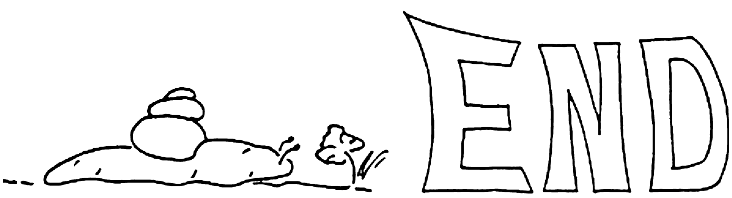
Given the graph of a function. Sketch the graph of the inverse function.
If you feel unsure, please consult the textbook again

------------------▷ 83

165

You have reached the end of this chapter

If all subject matter has been new to you, it will have taken you quite some time.
But you have finished it and made an important step forward.



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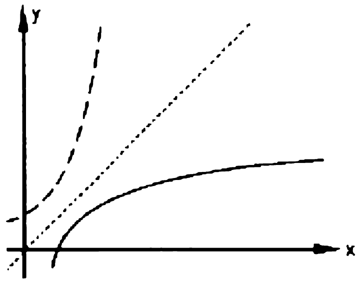

83

Give the geometrical procedure to obtain the inverse function:

………………………………………………………………

………………………………………………………………

------------------▷ 84

Please continue on page 1
(bottom half)