Chapter 4

Exponential, logarithmic and hyperbolic functions

0

------------------▷ 1

$\ln y = -ax$

44

If you have difficulties taking logarithms of an equation you often may use this trick. Try in an intermediary step to write both sides as powers of the same base.

Given $y = e^{a}$

Intermediate step

$e^{\ln y} = e^{a}$

Result: $\ln y = a$

Given: $y = e^{a+x}$

…………..=…………..

------------------▷ 45

4.1 Powers, Exponential Function

1

First you will study a section in the textbook. For many users this section will be known from school. But if the material covered is not a mere repetition of well-known facts, carefully take notes and copy the important rules into your notebook.

Study in the Textbook **4.1.1 Powers; Exponential Function**
4.1.2 Laws of Indices or Exponents
Textbook pages 69–71

When done, proceed to

-------------------▷ 2

$\ln y = a + x$

45

Take logarithms of the following equations

$y = e^{\frac{1}{x}}$ …………=………………

$y = 2^{a \cdot x}$ …………=………………

$y = 10^{(-x+5)}$ …………=………………

Choose appropriate bases

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Expand the following expressions

2

$a^4 = \ldots\ldots\ldots$

$b^{-2} = \ldots\ldots\ldots$

-------------------▷ 3

$\ln y = \frac{1}{x}$

46

$ldy = a \cdot x$

$\lg y = -x + 5$

We give some more exercises.
Hint: if exercises seem easy you do not need more of them. If exercises seem difficult you need more of them

$y = e^{(\alpha x+\beta)}$ …………=………………

$b \cdot y = e^{a \cdot x} \cdot e^{c \cdot x}$ …………=………………

$a \cdot y = 10^{0.1x}$ …………=………………

$y = e^{(\ln x - \ln a)}$ …………=………………

-------------------▷ 47

$a^4 = a \cdot a \cdot a \cdot a$

$b^{-2} = \dfrac{1}{b \cdot b}$

3

Now, let us repeat the vocabulary

The term b^m is called ……………….

b is called the ……………………….

m is called …… or ……………..

------------------▹ 4

$\ln y = \alpha x + \beta$

47

$\ln(by) = (a + c)x$ or $\ln y = (a + c)x - \ln b$

$\lg(ay) = 0.1x$ or $\lg y = 0.1x - \lg^{a}$

$\ln y = \ln x - \ln a$

It may be time to have a short break. The reader sketched will have a short break. What is he doing? Have a guess…………

------------------▹ 48

b^m is called power
b is named base
m is named exponent or index
Hint: These names should be known by heart

4

The definition of powers to a negative exponent is derived by looking at the results of consecutively dividing a given power by its base.
Express the following power as a fraction.

$x^{-3} = ...$

------------------▷ 5

Perhaps he recapitulates the new concepts of the preceding section. Perhaps he writes down the time for the end of his break.

48

------------------▷ 49

$x^{-3} = \frac{1}{x^3}$ 5

Let us reiterate. The expression

10^x is called …………..

10 is called ……………

x is called …. or ………

------------------▷ 6

4.2.1 Operations with logarithms 49

The basic reasoning of operations with logarithms is quite simple. All operations have to be done with logarithms instead of with the original values.
Thus a product of two values will be the sum of its logarithms.

READ **4.2.2 Operations with logarithms**
Textbook pages 78–79

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Power
Base
Exponent or index

6

Write down the power for:

a) base: x
Exponent: 3 ………….
Power: ………………

b) base A
Exponent x …………
Power: ……………..

------------------▷ 7

Can you write down

50

a) $\ln(a \cdot b) =$ ....

b) $\ln\frac{a}{b} =$ ....

c) $\lg(A \cdot B) =$ ....

d) $\lg\frac{y}{x} =$ ....

------------------▷ 51

a) x^3
b) a^x

7

It will be very useful if you understand the origin of the rules. In the following we will deliberately change the notations. The relationships remain unchanged. Transform the following terms:

Product:	$a^x \cdot a^y = \ldots\ldots.$	$A^{t_1} \cdot A^{t_2} = \ldots..$
Quotient:	$\dfrac{b^m}{b^n} = \ldots..$	$\dfrac{S^n}{S^m} = \ldots..$
Power:	$(x^n)^m = \ldots.$	$(a^x)^y = \ldots.$
Root:	$\sqrt[a]{x^b} = \ldots..$	$\sqrt[x]{a^y} = \ldots\ldots$

-------------------▷ 8

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a) $\ln(a \cdot b) = \ln a + \ln b$

51

b) $\ln\dfrac{a}{b} = \ln a - \ln b$

c) $\lg(A \cdot B) = \log A + \log B$

d) $\lg\dfrac{y}{x} = \lg x - \lg y$

In case of difficulties calculate the examples consulting the textbook.

-------------------▷ 52

Product:	a^{x+y}	$A^{(t_1+t_2)}$	8
Quotient:	b^{m-n}	s^{n-m}	
Power:	$x^{n\cdot m}$	$a^{x\cdot y}$	
Root:	$x^{\frac{b}{a}}$	$a^{\frac{y}{x}}$	

In case of difficulties consult the textbook and solve the exercise again. Do not be disturbed by the different notations.
In practice notations change in accordance with the given problems.
Now solve

a) $27^{o} =$ .... b) $(3^3)^0 =$ ....

c) $(2^2)^3 =$ .... d) $1^5 =$ ....

------------------▷ 9

In the next examples we deliberately use different notations. The objective is to obtain a certain familiarity with using different notations. 52

$\lg(x\cdot y) =$ ....

$ld(N_1 \cdot N_2) =$ ....

$\lg\frac{A\cdot B}{C} =$ ....

$\ln\frac{a\cdot\beta\cdot\gamma}{\delta} =$ ....

------------------▷ 53

a) 1 b) 1

c) 64 d) 1

9

Next, compute the values or simplify the expressions respectively.

a) $3^4 \cdot 3^{-3} = \ldots\ldots\ldots\ldots$

b) $10^6 \cdot 10^8 \cdot 10^{-1} = \ldots\ldots\ldots\ldots\ldots\ldots$

c) $b^{-m} = \ldots\ldots\ldots\ldots$

d) $e^{-1} = \ldots\ldots\ldots\ldots\ldots\ldots$

e) $4^{\frac{1}{2}} = \ldots\ldots\ldots\ldots$

------------------▷ 10

$\lg xy = \lg x + \lg y$

53

$ldN_1 \cdot N_2 = ldN_1 + +ldN_2$

$\lg \frac{AD}{c} = \lg A + \lg B - \lg C$

$\ln \frac{\alpha \cdot \beta \cdot \gamma}{\delta} = \ln \alpha + \ln \beta + \ln \gamma - \ln \delta$

Calculate

$\ln 5^x = \ldots\ldots\ldots\ldots$

$\lg x^2 = \ldots\ldots\ldots\ldots$

$\lg a^{\frac{1}{2}} = \ldots\ldots\ldots\ldots$

------------------▷ 54

10

a) 3

b) 10

c) $c\dfrac{1}{b^m}$

d) 2

e) $\dfrac{1}{e}$

If you had difficulties with the previous exercises, solve them consulting the textbook. You should gain a certain familiarity with these transformations.

If the tasks so far were easy go straight to -------------------▷ 17

If, however, you want to improve your efficiency, you are invited to go to -------------------▷ 11

54

$\ln 5^x = x \ln 5$

$\lg x^2 = 2 \lg x$

$\lg a^{\frac{1}{2}} = \frac{1}{2} \lg a$

Transform:

a) $\ln 2^x =$ ....

b) $\lg \sqrt{x} =$ ....

c) $\lg \sqrt[3]{x} =$ ....

d) $ld(4 \cdot 16) =$ ....

-------------------▷ 55

Solve

11

a) $\sqrt[x]{A} =$ .... b) $27^{\frac{1}{3}} =$ ....

c) $(y^2)^3 =$ .... d) $(0.1)^0 =$ ....

e) $10^3 \cdot 10^{-3} \cdot 10^2 =$ .... f) $\alpha^{-3} =$ ....

------------------▷ 12

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a) $\ln 2^x = x \ln 2$ b) $\lg\sqrt{x} = \frac{1}{2}\lg x$

55

c) $\lg\sqrt[3]{x} = \frac{1}{3}\lg x$ d) $ld(4 \cdot 16) = ld4 + ld16 = 6$

Can you give the rules for the general care base a

Multiplication: …………

Division: ……………….

Power: …………………

Root: ………………….

------------------▷ 56

a) $A^{\frac{1}{x}}$ b) 3 12

c) y^6 d) 1

e) 10^6 f) $\frac{1}{\alpha^3}$

Further exercises wanted ------------------▷ 13

No difficulties so far ------------------▷ 16

Multiplication: $\log_a AB = \log_a A + \log_a B$ 56

Division: $\log_a \frac{A}{B} = \log_a A - \log_a B$

Power: $\log_a A^m = m \log_a A$

Root: $\log_a \sqrt[n]{A} = \frac{1}{n} \log_a A$

Want to proceed ------------------▷ 58

Want more exercises ------------------▷ 57

Solve 13

a) $2^{-3} =$ .... b) $27^{0} =$ ....

c) $e^{0} =$ .... d) $3^{-1} =$ ....

e) $\alpha^{-3} =$ .... f) $y^{\frac{2}{3}} =$ ....

$\dashrightarrow$ 14

Transform 57

a) $\ln(C \cdot D) =$ ....

b) $\lg y^{2} =$ ....

c) $ld\, 2 \cdot 32 =$ ....

$\dashrightarrow$ 58

a) $\frac{1}{2^3} = \frac{1}{8}$ b) 1 14

c) 1 d) $\frac{1}{3}$

e) $\frac{1}{\alpha^3}$ f) $(\sqrt[3]{y})^2$

Transform following this example $x^n \cdot x^m = x^{m+n}$

a) $b^n b^m =$.......... b) $(y^n)^m =$...........

c) $\frac{A^n}{A^m} =$.......... d) $\sqrt[n]{C^m} =$.............

-------------------▷ 15

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a) $\ln C + LnD$ 58

b) $2\lg y$

c) 6

Transform

a) $ld\sqrt{x} =$...............

b) $\ln\left(e^{2x} \cdot e^{5x}\right) =$................

c) $\lg\frac{1}{10^x} =$..................

-------------------▷ 59

a) b^{n+m} b) A^{n-m} 15

c) $y^{n\cdot m}$ d) $C^{\frac{m}{n}}$

Last exercise:

a) $4^{\frac{1}{2}}$ b) $(3^0)^2$

c) $3^4 \cdot 3^{-3}$ d) $10^{-6} \cdot 10^{-8} \cdot 10^1$

e) e^{-1}

We do not give the answers this time. In case of doubt ask a fellow student or consult the textbook again.

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a) $\frac{1}{2} ldx$ 59

b) $7x$

c) $-x$

Calculate by taking logarithms

$C = 10^{3x+1}$ $x = \ldots\ldots\ldots\ldots$

$A = e^{(r\cdot t)}$ $t = \ldots\ldots\ldots\ldots$

$16 = 2^{x+2}$ $x = \ldots\ldots\ldots\ldots$

------------------▷ 60

4.1.1 Exponential function

16

The exponential function is a basic requisite for further studies.

Study in the textbook
4.1.3 Binomial theorem
4.1.4 Exponential function
Pages 71–73

Having done this ------------------▷ 17

60

$$x = \frac{1}{3}(\lg C - 1)$$

$$t = \frac{\ln A}{r}$$

$$x = 2$$

If you encounter difficulties calculating exercises there is a golden rule. Write down the given exercises on a separate sheet and go back to the textbook.

Try to solve the problem with reference to the textbook and exercise at the same time.
You must not divide your attention by turning over pages many times.
You memorize the operations with logarithms much easier if you understand the relations with the laws of exponents treated earlier

------------------▷ 61

17

The function $y = 10^x$ is called ………………….

Which of the following functions increases most rapidly for x $\rightarrow \propto$?

Insert values for

$x = 10$, $x = 100$, $x = 1000$

□ $y_1 = x^{100}$

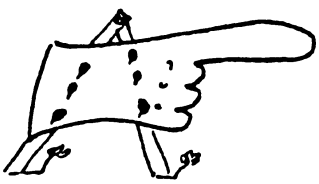
□ $y_2 = 10^x$

------------------▷ 18

61

Can you express the basic reasoning of the operations with logarithms in your own words? Perhaps you may explain this for a fellow student.

This is the advantage of working in a team of fellow students. You often need to explain something to others. It is not enough to understand subject matter. You must be able to express it in your own words. At the end you will need this competence during your examinations.



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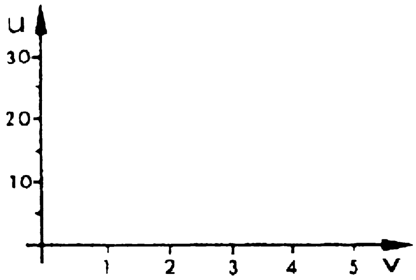
Exponential function

18

$y_2 = 10^x$

Hint: For $x = 1000$ we obtain for $y_1 : (1000)^{100} = 10^{300}$ and for $y_2 : 10^{1000}$
y_2 exceeds y_1 significantly

If we substitute the familiar notations x and y by other notations the mathematical relationships do not change.
Complex equations often seem difficult if unfamiliar notations are used.
In these cases it often helps to substitute the given notations by x and y and to solve the seemingly easier equations and then resubstitute.



Sketch the function $u = 2^v$

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4.2.2 Logarithmic function

62

READ **4.2.3 Logarithmic function**
Textbook pages 79–80

Having done

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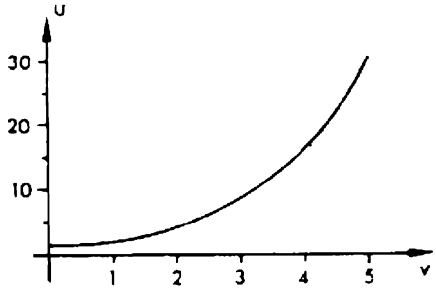

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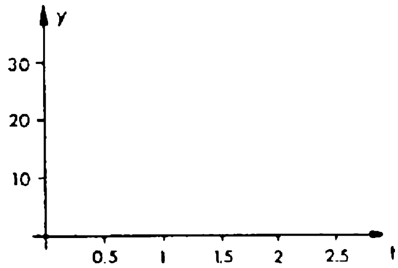
Sketch the exponential function
$y = 2^{at}$ with $a = 2$



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At what point intersect all logarithmic functions?

63

$x =$ ....

$y =$ ....

Has the logarithmic function a pole?

☐ yes

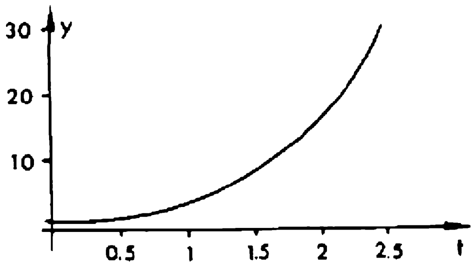
☐ no

Has the logarithmic function an asymptote

☐ yes

☐ no

------------------▷ 64



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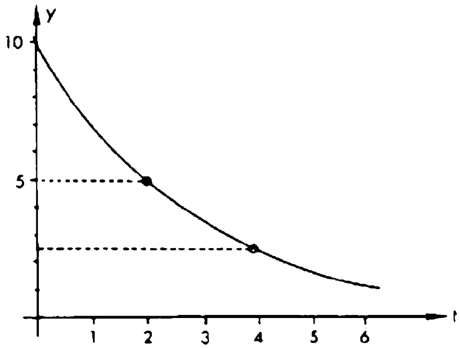


Solve the following task.
In case of difficulties consult the textbook.
The plot represents the general exponential function

$y = A \cdot e^{-\frac{t}{t_h}}$

Determine A and t_h (t_h = half life value)

Use the given points.

$y =$ ………………..

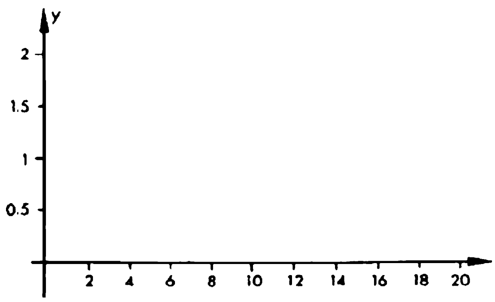
------------------▷ 21

Intersection of all logarithmic functions at $x = 1$ $y = 0$

64

Pole at $x = 0$

No asymptote



Sketch the graph of

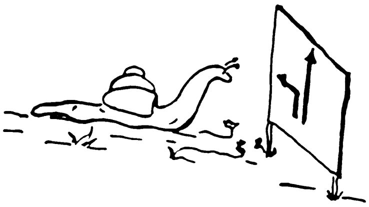
$y = \lg x$

$y = \ln x$

------------------▷ 65

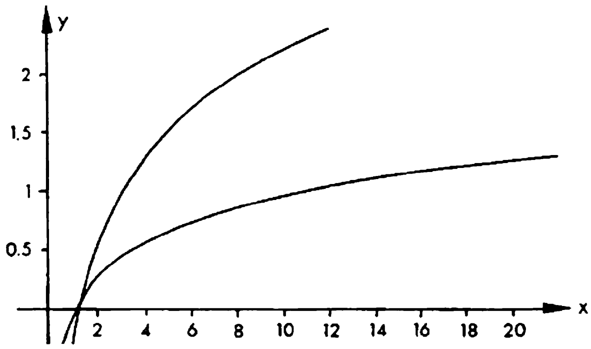
$y = 10 \cdot 2^{-\frac{t}{2}}$

21



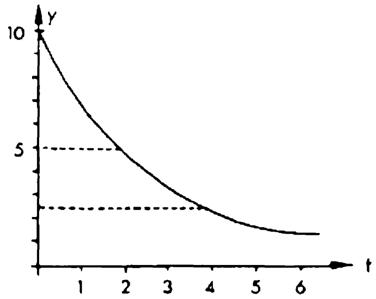
All correct --------------------▷ 24

Explanation wanted --------------------▷ 22



65

--------------------▷ 66



In the textbook this exponential function is explained. t is the time. The initial value for t = o is A = ……………..
From the plot you may read that the curve decreases to half of its initial value at t_h = ...

22

Hint:

At $t = 0$ the term $A \cdot e^{\frac{t}{th}} = A \cdot e^0 =$ ....

Thus, the function is $y =$ ………………

-------------------▷ 23

The logarithmic function is the inverse of the exponential function.
Is the exponential function the inverse of the logarithmic function?

66

☐ yes -------------------▷ 68

☐ no -------------------▷ 67

$A = 10$

23

$t_h = 2$

$y = 10 \cdot 2^{-\frac{t}{2}}$

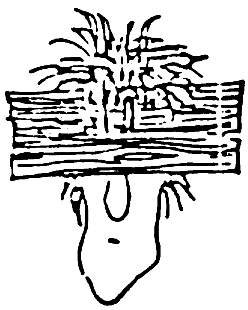
Hint: The values have been taken from the graph.

Since A = 10 and $t_h = 2$ we insert into $y = A \cdot e^{-\frac{t}{t_h}}$ and obtain $y = 10 \cdot 2^{-\frac{t}{2}}$

------------------▷ 24

Sorry you are wrong. We obtain the inverse of a function by reflecting it in the line $y = x$ which bisects the first quadrant and vive versa. Go back to section 3.4 "inverse functions" and read it again. The relation is symmetrical.

67



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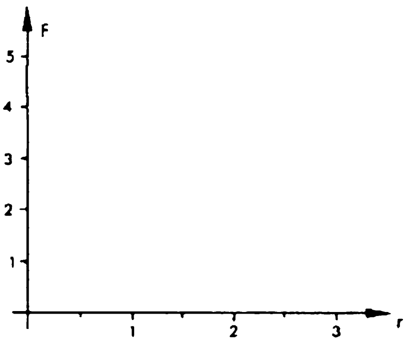
24



Sketch the exponential function given below

$F = e^{0.5t}$ $e = 2.72$

In case you are not familiar with this notation you may substitute F by y and t by x
Perhaps the equation will be more familiar to you

------------------▷ 25

68

Yes, you are right. The exponential function is the inverse function of the logarithmic function.
We remember that we get the inverse function by reflecting the original function in the line $y = x$ which bisects the first quadrant.

Not every function has an inverse function.
If possible give the inverse function for

$y_1 = 3^{2x}$ $y_1^{-1} = \ldots\ldots\ldots\ldots\ldots$

$y_2 = 4x^2$ $y_2^{-1} = \ldots\ldots\ldots\ldots$

Hint:
Remember the definition of a function.

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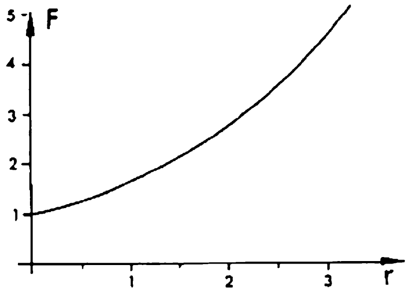


25

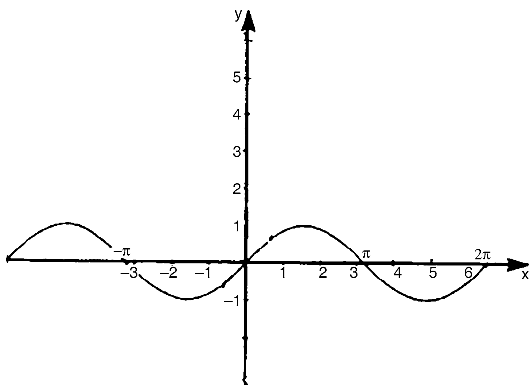
We suggest a break. You may work with fairly good concentration for 20 minutes up to 60 minutes.
There are great differences in optimal individual working periods.
If you are interested in your work these periods may be longer.
Find out how long you may work with concentration. It is important to divide your tasks and to have a short break - and to end the short break in due time..

------------------▷ 26

$y_1^{-1} = \frac{1}{2}\log_3{}^x$

69

y_2 has no inverse function since the expression $y_2^{-1} = \pm\frac{1}{2}\sqrt{x}$ does not represent a function because it is ambiguous.



On the other side the sine curve is plotted.
You may reflect it in the line $y = x$
which bisects the first quadrant.
Sketch the reflected curve. Is the reflected curve a function?

------------------▷ 70

26

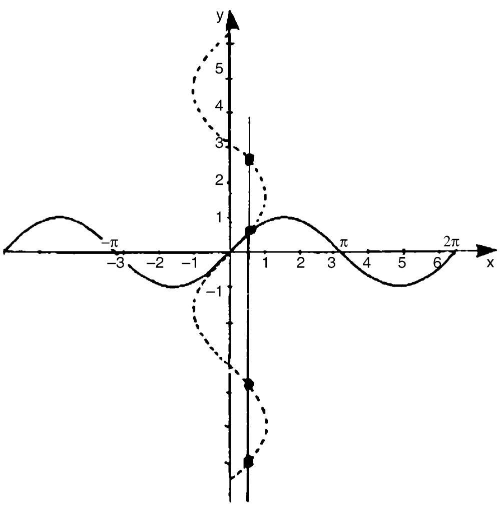
Short breaks have a duration of 5–15 minutes. With longer breaks the difficulties of warm up rise again.
It is of some importance what you do during the break. Mark appropriate activities

☐ Water your flowers, have a cup of tea, prepare a coffee, do some physical exercise

☐ solve mathematical problems, read a different chapter in your textbook.

------------------▷ 27

70



The reflected curve is ambiguous. It is a relation not a function. We can get a function if we restrict the domain and the codomain.

$$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

You know this function already:
$y = arc\sin x$ and you know its meaning.
Complete:
y is the angle …………………………..

…………………………..

Sketch in the plot above with a heavy line the function for the restricted domain and codomain.

------------------▷ 71

Appropriate activities are to water flowers, prepare tea or coffee, do some physical exercise. 27

Learning and memorizing a certain matter will be prevented if you concentrate during your break on a similar matter. Solving other mathematical problems is very similar to your learning. It has a negative effect on your learning. This phenomenon is well known in psychology and is named interference.
Another suggestion. Write down on a separate sheet of paper the planned end of your break.

Now enjoy your break.

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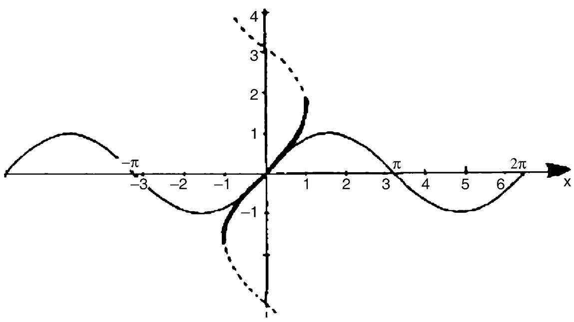
y is the angle whose sine is x. 71



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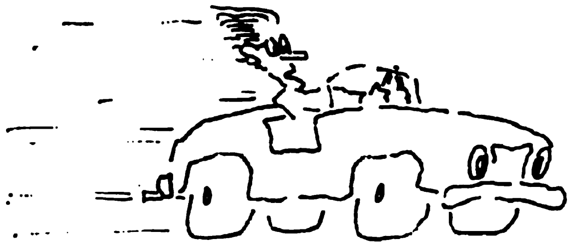
Short break 28



Having ended your break go to ------------------▷ 29

4.3 Hyperbolic functions and inverse Hyperbolic Functions 72

Since these functions are used in advanced mathematics chapters you may skip this section for the time being. Especially if you found it difficult to master the preceding two introductory chapters.
In this case you may return to this section later.
But if all of the preceding material was known you should study this section now.

I want to skip section 4.3 ------------------▷ 88

I want to study the section on hyperbolic functions

READ **4.3. Hyperbolic functions and inverse hyperbolic functions**
Textbook pages 80–84

Having done ------------------▷ 73

Before continuing compare the actual time with the time you planned. There may be a difference. Breaks tend to increase. This does not matter. But in the long run these differences between planned lengths of breaks and the real lengths should not increase too much. 29

-------------------▷ 30

Give the definition of 73

sinh x = ...

-------------------▷ 74

4.2 Logarithm, logarithmic function

30

Logarithms are quite difficult when learned for the first time; however if you are familiar with this matter you will proceed quickly.

READ **4.2.1 Logarithm**
Textbook pages 76–77

Then go to

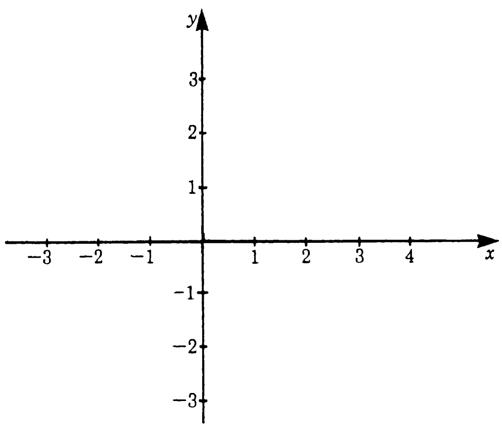
------------------▷ 31

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

74

Sketch $y_1 \sinh 2x$ with a dashed line

and $y_2 \sinh \dfrac{x}{2}$ with a dotted line



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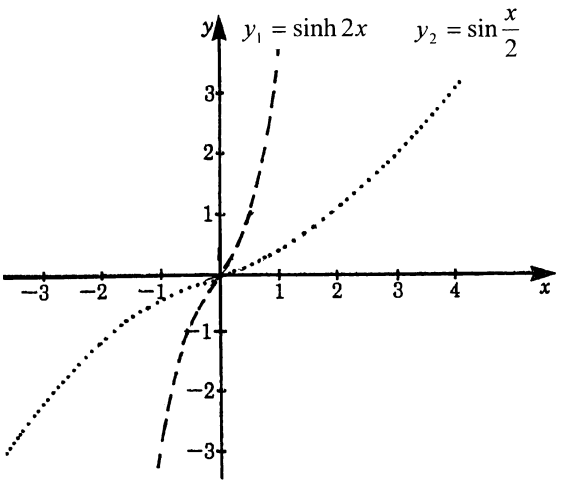
Taking logarithms is a new operation. To take logarithms is to solve the equation $y = a^x$ for x. This means given: ………….. and ……………
wanted ………..…

31

------------------▷ 32

75



Give the definition of $\cosh x =$ ....

------------------▷ 76

$y = a^x$ **32**

given: a and y

wanted: x

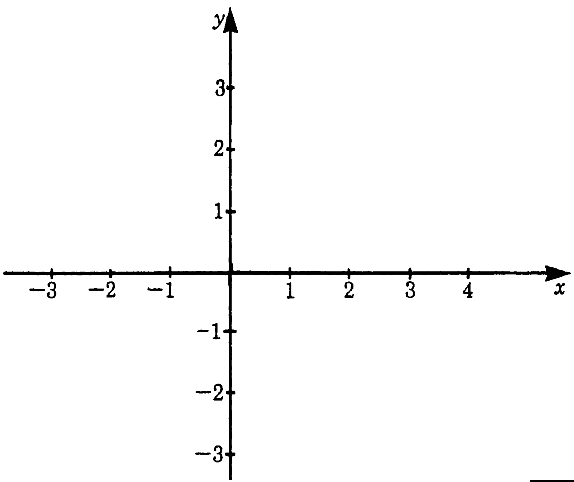
In another notation the task of taking logarithms can be expressed as well:
The equation $a^y = x$ is to be solved for y.
Up to now we are unable to find a solution.
Thus, we must create a new operation. In mathematics this new operation is called “Taking logarithms.”

------------------▷ 33

$\cosh x = \dfrac{e^x + e^{-x}}{2}$ **76**

Sketch $y_1 = \cosh 2x$ with a dashed line

and $y_2 = \cosh \dfrac{x}{2}$ with a dotted line



------------------▷ 77

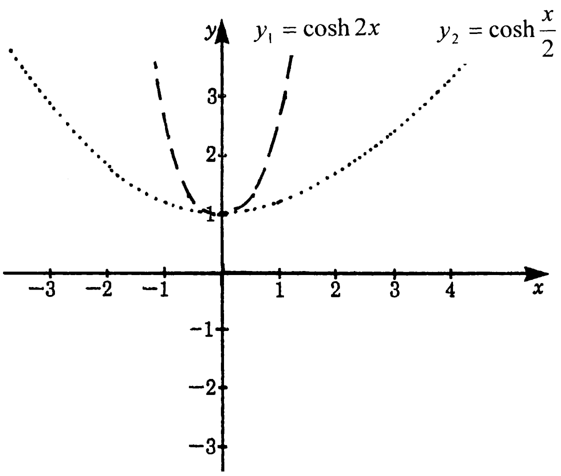
Let us take the equation $a^y = x$ 33
In the textbook you found the following definition:

The logarithm of a given number x to a base a is the exponent of the power to which this base must be raised to equal this number x.

For this exponent we use the symbol
$\log_a x$

In other words: The term "$\log_a x$" is a power or an exponent.
$a^{(\log_a x)} = ...$

------------------▷ 34



77

Calculate $(\cosh 2x)^2 - (\sinh 2x)^2 = ...$

------------------▷ 78

$a^{(\log_a x)} = x$ 34

You must memorize:
For a given base
the logarithm of a certain number is the exponent to equal this number.

The definition and the meaning of logarithms are important and you must be familiar with them. The logarithms for base 10 are called ……….. and abbreviated ……………….

------------------▷ 35

$(\cosh 2x)^2 - (\sinh 2x)^2 = 1$ 78

Give the definition of $\tanh x$. Try first to answer without consulting the textbook

$\tanh x =$ .... = ... = ...

------------------▷ 79

common logarithm abbreviation: lg

35

Let us regard common logarithms.

Calculate:

$10^{\lg 5} = \ldots\ldots\ldots\ldots\ldots$

$10^{\lg 20} = \ldots\ldots\ldots\ldots\ldots$

$10^{\lg 3.14} = \ldots\ldots\ldots\ldots\ldots$

------------------▷ 36

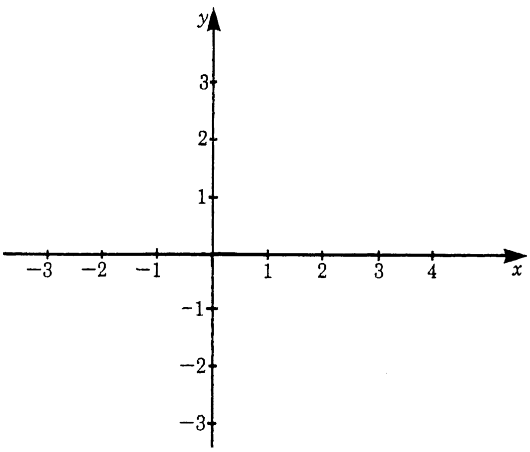
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1-e^{-2x}}{1+e^{-2x}}$$

79

Try to sketch

$y_1 = \tanh 2x$ with a dashed line and

$y_2 = \tanh \dfrac{x}{2}$ with a dotted line



------------------▷ 80

$$10^{\lg 5} = 5$$
$$10^{\lg 20} = 20$$
$$10^{\log 3.14} = 3.14$$

36

Logarithms with base 2 are called dyadic logarithms or logarithms to the base of 2 and abbreviated …………….

Calculate

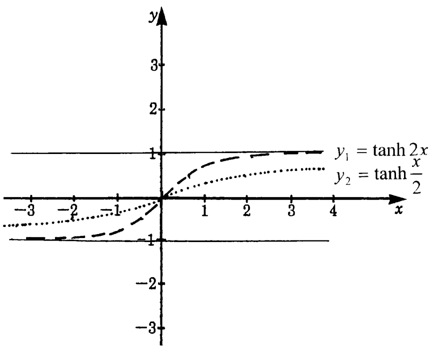
$2^{ld4} =$ ....

$2^{ld100} =$ ....

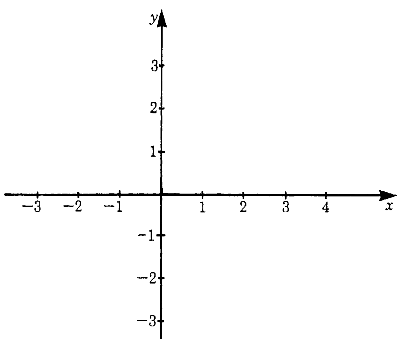
$2^{ldb} =$ ....

------------------▷ 37

80



Try to sketch $y_1 = \coth x$ with a dashed line and $y_2 = \coth \frac{x}{2}$ with a dotted line



------------------▷ 81

Logarithms to the base of 2 are abbreviated ld

$2^{ld4} = 4$

$2^{ld100} = 100$

$2^{ldb} = b$

37

Logarithms with base e are called ………….. and abbreviated ……………….

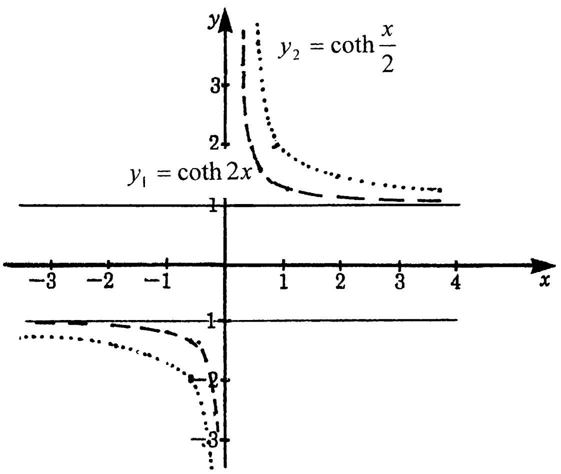
Calculate:

$e^{\ln 6} = ...$

$e^{\ln a} = ...$

$e^{\ln 10} = ...$

------------------▷ 38



81

In the textbook we give the definition of the inverse hyperbolic functions.

If you wish to derive one of them ------------------▷ 82

If you want to skip derivation of $\sinh x^{-1}$ ------------------▷ 87

Natural Logarithms ln 38

$e^{\ln 6} = 6$

$e^{\ln a} = a$

$e^{\ln 10} = 10$

With logarithms the base has always to be defined. Give the names of the logarithms with the following base.

base 2: ………………..

base *e*: ………………..

base 10: ………………

------------------▷ 39

Given the hyperbolic sine: $f(x) = \sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ 82

We may write it $y = \frac{1}{2}\left(e^x - e^{-x}\right)$

To obtain the inverse function we change y and x

$x =$ ……………………

------------------▷ 83

base 2: dyadic logarithms or logarithms to the base of 2
base e: natural logarithms
base 10: common logarithms

39

If you want to take logarithms you have three options

1. You use your pocket calculator or your computer. This is convenient and precise.

2. You use a plot of the logarithmic function. This is convenient but not precise.

3. You use a table. This is precise but inconvenient.

For some values you can calculate the logarithms without help

$ld 2 = \ldots\ldots\ldots\ldots\ldots\ldots$

$\ln e^{x} = \ldots\ldots\ldots\ldots\ldots\ldots$

$\lg 100 = \ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 40

83

$x = \frac{1}{2}\left(e^{y} - e^{-y}\right)$

$2x = \left(e^{y} - e^{-y}\right)$

Multiplying by e^{y} gives

$2x \cdot e^{y} = \ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 84

40

$ld\,2 = 1$ since $2^1 = 2$

$\ln e^x = x$ since $e^x = e^x$

$\lg 100 = 3$ since $10^3 = 100$

An operation used later on is to take logarithms of equations. In this case the operation is to be applied to both sides of the equation.
By this operation equations sometimes simplify.
Example: given $e^y = e^{ax}$
The base is (and must be) the same for both sides. In this case the exponents on both sides must be equal:
$y = ax$
Thus, we have just taken the logarithm of the equation since $\ln e^y = y = \ln e^{ax} = ax$
Take the logarithms
$e^a = e^{b+c}$

…………..=…………….

-------------------▷ 41

84

$2xe^y = \left(e^{2y} - 1\right)$

Substituting $e^y = a$

gives

$2xa = ...$

-------------------▷ 85

a = b + c

41

Take the logarithm of the equation

$10^y = 10^{bx}$

Using common logarithms we get

$\lg 10^y = \lg 10^{bx}$

Thus $y = bx$

Take logarithms of the following equations

$2^y = 2^{cx}$ $y =$…………………

$e^a = e^{\omega(t+t_0)}$ $a =$………………….

------------------▷ 42

$2xa = a^2 - 1$

85

Thus, we get a quadratic function for a.
We already know to solve it:

$a = x + \sqrt{x^2 + 1}$

Since $a = e^y$ we obtain

$e^y = x + \sqrt{x^2 + 1}$

To obtain the inverse function y we take the logarithm and obtain $y =$……………..

------------------▷ 86

$y = cx$ $\qquad$ $a = \omega(t + t_0)$ **42**

To take logarithms of an equation means to regard exponents if the bases are equal.
Example: $2^7 = 2^{x+1}$
We take the logarithms: $\quad ld2^7 = ld2^{x+1}$

$$7 = x + 1$$
$$x = 6$$

Calculate $\quad 10^{(2y+1)} = 10^{x-3}$

$$y = ...$$

-------------------▷ 43

$y = \ln\left(x + \sqrt{x^2 + 1}\right)$ **86**

Well done.

If you worked through this rather rough matter you will use hyperbolic functions later on in advanced mathematics.

-------------------▷ 87

$y = \frac{1}{2}(x-4)$ 43

In the foregoing examples we had on both sides of the equation exponents of the same base. This is not always the case.

Example $y = e^{-ax}$. Can you take the logarithm? You may think this is impossible. But in this case a trick helps.

Write down y as a power of e.

$y = e^{\ln y}$

Thus, we have on both sides the same base

$e^{\ln y} = e^{-ax}$

Now you can take the logarithm of the equation:

…………..=…………….

-------------------▷ 44

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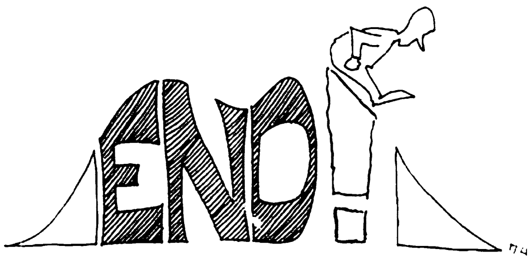
87



You have reached the end of chapter 4