Chapter 7
Applications of Integration

-------------------▷ 1

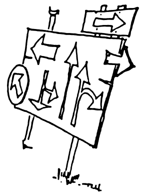
$$V = \pi \left[x + x^2 + \frac{x^3}{3} \right]_a^b = \pi \left[b + b^2 + \frac{b^3}{3} - a - a^2 - \frac{a^3}{3} \right]$$ 40

The integrals in the last example have been standard. But as a rule these integrals are quite cumbersome. Nowadays they are solved in pratice with the help of computer programs like EULER/MAXIMA or Mathematica or Mathlab.

-----------------▷ 41

This chapter demonstrates in detail the use of integral calculus to solve special problems in physics and mechanics. It may be skipped for the time being and can be used later on as a reference when problems are encountered. 1

Thus this study guide will be divided in separate units for each topic discussed.
Choose the topic you want to study:



Areas ----------------▷ 2

Lengths of curves ----------------▷ 30

Surface area and volume of a solid of revolution ----------------▷ 33

Applications to mechanics ----------------▷ 41

The Theorems of Pappus
Moment of inertia, second moment of area ----------------▷ 60

For the time being I choose to proceed with chapter 8,
"Taylor series and power" series and will skip this study guide proceed to chapter 8

7.4 Applications to mechanics

41

The basic physics concepts of the next sections will be known from physics lessons. However, the calculation of examples may be new.

READ **7.4.1 Basic concepts of mechanics**
7.4.2 Center of mass and centroid
Textbook pages 208–211

----------------▷ 42

7.1 Areas

2

The integral calculus has been introduced in chapter 6 by solving the area problem. Thus the first following example will be easy to read. In the second example the notation is changed, so you have to read quite carefully.

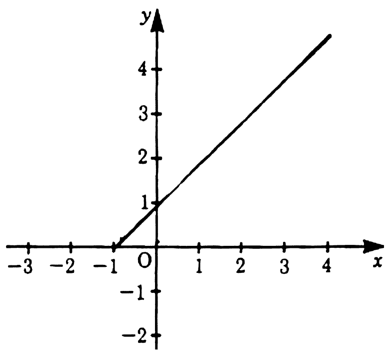
READ **7.1 Areas**
Textbook pages 191-194

-----------------▷ 3

42

Given a straight line $y = 1 + x$
It will be rotated about the x-axis between
$x_1 = a$ and $x_2 = b$, thus generating a solid.
Calculate the solid´s center of mass which may also be called centroid.

$x_C =$ ....



Solution found -----------------▷ 44

Hints wanted -----------------▷ 43

Given a function $f(x) = \frac{C}{x}$.

3

Calculate the area between x = 1 and x = 5. First give the formal solution in form of a ……….integral. Let C be C = 2

A = ……….

-----------------▷ 4

43

The center of mass or centroid is given by two values: $\bar{x}_C$ and $\bar{y}_C$.

For the given solid it follows from symmetry that $\bar{y}_C = 0$, since the solid is generated by rotating y = 1 + x around the x-axis.

There remains the task of calculating $\bar{x}_C$.

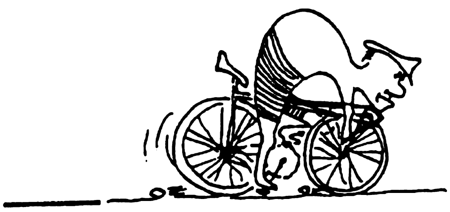
Write down the formula 7.11a.

Insert y = 1 + x and solve the definite standard integrals regarding the boundaries.

$\bar{x}_C =$ ....

The calculation of the area A is standard.

In case of further doubts study the textbook again.



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definite integral

$A = C[\ln x]_1^5$

4

Using your calculator evaluate

$A = C[\ln x]_1^5 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

------------------▷ 5

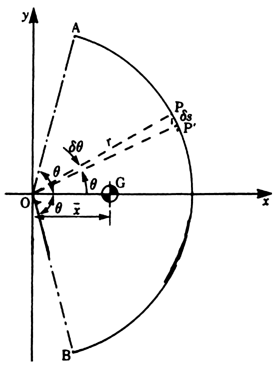
$$\bar{x}_C = \frac{1}{A}\int_a^b (1+x)x\,dx = \frac{1}{A}\left[\frac{b^2}{2} + \frac{b^3}{3} - \frac{a^2}{2} - \frac{a^3}{3}\right]$$

44

A is given by $A = \int_a^b y\,dx = \int_a^b (1+x)\,dx = \left[b + \frac{b^2}{2} - a - \frac{a^2}{2}\right]$

A circular plate is cut into a circular sector of r = 1m and included angle of 2Θ. Find the position of the centroid or $\bar{x}_C$ along the axis of symmetry

$\bar{x}_C = \ldots\ldots\ldots\ldots$



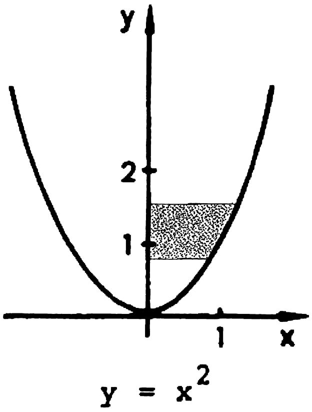
Solution found ------------------▷ 49

Hint wanted ------------------▷ 45

$A = 2 \cdot [\ln x]_1^5 = 2(\ln 5 - \ln 1) = 2 \cdot \ln 5$ 5

$= 2 \cdot 1.6094 = 3.2189$

The figure below shows the well known parabola
$y = x^2$.
The area shaded is named ………… .



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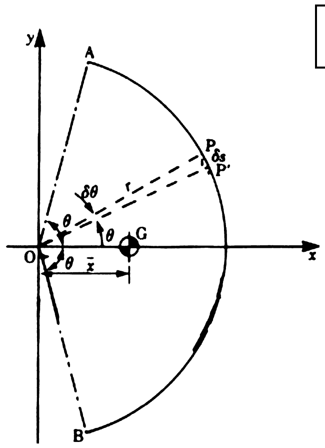
This problem is related to the example treated in the textbook. A thin strip was bent into a circular arc. The arc subtended an angle at the center. The position of the center of mass has been calculated to be 45

$$\bar{x}_C = \frac{r \sin \Theta}{\Theta}$$



Referring to the figure the circular sector is now given by the area ABO

First let us calculate its area A:

A = ………………

Solution -----------------▷ 47

Hint -----------------▷ 46

Complementary area

6

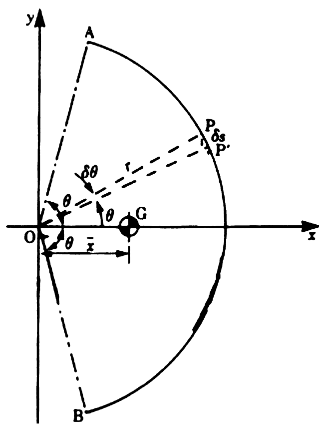
Calculate the complementary area for $y_1 = 1$ and $y_2 = 3$.

A = ……………

Solution found -----------------▷ 8

Help wanted -----------------▷ 7

46



We want to calculate the area of the circular sector ABC.
Using polar coordinates we get

$$A = \int_{-\Theta}^{+\Theta} \frac{R^2 d\Theta}{2} = ...$$

-----------------▷ 47

To solve $A_C = \int_{y_1}^{y_2} x\, dy$ Given $y = x^2$ **7**

First we must find $x = g(y)$

In this case

x = ……………

Now solve

$$A_C = \int_1^3 xdy$$

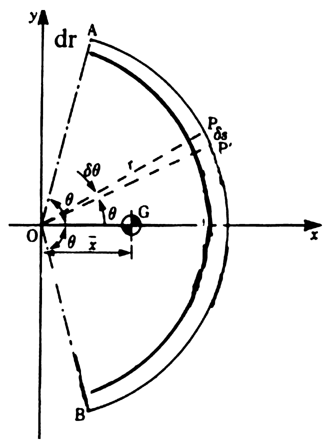
$$A_C = ...$$

-----------------▷ **8**

$A = R^2\Theta$ **47**

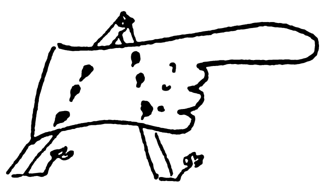
The first moment of the circular sector is $A \cdot \bar{x}_C$.
This must equal the sum of moments of all circular strips of width dr.
A strip has the first moment

$$\bar{x} \cdot = \frac{r \sin\Theta}{\Theta} \Delta A_s$$



The area ΔA_s of a strip is $\Delta A_s = ...$

Hint: The length of the strip is $r \cdot 2\Theta$
Its width is dr.



-----------------▷ **48**

8

$x = +\sqrt{y}$

$$A_C = \int_1^2 \sqrt{y}\,dy = \frac{2}{3}\left[y^{\frac{3}{2}} \right]_1^2 = \frac{2}{3}\left[2^{\frac{3}{2}} - 1 \right] = \frac{2}{3}1.828 = 1.219$$

If you had difficulties solving the example in the textbook regarding thermodynamics we suggest to calculate the example substituting p by y and V by x. Then try again to calculate the example following the text given in the textbook.

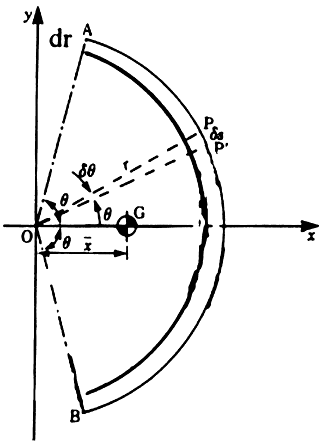
----------------▷ 9

48

$\Delta A_s = r \cdot 2\Theta \cdot \Delta r$ or $dA_s = r \cdot 2\Theta dr$

Now we sum up the moments of all strips from
r = 0 to r = R
This must equal $A \cdot \bar{x}$:

$$A \cdot \bar{x} = \int_0^R \frac{r \sin \Theta}{\Theta} \cdot r 2\Theta dr = ...$$



----------------▷ 49

7.1.1 Areas for parametric functions

9

This section reguires knowledge of parametric functions which have been introduced in chapter 5 section 5.10.
It may be helpful to rehearse this section before proceeding.
In case of difficulties go back to section 5.10. The cycloid which is discussed in the following has been introduced at the end of section 5.10

Now study **7.1.1 Areas of parametric functions**
Textbook pages 194–195

----------------▷ 10

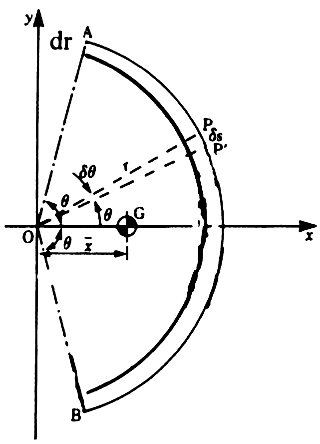
49

$$A\bar{x} = \frac{2R^3}{3} \cdot \sin\Theta$$

Since we calculated A in frame 47 to be $A = R^2\Theta$

We finally get

$$\bar{x} = \frac{2}{3} R \cdot \frac{\sin\Theta}{\Theta}$$



----------------▷ 50

Following the text given, calculate the area of a half circle. The circle of radius R is given in parametric form by 10

$x = R \cdot \cos\varphi$
$y = R \cdot \sin\varphi$

A = ...

Solution -----------------▷ 13

Help -----------------▷ 11

7.4.1 The Theorems of Pappus

50

The Theorems of Pappus show that knowledge of the center of mass often helps to solve problems.

READ **7.4.3 The Theorems of Pappus**
Textbook pages 211–213

-----------------▷ 51

The parameter is φ.
Note: In the text the parameter is t.
The text shows that

$$A = \int_{\varphi_1}^{\varphi_2} g(\varphi)\frac{dx}{d\varphi}d\varphi$$

Given $y = R\sin\varphi$ $x = R\cos\varphi$

Remember that we start integrating at x = 0 which corresponds to $\varphi_1 = \frac{\pi}{2}$ and proceed to $x = R$ coresponding to $\varphi_2 = 0$

Thus we get A = ……………………..

Solution ----------------▷ 13

Further help wanted ----------------▷ 12

11

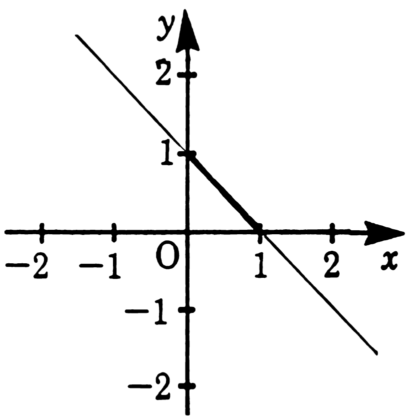
Given a straight line

$y = 1 - x$

We regard the domain $0 \le x \le 1$

Rotating the line around the x-axis generates a cone.
Calculate the surface of the cone.

S = ……………………..



Solution found ----------------▷ 54

Hint welcome ----------------▷ 52

51

Given $A = \int_{\varphi_1}^{\varphi_2} g(\varphi) \cdot \frac{dx}{d\varphi} d\varphi$

12

Since $y = R\sin\varphi$ and $x = R\cos\varphi$

$$\frac{dx}{d\varphi} = -R\sin\varphi$$

Boundaries: We integrate from $x = 0$ and $\varphi = \frac{\pi}{2}$ to $x = \mathrm{R}$ and $\varphi = 0$

Thus we get

$$A = \int_{\frac{\pi}{2}}^{0} R\sin\varphi(-R\sin\varphi)d\varphi = R^2 \int_{0}^{\frac{\pi}{2}} \sin^2 \varphi d\varphi$$

$$\mathrm{A} = R^2 \frac{1}{2}[\varphi - \sin\varphi \cdot \cos\varphi]_0^{\frac{\pi}{2}} = \ldots\ldots\ldots\ldots\ldots$$

----------------▷ 13

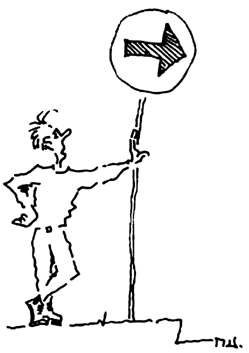
You need the length L of the curve, which in our case is

52

L = ………………..

And you need the ordinate of the center of mass of the curve, which is obvious

$\bar{y} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$



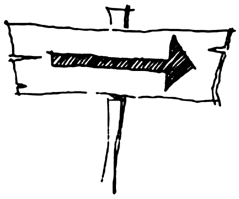
----------------▷ 53

$$A = \frac{R^2\pi}{4}$$

13

A result well known for a quarter of a circle.

In case of difficulties try again to solve the last exercise on your own



----------------▷ 14

$L = \sqrt{2}$

53

$\bar{y} = \frac{1}{2}$

Now you apply the first theorem of Pappus to obtain the surface S of the cone

S = ……………………………

----------------▷ 54

7.1.2 Areas in polar coordinates

14

Polar coordinates have not been introduced yet. They will be introduced in chapter 13, section 13.4. They are quite easy to understand. If you want to study the following section, you have to study section 13.4 before. It is a short section without greater difficulties.
Having done read

7.1.2 Areas in polar coordinates
Textbook pages 195–196

----------------▷ 15

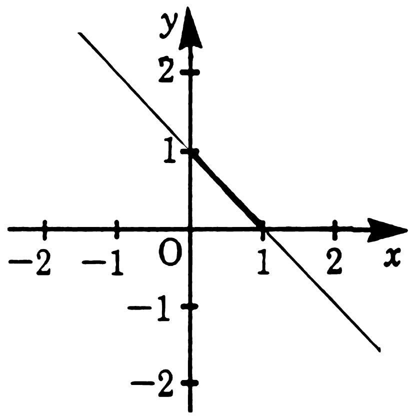
$S = \pi\sqrt{2}$

54

Now let us calculate the volume of the cone generated using the second theorem of Pappus.

Given $y = 1 - x$, domain $0 \le x \le 1$

V = …………………….



Solution found ----------------▷ 57

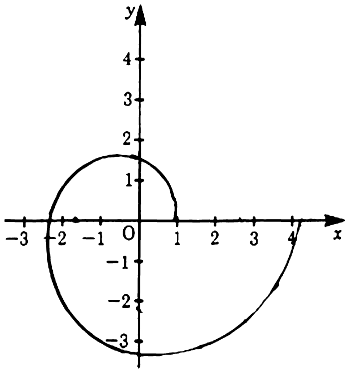
Help wanted ----------------▷ 55

Given a spiral

$r = (1 + 0.5\varphi)$

Calculate the area $0 \le \varphi \le 2\pi$

A = …………



15

Solution found ----------------▷ 17

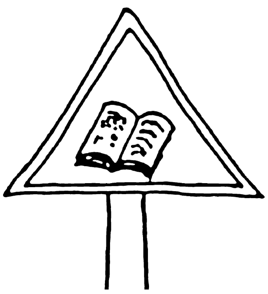
Hint wanted ----------------▷ 16

From the textbook we know that

55

V = …………………….

Hint: In case of doubts try to understand the derivation of this formula reading the section in the textbook again.



----------------▷ 56

It is fundamental to understand the formula. Go back to the textbook and read once more the derivation of the formula $A = \frac{1}{2}\int_{\varphi_1}^{\varphi_2} r^2 d\varphi$ 16

Given $r = (1 + 0.5\varphi)$

The boundaries are $\varphi_1 = 0$ $\varphi_2 = 2\pi$

Now insert *r* into the integral which is easy to solve.

A = …………

-----------------▷ 17

$V = 2\pi\,\bar{y}\cdot A = \pi\int_a^b y^2 dx$ 56

Given is y = 1 - x and a = 0 and b = 1

It is quite easy to insert the given values and to solve the integral.

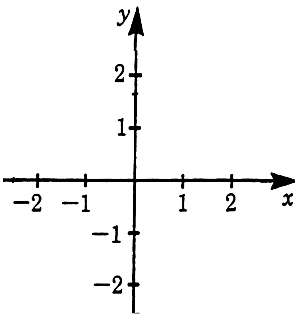
V = ……………………

-----------------▷ 57

$$A = \frac{1}{2}\int_0^{2\pi}(1+0.5\varphi)^2\,d\varphi = \frac{1}{2}\int_0^{2\pi}\left(1+\varphi+\frac{1}{4}\varphi^2\right)d\varphi = \frac{1}{2}\left[\varphi+\frac{\varphi^2}{2}+\frac{1}{4\cdot 3}\varphi^3\right]_0^{2\pi} = \pi+\pi^2+\frac{1}{3}\pi^3$$

17

Calculate the total area bounded by the curve $r = 1+\cos\varphi$ and the $x = axis$
First try to sketch the curve



Now calculate the total area

A = ………………………..

Solution found ----------------▷ 20

Hint needed ----------------▷ 18

$$V = \frac{\pi}{3}$$

57

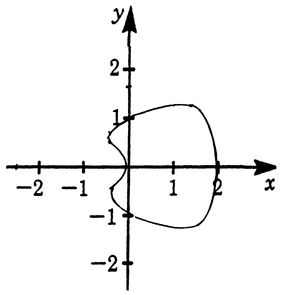
Find the center of mass of a half circle of radius R.

$\bar{y} =$ ....

Solution found ----------------▷ 60

Help welcome ----------------▷ 58

18



The area consists of two parts whose areas are equal.

Since the total area is asked for, you may calculate the first part and double the result.
Or you calculate both parts using the appropriate boundaries.
For the first part you get

$$A_1 = \frac{1}{2}\int_0^{\pi}(1+\cos\varphi)^2\,d\varphi = \frac{1}{2}\int_0^{\pi}\left(1+2\cos\varphi+\cos^2\varphi\right)d\varphi = \ldots\ldots\ldots\ldots\ldots\ldots$$

$$A_1 = \frac{1}{2}\left[\varphi + 2\sin\varphi + \frac{1}{2}(\varphi + \sin\varphi\cdot\cos\varphi)\right]_0^{\pi}$$

Thus

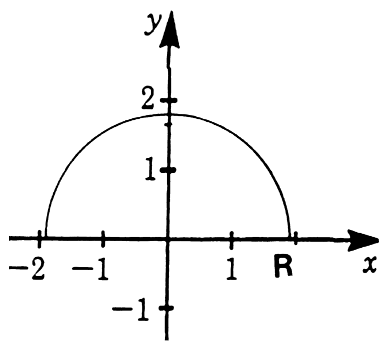
$A_1 = \ldots\ldots\ldots$

----------------▷ 19

58

Wanted is the center of mass of a half circle. Rotating about the x-axis it generates a full sphere.
Pappus´ second theorems states

$V = 2\pi\,\bar{y}\cdot A$



The volume V of a sphere is known to be $V = \frac{4}{3}\pi R^3$ and the area A of a half circle is known to be

$A = \frac{1}{2}R^2\pi$.

Thus we insert and solve for $\bar{y}$

$\bar{y} = \ldots\ldots\ldots\ldots\ldots\ldots$

----------------▷ 59

$A_1 = \frac{1}{2}\pi + \frac{1}{4}\pi = \frac{3}{4}\pi$ 19

Thus the total area is A = …….

Now try the other way. The result must be the same.

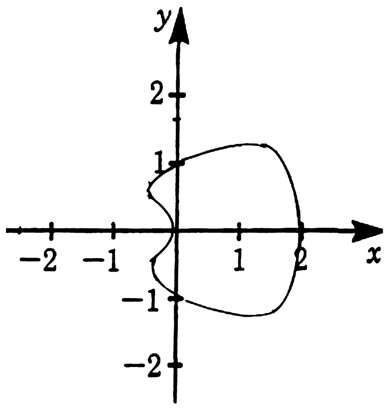
A = …….

--------------------▷ 20

$\bar{y} = \frac{4}{3\pi} \cdot R$ 59

--------------------▷ 60

$A = \frac{3}{2}\pi$



20

----------------▷ 21

7.4.2 Moment of inertia; second moment of area

60

The basic concepts introduced in this section will be known from physics lectures. In case you are not familiar with these concepts take notes of all new concepts and theorems. You will use your notes while working with the exercises and examples.
Since the section is a bit extended, study the first four pages including perpendicular and parallel axis theorems.
If you have difficulties with the first example, you will be given hints in the study guide later on.

READ **7.4.4 Moment of inertia, second moment of area**
Moment of inertia
Perpendicular and parallel axis theorems
Textbook pages 213–218

----------------▷ 61

7.1.3 Areas of closed curves

21

This section will be easy to understand since no new concepts are involved.

READ **7.1.3 Areas of closed curves**
Textbook pages 197–198

----------------▷ 22

61

In the textbook the first example demonstrated the calculation of the moment of inertia of a disc.

Difficulties may have been due to the solution of the integral which is indeed tricky do solve

$$\int_0^{\frac{\pi}{2}} \cos^2 \Theta \sin^2 \Theta d\Theta = \frac{\pi}{16}$$

I knoww how to solve the integral ----------------▷ 65

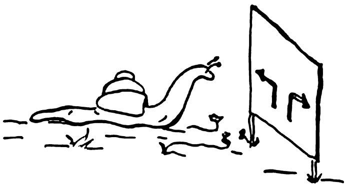
Detailed solution wanted ----------------▷ 62

Calculate the area bounded by the following curves: 22

$y^2 = 4x$ and $x^2 = 6y$

This is an intriguing question at first sight.

A = ……



Solution found -----------------▷ 29

Hints wanted -----------------▷ 23

To solve: 62

$$I_x = 4\rho R^4 h \cdot \int_0^{\frac{\pi}{2}} \cos^2 \Theta \sin^2 \Theta d\Theta$$

We remember the addition theorem. See appendix of chapter 3:

$\sin 2\Theta = 2 \sin \Theta \cdot \cos \Theta$

Inserting into the integral gives

$$I_x = 4\rho R^4 h \int_0^{\frac{\pi}{2}} \quad = \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

-----------------▷ 63

Given $y_1^2 = 4x$ and $x^2 = 6y_2$ **23**

First we transform these equations into a form we can integrate:

$y_1 = ...$ $y_2 = ...$

-----------------▷ **24**

$$I_x = 4\rho R^4 h \int_0^{\frac{\pi}{2}} \left(\frac{\sin 2\Theta}{2}\right)^2 d\Theta$$ **63**

Now we substitute $2\Theta = \varphi \quad 2d\Theta = d\varphi$

New upper boundary: π

$$I_x = 4\rho R^4 h \int_0^{\pi} \frac{\sin^2 \varphi}{4} \frac{d\varphi}{2}$$

You know how to solve this integral:

$$I_x = 4\rho R^4 h \cdot \frac{1}{8} \cdot [..................]_0^{\pi}$$

-----------------▷ **64**

$y_1 = 2\sqrt{x}$ and $y_2 = \dfrac{x^2}{6}$ 24

These forms can be integrated because they are standard.
Thus we get the expressions

$A_1 = \ldots\ldots\ldots$ $A_2 = \ldots\ldots\ldots$

----------------▷ 25

$$I_x = 4pR^4h \cdot \frac{1}{8}\left[\frac{\varphi}{2} - \sin\varphi \cdot \cos\varphi\right]_0^{\pi} = 4\rho R^4 h \frac{1}{8} \cdot \frac{\pi}{2} = \frac{\rho R^4 h \cdot \pi}{4}$$ 64

----------------▷ 65

$$A_1 = \int_{x_0}^{x_1} y_1 dx = 2\int_{x_0}^{x_1} \sqrt{x} dx = \left[\frac{4}{3}x^{\frac{3}{2}}\right]_{x_0}^{x_1}$$ 25

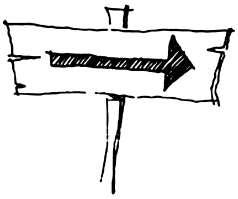
$$A_2 = \int_{x_0}^{x_1} y_2 dx = \int_{x_0}^{x_1} \frac{x^2}{6} dx = \left[\frac{x^3}{3 \cdot 6}\right]_{x_0}^{x_1}$$

Now the boundaries x_0 and x_1 have to be determined. Given $y_1 = 2\sqrt{x}$ and $y_2 = \frac{x^2}{6}$.

The first intersection point is $x_0 = 0$.

For the second intersection point the y-values must coincide. This gives $y_1 = y_2$ and therefore the equation

……. = ……….



-----------------▷ 26

Given a rod of length 2m and $\rho = 0.5 \frac{kg}{m}$. 65

Calculate its moment of inertia if rotated about the

a) center of gravity
b) one end

$I_{center\,of\,gravity}$ = ...

I_{end} = ...

Solution found -----------------▷ 69

Help welcome -----------------▷ 66

$2\sqrt{x_1} = \frac{x_1^{\,2}}{6}$ 26

Thus we get the second limit

$x_1 = ...$

Solution -----------------▷ 28

Further hint -----------------▷ 27

The total mass of the rod is $M = \rho L = 1kg$. 66

Remember how to calculate the moment of inertia about an axis denoting by x the distance of a mass element dm from the axis.

I = …………………

-----------------▷ 67

Given $2\sqrt{x_1} = \frac{x_1^{\ 2}}{6}$ Wanted x_1 27

We transform

$$12 = \frac{x_1^{\ 2}}{\sqrt{x_1}} = x_1^{\frac{3}{2}}$$

Next transformation:

$$12^2 = x_1^{\ 3} = 144$$

$$x_1 = \ldots\ldots\ldots\ldots\ldots$$

-----------------▷ 28

$I = \int x^2 dm$ 67

Since $dm = \rho dx$ you can now calculate

$$I_{center\, of\ gravity} = \int_{-1}^{+1} \rho \cdot x^2 dx = \ldots\ldots\ldots\ldots\ldots$$

-----------------▷ 68

$x_1 = \sqrt[3]{144}$ 28

Now we can insert $x_0 = 0$ and $x_1 = \sqrt[3]{144}$ into the results obtained before (see frame 25)

$A_1 = \left[\frac{4}{3}x^{\frac{3}{2}}\right]_0^{x_1}$ and $A_2 = \left[\frac{x^3}{3\cdot 6}\right]_0^{x_1}$

$A = ...$

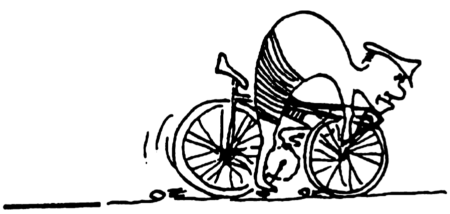
-----------------▷ 29

$$I_x = \rho\int_{-1}^{+1} x^2 dx = \rho\left[\frac{x^3}{3}\right]_{-1}^{+1} = \frac{2\cdot\rho}{3} = \frac{1}{3}kgm^2$$ 68

For a rotation about one end you follow same reasoning. But this time the axis of rotation is shifted to x = 0.

The boundaries of the integral have to be changed.

$I_{end} = ...$



-----------------▷ 69

$A = 8$

29

Again it is up to you to study further sections of applications of integral calculus or to skip these for the time being and to return later on when this material may be needed.

Thus choose

7.2. Lengths of curves -----------------▷ 30

7.3 Surface area and volume of a solid of revolution -----------------▷ 33

7.4 Applications to mechanics -----------------▷ 41

7.4.1 The Theorems of Pappus -----------------▷ 50

7.4.2 Moment of inertia, second moments of area -----------------▷ 60

---

a) $I_{center\,of\,gravity} = \frac{1}{3} kgm^2$

69

b) $I_{end} = \frac{8}{3}\rho = \frac{4}{3} kgm^2$

Let us now calculate the moment of inertia of the rod using the parallel axis theorem (Steiner´s Theorem).

Given the same rod of length 2m whose mass per meter is $\rho = 0.5 \frac{kg}{m}$

We just calculated its moment of inertia for an axis through its center of gravity to be $I_{center} = \frac{1}{3} kgm^2$

Calculate its moment of inertia if it is rotated around an end.

$I_{end} = \ldots\ldots\ldots\ldots\ldots\ldots$

-----------------▷ 70

7.2 Lengths of curves

30

This section needs careful reading. Try to follow the transformations, executing them on a separate sheet. Basically we apply the theorem of Pythagoras to a small triangle.

READ **7.2. Lengths of curves**
Textbook pages 198–202

----------------▷ 31

$I_{end} = \frac{4}{3} kgm^2$

70

It is the same result calculated before.

Now proceed to the second part of section 7.4.4 which will be of interest to civil engineers.

READ **7.4.4 Moments of inertia; second moment of inertia**
Radius of gyration
Second moment of inertia
Center of pressure
Textbook pages 218–223

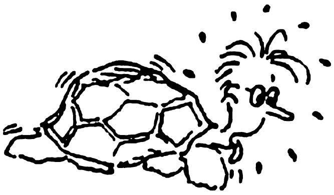
----------------▷ 71

31

Try to solve the first example given in the textbook again, this time on your own without using the book.
Calculate the length of a circle of radius R.

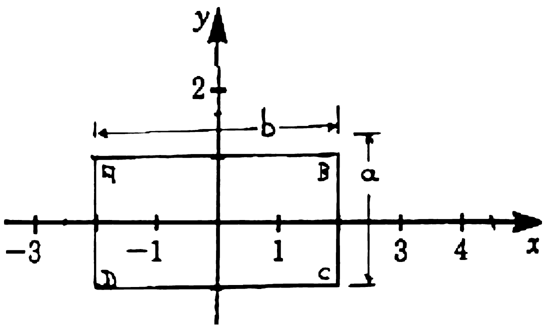
L = ………………….

In case of difficulties solve the problem using the textbook



----------------▷ 32

71



Given a rectangle ABCD with sides a = 2 and b = 4 and mass $\rho = 0.1kg$ per square unit.
Determine the moment of inertia about the z-axis.

I_z = ...

Solution found ----------------▷ 75

Hints welcome. The problems seems intriguing to me ----------------▷ 72

$L = 2\pi R$ 32

Now again it is up to you to choose:

7.3 Surface area and volume of a solid of revolution -----------------▷ 33

7.4 Applications to mechanics -----------------▷ 41

7.4.3 The Theorems of Pappus -----------------▷ 50

7.4.4 Moment of inertia, second moment of area -----------------▷ 60

If you want to skip all sections for the time being Proceed to chapter 8

The given problem can be solved using the perpendicular axis theorem. 72
Determining the moments of inertia about the x-axis and the y-axis is quite easy.

$I_x =$ ....

$I_y =$ ....

Solution found -----------------▷ 75

Detailed solution -----------------▷ 73

7.3 Surface area and volume of a solid of revolution

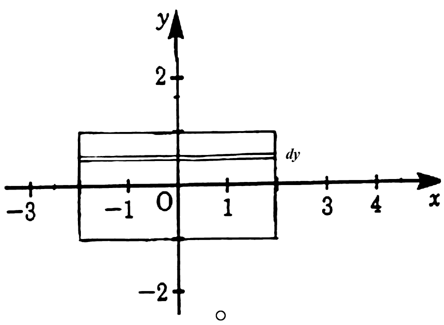
33

This section requires knowledge of the preceeding section i.e. lengths of curves.
It is of interest mainly to engineers and consists of worked out examples. It is worthwhile to follow the examples given in the textbook calculating on a separate sheet.

READ **7.3 Surface Area and Volume of a solid of Revolution**
Textbook pages 202–208

----------------▷ 34

73



Regarding a strip parallel to the x-axis

$dm = \rho \cdot 4 \cdot dy$

$dI = y^2 \cdot dm = y^2 4\rho dy$

$I_x = 4\rho \int_{-1}^{+1} y^2 dy = 4\rho \left[\frac{y^3}{3}\right]_{-1}^{+1} = ...$

Now determine the moments of inertia about the x-axis following the same reasoning regarding a strip parallel to the y-axis

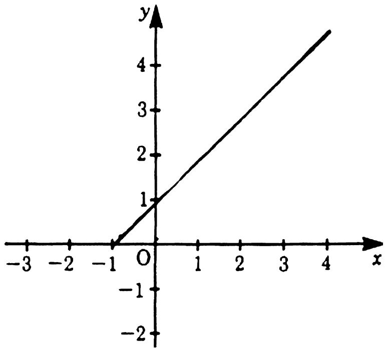
$dm = ...$

$I_y = ...$

----------------▷ 74

Given a straight line

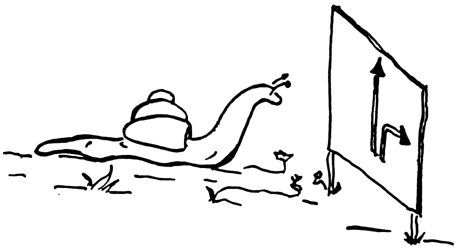
$y = 1 + x$



34

It is rotated about the x-axis.
Calculate the area of the surface thus generated between the boundaries $x_1 = a$ and $x_2 = b$.

A = ……………………



Solution found ----------------▷ 38

Hints wanted ----------------▷ 35

$I_x = \frac{8}{3}\rho$

74

$dm = \rho \cdot 2 \cdot dy$

$I_y = \frac{32}{3}\rho$

We remember the perpendicular axis theorem

$I_z = I_x + I_y$

Thus regarding our results we obtain

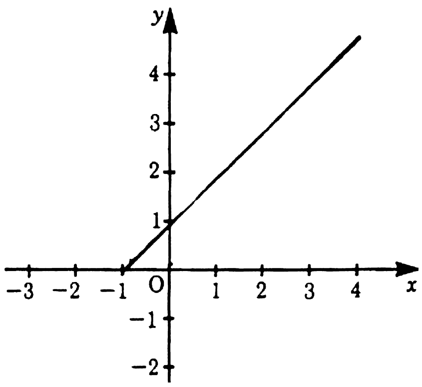
$I_z =$ ....

----------------▷ 75

Given $y = 1 + x$

Look into the textbook and repeat the formula for the surface generated:

A = …………………..



35

----------------▷ 36

75

$$I_z = \frac{40}{3}\rho$$

Now try to solve the example given on page 220 (fig. 7.26) in the textbook.
In case of difficulties look at the detailed calculation given there.

----------------▷ 76

36

$$A = 2\pi \int_a^b y \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx$$

Given $y = 1 + x$

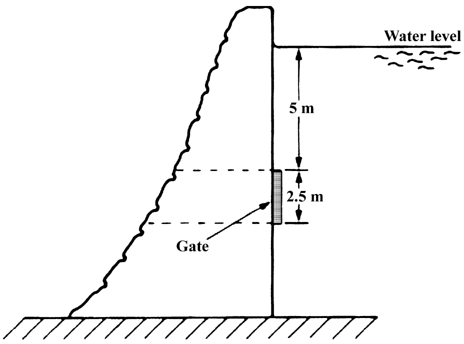
Inserting you get

A = …………………..

-----------------▷ 37

76

The exercise 22 at the end of this chapter reads as follows:



A rectangular plate of base 5m and height 8m is immersed in a lake. Calculate the total pressure on the plate and the depth of the centre of pressure if the plate is vertical.
Density of water = 1000 kg/m^3

Try to answer the first question. We denote the depth x and the width of the gate a
Total pressure on the gate

F = ……………………..

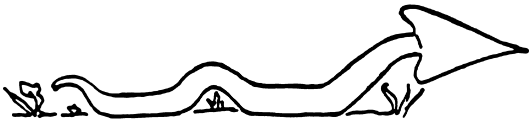
Solution found -----------------▷ 78

Hints wanted -----------------▷ 77

$$A = 2\pi \int_a^b (1+x) \cdot \left(2^{\frac{1}{2}} \right) dx$$ 37

Now it is quite easy to solve the integral

A = ……………………



----------------▷ 38

We regard a horizontal strip of width a = 5m and height dx in the depth x: 77
The force on the strip is:

$$dF = \rho \cdot g \cdot x \cdot dx \cdot a$$

The total force on the gate:

$$F = \rho \cdot g \int_5^{7.5} x dx \cdot a$$

Now solve the integral, insert the boundaries and calculate the result:

F = ………………….

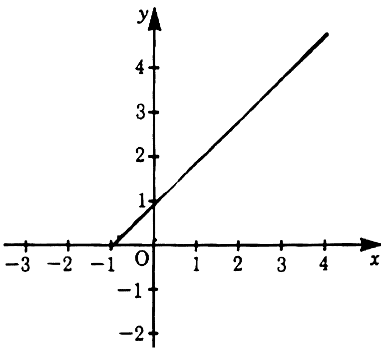
----------------▷ 78

$$A = 2 \cdot \sqrt{2}\pi \left[x + \frac{x^2}{2} \right]_a^b = 2\sqrt{2}\pi \left[b + \frac{b^2}{2} - a - \frac{a^2}{2} \right]$$ 38

Given the same straight line
$y = 1 + x$

It is rotated about the x-axis.
Give the volume of the solid generated. The boundaries are $x_1 = a$, $x_2 = b$

V = …………………………..



Solution found -----------------▷ 40

Hint wanted -----------------▷ 39

F = 230kN 78

Now solve the second question. Calculate the position x_C of the center of pressure.
We regard a horizontal strip of width a = 5m and height dx in the depth x: Its moment regarding the line of the water level is

$$dF \cdot x = a \cdot dx \cdot \rho \cdot gx^2 = a\rho gx^2 dx$$

Thus

$$F \cdot x_C = a\rho g \int_5^{7.5} x^2 dx$$

$F \cdot x_C =$ ....

-----------------▷ 79

39

Proceed as calculating the exercises before.
Look into the textbook for the formula, insert and integrate the standard integrals.

V = ……………………………..

-----------------▷ 40

Please continue on page 1
(bottom half)

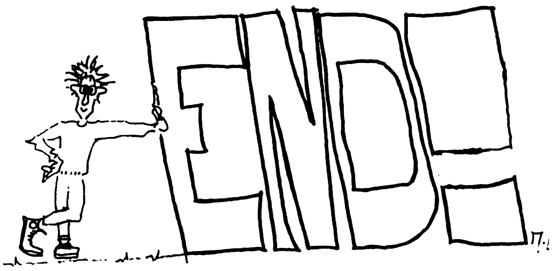
79

$$F \cdot x_C = a\rho g \left[\frac{x^3}{3} \right]_5^{7.5} \cong 1456 kNm$$

Using this result we obtain

$$x_C = 6.33m$$

By now you have reached the end of this chapter which posed some tricky calculations.
But after all you mastered the stuff successfully: Congratulation!



of chapter 7