Solutions to Problems

Chapter 1

1. Determine analitically the melt temperature at surface for a wet peridotite having 0.05% water content;

Solution:

From Figure 1.4, we see that the melt temperature at the 0.05% wt water peridotite solidus (104 km depth) is 1332°C. Considering that $(\partial T/\partial z)_S \cong 1$ °K/km for a basaltic magma, the melt temperature at surface will be $T(0) \cong 1228$°C.

2. Solve Equation 1.8 assuming a linear decrease of the coefficient of thermal expansion α with the depth z;

Solution:

For a linear decrease of the coefficient of thermal expansion, α, with the depth, z, we have that:

$$\alpha(z) = az + b$$

In this instance, Equation 1.8 assumes the form:

$$\left(\frac{\partial T}{\partial z}\right)_S = \frac{(az+b)gT(z)}{c_p}$$

Solving for z gives the following general solution:

$$T(z) = T(0)\exp\left(\frac{gz(az+2b)}{2c_p}\right)$$

Using the values for α at 0.1 MPa and 10 GPa, we obtain: $a = -5\times10^{-8}$ K^{-1}m^{-1} and $b = 5\times10^{-5}$ K^{-1}.

3. Determine the thickness of new oceanic crust formed from a 100 km wide melting regime, assuming that the degree of melting increases linearly from nearly zero at z = 50 km to 24% close to the Earth's surface and that the fraction of retained melt is 0.1%/km;

Solution:

The average degree of melting is $\bar{F}$ = 0.12. However, in 50 km 5% of the generated melt is retained. Therefore, the thickness of the new oceanic crust will be:

$$H = (1 - 0.05)\times\frac{w\bar{F}}{\pi} \cong 3.6\,\mathrm{km}$$

Chapter 2

1. An Eulerian reference frame is a geocentric reference frame obtained rotating an Euler pole to the North pole. Determine the equation of transformation from geographic to Eulerian latitude;

Solution:

If an Euler pole has geographic coordinates (θ_e, ϕ_e), a transformation of the geographic coordinates of a point, (θ,ϕ), to the Eulerian reference frame is obtained by rotating the corresponding position vector $\boldsymbol{r}(\theta,\varphi)$ about an Equatorial pole placed at (0°,ϕ_e + 90°) by an angle $-\theta_e$.

2. The number of edges in a circuit with p plates is always $e = p - 1$. Explain why;

Solution:

A plate circuit is a tree containing p nodes. When p = 2, we have clearly $e = 1 = p - 1$. Now we can prove by induction that $e = p - 1$ for any tree. Let us assume that the equality is true up to some order $p - 1$ and consider a tree having p nodes. If we remove from this tree the edge that links two adjacent nodes x and y, we obtain two disjoint subtrees, T_1 and T_2, each having order $p_i < p$. Therefore, we have:

$$e_i = p_i - 1 \ ; \ i = 1{,}2$$

Consequently, taking into account that $p = p_1 + p_2$, we obtain:

$$e = e_1 + e_2 + 1 = p_1 + p_2 - 1 = p - 1$$

This proves that $e = p - 1$ for any tree.

3. Show that the relation $e = p - 1$ is compatible with (2.34), e and p being the number of edges and the number of plates in a circuit;

Solution:

To prove that $e = p - 1$ is compatible with (2.34), we must show that $e \le c$, where c is the number of conjugate boundaries. Substituting $e = p - 1$ in (2.34) gives:

$$b = 3(e - 1)$$

Therefore, by (2.55) we have:

$$e = \frac{1}{3}b + 1 = \frac{1}{2}c + 1 < c$$

This proves that the identity is compatible with (2.34).

4. Given the three–plates system formed by the Pacific, North American, and Juan de Fuca plates, determine the relative velocity vector of Juan de Fuca with respect to N. America at (46.5°N,125.8°W) using the data in Table 2.5;

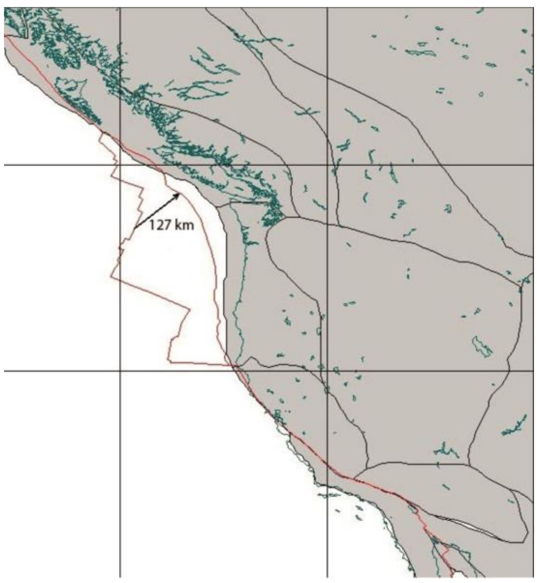
Solution:

From Table 2.5, we select the instantaneous Euler vectors of the Juan de Fuca and North American plates in the NNR reference frame. They are, respectively, $\boldsymbol{\omega}_{JDF}$ = (0.006529,0.011291,–0.010270) and $\boldsymbol{\omega}_{NAM}$ = (0.000612,–0.003564,–0.000292). Therefore, the Euler vector of relative motion of Juan de Fuca with respect to N. America is: $\boldsymbol{\omega}_{JDF,NAM} = \boldsymbol{\omega}_{JDF} - \boldsymbol{\omega}_{NAM}$ = (0.005917,0.014855,–0.009978). To determine the relative velocity vector at (46.5°N,125.8°W), we convert this geographic location into a position vector using (2.27). Assuming an Earth mean radius R = 6371 km, the result is: $\boldsymbol{r}$ = (–2565.3,–3556.9,4621.4). The corresponding linear velocity $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$ will be given by: $\boldsymbol{v}$ = (33.2,–1.7,17.1). Therefore, the velocity of convergence is v = 37.3 mm/yr directed N48E.

5. Assuming that the spreading asymmetry is zero along the Juan de Fuca Ridge, how long time is required for a point at (46.9°N,129.4°W) along the ridge to enter the Cascadia Trench and what is the predicted location of ridge subduction?

Solution:

From the MORVEL velocity model (Table 2.3), we can easily determine the velocity vector of the Juan de Fuca plate with respect the North America plate at (46.9°N,129.4°W). We obtain: v = 41.32 mm/yr, direction N41E. The distance between the Juan de Fuca Ridge and the Cascadia Trench along this direction is ΔX = 127 km, and the velocity vector does not change significantly along the path (see figure). Therefore, the time that is required for subduction of the ridge segment is $\Delta T = \Delta X / v \cong 3\,\mathrm{Ma}$.

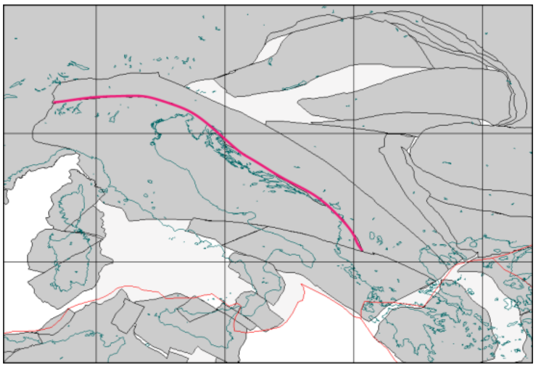


6. Subduction of the Capricorn plate beneath Sundaland along the Sumatra Trench is highly oblique. Strike–slip motion along the Sumatran Fault, which is parallel to the trench in the forearc region, determines partitioning of such oblique subduction into a trench–normal component and a trench–parallel component. Determine the slip rate and the sense of shear along the Sumatran Fault at (2.5°S,101.5°E);

Solution:

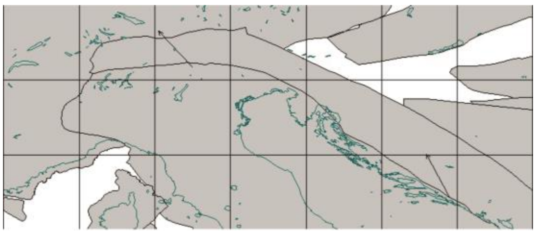
The relative velocity vector between the Capricorn plate and the Sundaland plate at (2.5°S,101.5°E) is v = 57.68 mm/yr, N20E. To obtain the trench–parallel component of this vector, we project $\boldsymbol{v}$ onto the local strike of the Sumatra Trench, which is N59W. We obtain a slip rate of 11 mm/yr and the predicted sense of shear along the Sumatran Fault is right lateral.

7. The Periadriatic Line in northern Italy and Croatia is a wide E–W and NW–SE structure that accommodated strike–slip motion between Africa and Europe in the geologic past (see figure). What would be the style of this fault at (46.4°N,11°E) and (42.8°N,17.8°E) if it were a present day plate boundary?

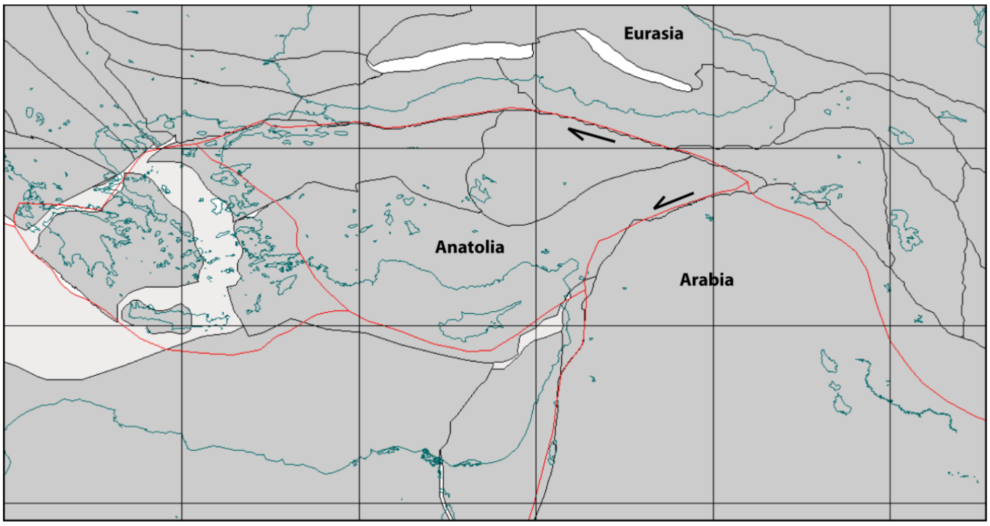


Solution:

From the MORVEL velocity model (Table 2.3), we obtain the velocity vectors of Africa relative to Europe at (46.4°N,11°E) and (42.8°N,17.8°E) (see figure). The style of the Periadriatic Line at these locations would be that of a sinistral transpressional fault.

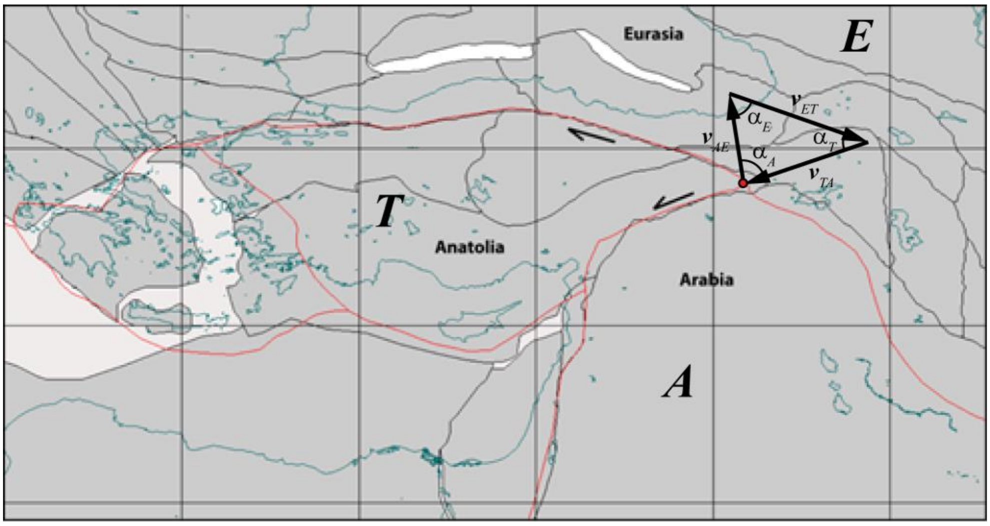


8. Anatolia is a small microplate between Arabia and Europe in the eastern Mediterranean, whose N and SE boundaries are transcurrent faults (see figure). Starting from the relative velocity of Arabia with respect to Europe, calculate the westward escape velocity of this microplate along its strike–slip boundaries;



Solution:

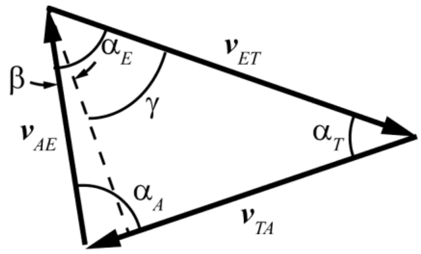
Let $\boldsymbol{v}_{AE}$, $\boldsymbol{v}_{ET}$, and $\boldsymbol{v}_{TA}$ be the relative velocity vectors of Arabia relative to Europe, Europe relative to Anatolia, and Anatolia relative to Arabia, respectively, at the triple junction of the three plates (see figure).



By (2.36), we have that:

$$\boldsymbol{v}_{AE} + \boldsymbol{v}_{ET} + \boldsymbol{v}_{TA} = 0$$

Let α_A, α_E, and α_T the angles of the velocity triangle at the triple junction.



We have:

$$\beta = 90 - \alpha_A \ ; \ \gamma = \alpha_A + \alpha_E - 90$$

Therefore,

$$v_{ET} = \frac{v_{AE} \sin \alpha_A}{\cos \gamma} = \frac{v_{AE} \sin \alpha_A}{\sin(\alpha_A + \alpha_E)}$$

$$v_{TA} = v_{ET} \cos \alpha_T + v_{AE} \cos \alpha_A = v_{AE} \left[\frac{\sin \alpha_A \cos \alpha_T}{\sin(\alpha_A + \alpha_E)} + \cos \alpha_A \right]$$

The strike of the eastern segment of the North Anatolian fault (NAF) is N65W, while the strike of the East Anatolian fault (EAF) is N115W. The relative velocity vector of Arabia with respect to Europe at the triple junction between Arabia, Eurasia and Anatolia is v_{AE} = 22.57 mm/yr, N9W. Therefore, α_T = 50°, α_A = 74°, and α_E = 56°. Consequently, v_{ET} = 28.32 mm/yr and v_{TA} = 24.43 mm/yr.

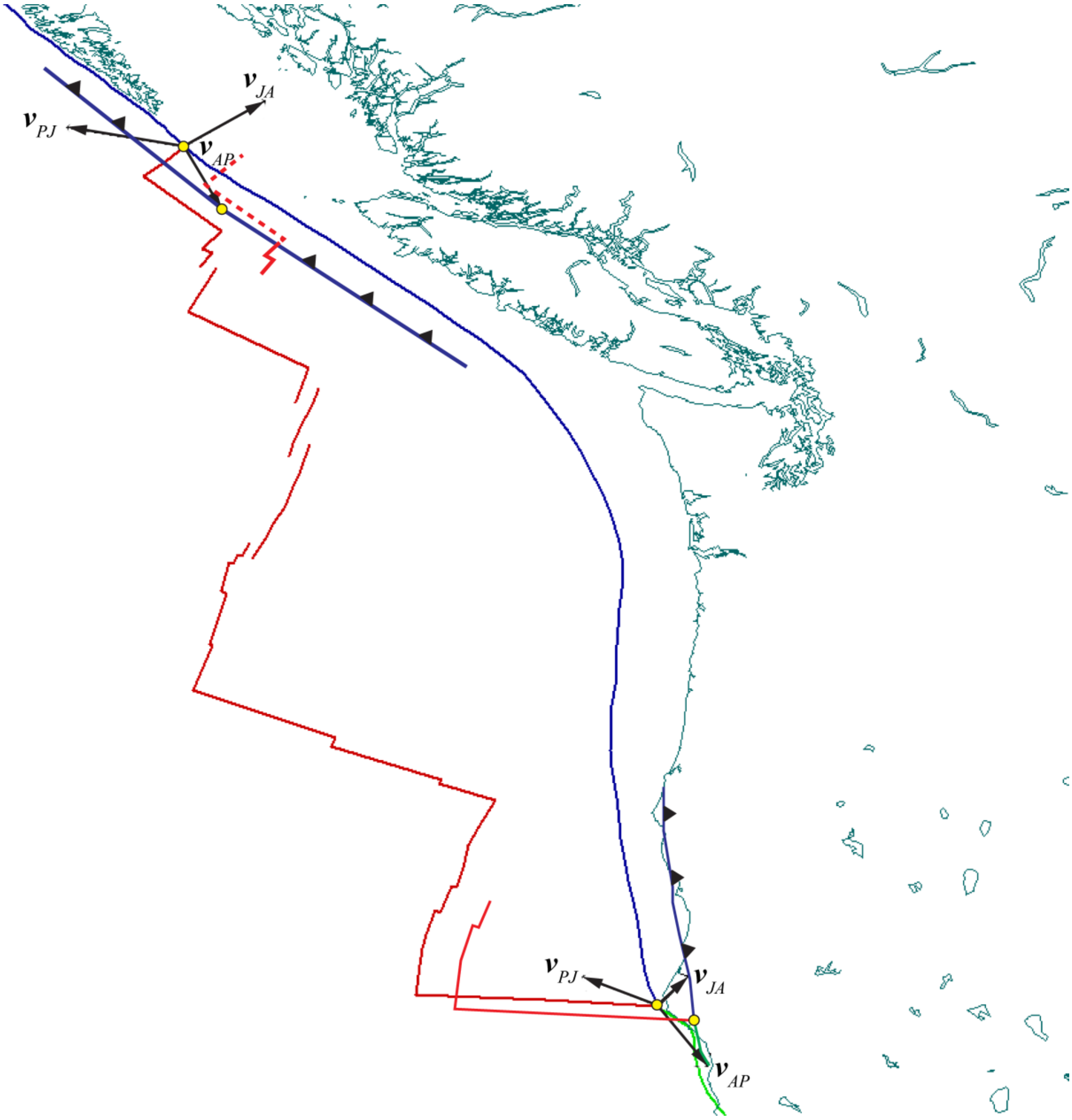
9. Determine the evolution of the Pacific–N. America–J. de Fuca triple junctions in the Pacific reference frame, and describe the geological setting around the region where the corresponding plate boundaries meet;

Solution:

The northern triple junction is a TTR triple junction at (51.52°N,130.66°W), while the southern one is a TRF triple junction at (40.32°N,124.48°W).

The velocity vectors at the northern TJ are (see figure below):

$$v_{AP} = 49.84 \text{ mm/yr, N160E}$$
$$v_{PJ} = 54.66 \text{ mm/yr, N75W}$$
$$v_{JA} = 48.67 \text{ mm/yr, N47E}$$



Evolution of the Pacific–N. America–J. de Fuca triple junctions (yellow circles). Red lines form the Pacific (*P*) – J. de Fuca (*J*) boundary. The blue line is the convergent boundary between J. de Fuca and N. America (*A*) (Cascadia subduction zone). The green line is the southern strike–slip boundary between N. America and Pacific.

At the southern TJ, the velocity vectors are:

$$v_{AP} = 50.98 \text{ mm/yr, N149E} \;;\; v_{PJ} = 47.29 \text{ mm/yr, N63W} \;;\; v_{JA} = 27.35 \text{ mm/yr, N35E}$$

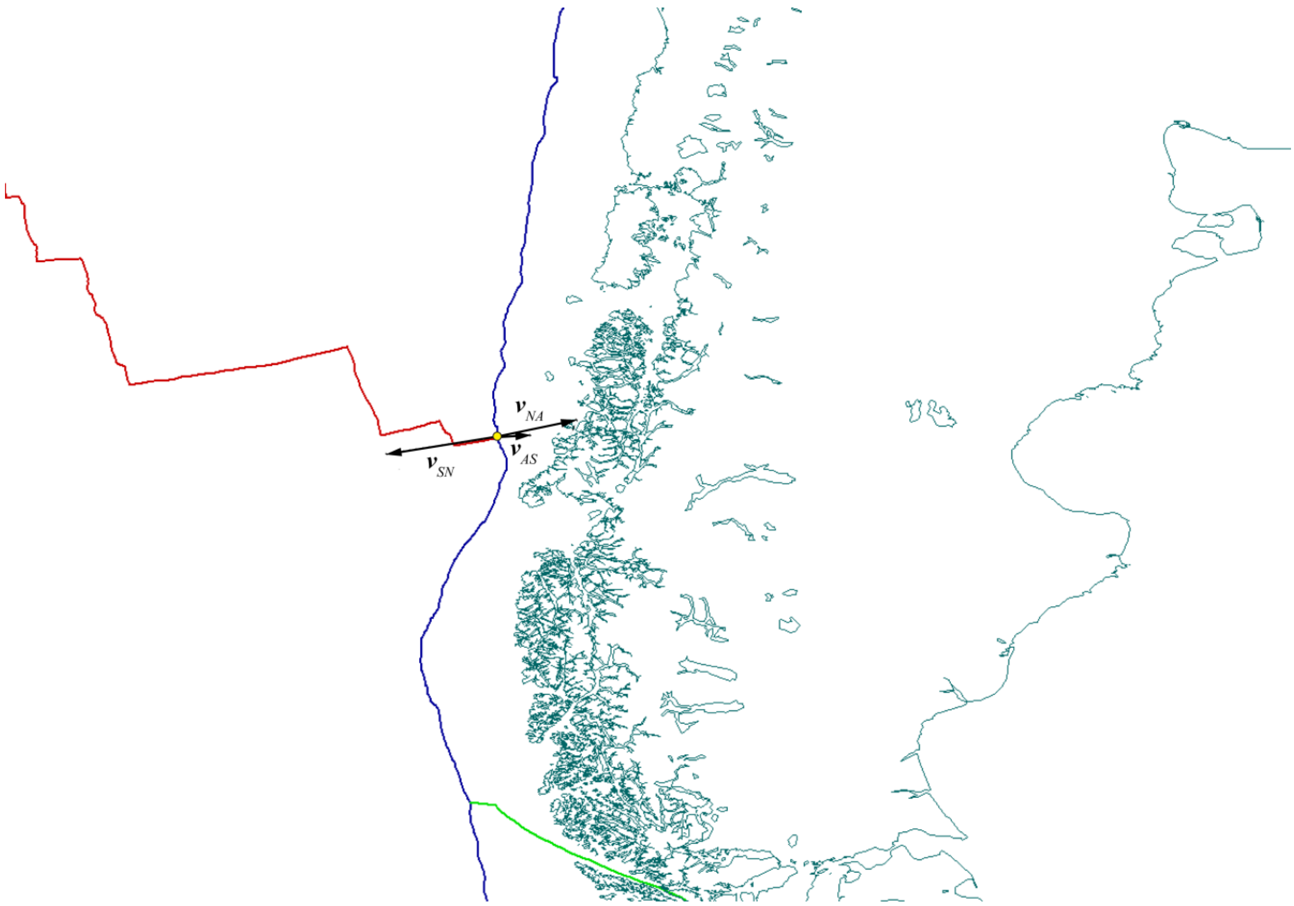
As shown in the figure above, the northern TJ migrates southeastward and this motion is accompanied by ridge subduction. The region where a spreading segment is subducted is characterized by absence of subduction! (think about this).The southern TJ migrates ESE along the strike–slip boundary. In this instance, there is no ridge subduction.

10. Determine the time interval of stability of the triple junction between Nazca, Antarctica, and S. America, and the subsequent migration path;

Solution:

This is a TTF triple junction at (45.84°S,75.98°W). The relative velocity vectors between Nazca (*N*), Antarctica (*A*), and S. America (*S*) at the triple junction are (see figure):

$$v_{AS} = 20.57 \text{ mm/yr, N87E} \; ; \; v_{SN} = 72.35 \text{ mm/yr, N103W} \; ; \; v_{NA} = 52.22 \text{ mm/yr, N73E}$$



The triple junction is stable and relative motions determine subduction of a transform segment of the Antarctica – Nazca ridge. The length of this segment is $\Delta X \cong 56$ km. Therefore, the closest spreading segment will reach the trench after a time interval $\Delta T = \Delta X / v_{NS} \cong 0.77$ Myrs.

Chapter 3

1. Calculate the maximum diamagnetic magnetization of 1 kg Hydrogen in an external field B with magnitude $B = 0.1$ T;

Solution:

By (3.50), we have that the diamagnetic magnetization is given by:

$$\boldsymbol{M} = -\frac{NZe^2}{6m_e V}\langle r^2\rangle \boldsymbol{B}_{ext}$$

where $Z = 1$ in the case of Hydrogen, $m_e = 9.10938291 \times 10^{-31}$ kg, $e = 1.6021765 \times 10^{-19}$ C, $<r> = a_0$ (Bohr radius) $= 5.2917721092 \times 10^{-11}$ m. The molar mass of H is 1.00794 g/mol. Therefore, 1 kg hydrogen contains $n = 992.122546977$ mol, that is, $N = 6.02214179 \times 10^{23} \times 992.122546977 = 5.97 \times 10^{26}$ atoms. To determine the volume V, we assume STP conditions ($T = 25$°C, $P = 0.1$ MPa). In this instance, by the ideal gas law we have:

$$V = \frac{NkT}{P}$$

where $k = 1.3806488(13)\times10^{-23}$ [JK^{-1}] is the Boltzmann constant. Therefore, we have: $V \cong 24.55$ m^3. Finally, we obtain $M \cong 3.2 \times 10^{-5}$ A/m.

2. The saturation magnetization per unit mass for hematite is $M_s \cong 0.48$ A m^2 kg^{-1}. Assuming that a hematite assemblage is formed by 10 μm SD grains, determine the magnetic moment of a single grain at saturation;

Solution:

The density of hematite is $\rho = 5.26\times10^{3}$ kg m^{-3}. A spherical SD grain having diameter $D = 10$ μm has volume $V = (1/6)\pi D^3 = 5.24\times10^{-16}$ m^3. Therefore, its mass is $m = \rho V = 2.76\times10^{-12}$ kg. Finally, the magnetic moment of a single grain at saturation will be $mM_S = 1.32\times10^{-12}$ A m^2.

3. A compass needle having volume $V = 20\times10^{-9}$ m^3 has magnetization $M = 300$ kA/m. Assuming that at some location the Earth's magnetic field has strength $F = 40000$ nT and inclination $I = 35°$, determine the maximum torque exerted on the needle;

Solution:

The needle has magnetic moment $m = MV = 6\times10^{-3}$ A m^2. By (3.15), the maximum torque exerted on the needle is $N_{max} = mF = 2.4\times10^{-7}$ Nm.

4. Determine the maximum magnitude of dipolar interaction energy for hematite;

Solution:

The cell parameter for hematite is $a = 5.038$ Å, while the number of nearest in–plane neighbors is $n = 3$ (see Fig. 3.19). Therefore, by (3.76) we have that:

$$U_{m,\max} = n\frac{\mu_0}{4\pi}\frac{\mu_B^2}{a^3} = 2.02\times10^{-25}\ \text{J}$$

Chapter 4

1. Write a computer program to convert from geographic to geomagnetic coordinates and vice versa;

Solution:

This program can be designed using standard rotation matrices (2.18). Let $\boldsymbol{R}(\theta,\phi,\omega)$ be the rotation matrix about the axis having pole at (θ,ϕ) by angle ω. Let (θ_0,ϕ_0) be the geographic coordinates of the geomagnetic North Pole. We know (see §2.3) that the x–axis of the geomagnetic coordinate system is chosen in such a way that the prime meridian passes through the geographic South Pole. Therefore, the direct transformation from geographic to geomagnetic coordinates can be performed by the following transformation:

$$\boldsymbol{T}(\theta_0,\phi_0) = \boldsymbol{R}(90,0,-\phi_0)\boldsymbol{R}(0,\phi_0+90,-\theta_0)$$

Therefore, the inverse transformation from geomagnetic to geographic coordinates is given by:

$$\boldsymbol{T}^{-1}(\theta_0,\phi_0) = \boldsymbol{R}(0,\phi_0+90,\theta_0)\boldsymbol{R}(90,0,\phi_0)$$

2. Determine an expression for the field components starting from the spherical harmonic expansion (4.93);

Solution:

By (4.73) we have that:

$$\begin{cases} X = -B_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial \theta} \\ Y = B_\phi = \dfrac{1}{r\sin\theta}\dfrac{\partial V}{\partial \phi} \\ Z = -B_r = -\dfrac{\partial V}{\partial r} \end{cases}$$

Therefore, from the spherical harmonic expansion (4.93) we obtain:

$$X = -\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[g_n^m \cos m\phi + h_n^m \sin m\phi\right]\frac{\partial P_n^m}{\partial \theta}$$

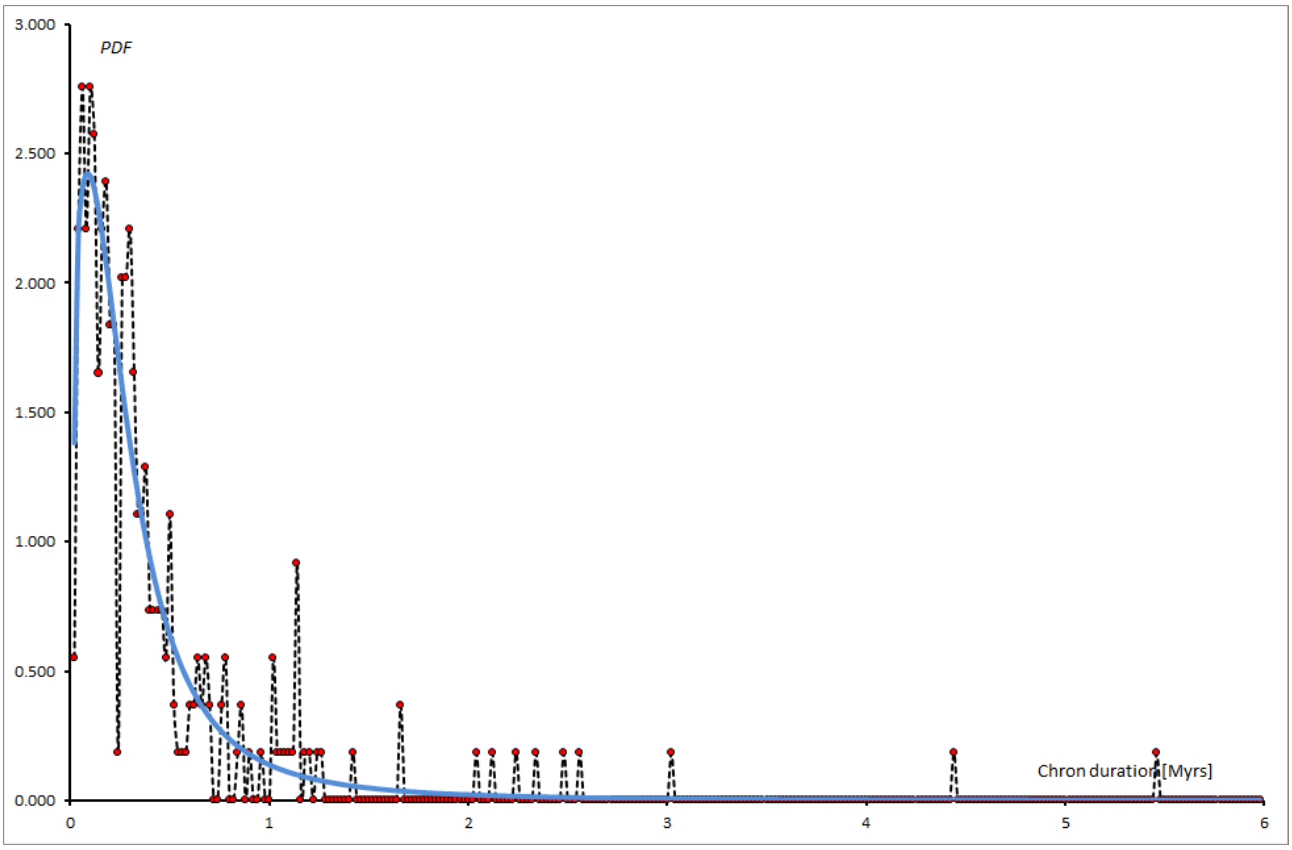
$$Y = -\frac{1}{\sin\theta}\sum_{n=1}^{\infty}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n} m\left[-g_n^m \sin m\phi + h_n^m \cos m\phi\right]P_n^m(\cos\theta)$$

$$Z = \sum_{n=1}^{\infty}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[g_n^m \cos m\phi + h_n^m \sin m\phi\right]P_n^m(\cos\theta)$$

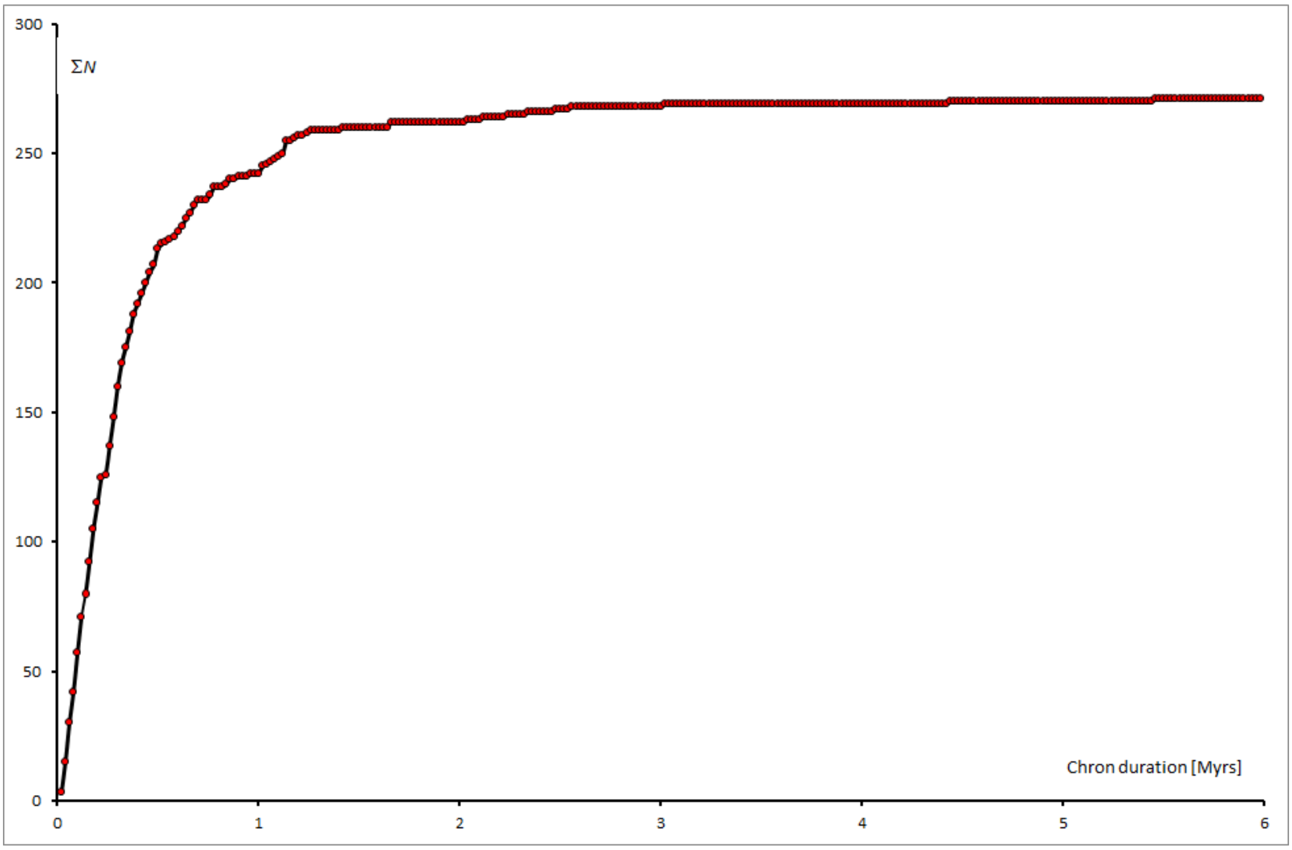
3. Download the MS Excel worksheet CK-GTS2004.xls, containing a combined time scale Cande & Kent (1995) – Gradstein et al. (2004). Plot the frequency distribution and the cumulative frequency distribution of the lengths of polarity intervals. What kind of distribution results?

Solution:

Using 0.02 Myr bins we obtain the following PDF of the frequency distribution (dashed line), which can be fitted, for example, by a four–parameters Dagum distribution PDF (blue line):



The data used to build this plot are in the file *Solution 4.3.xls*. The corresponding cumulative distribution is shown in the plot below.

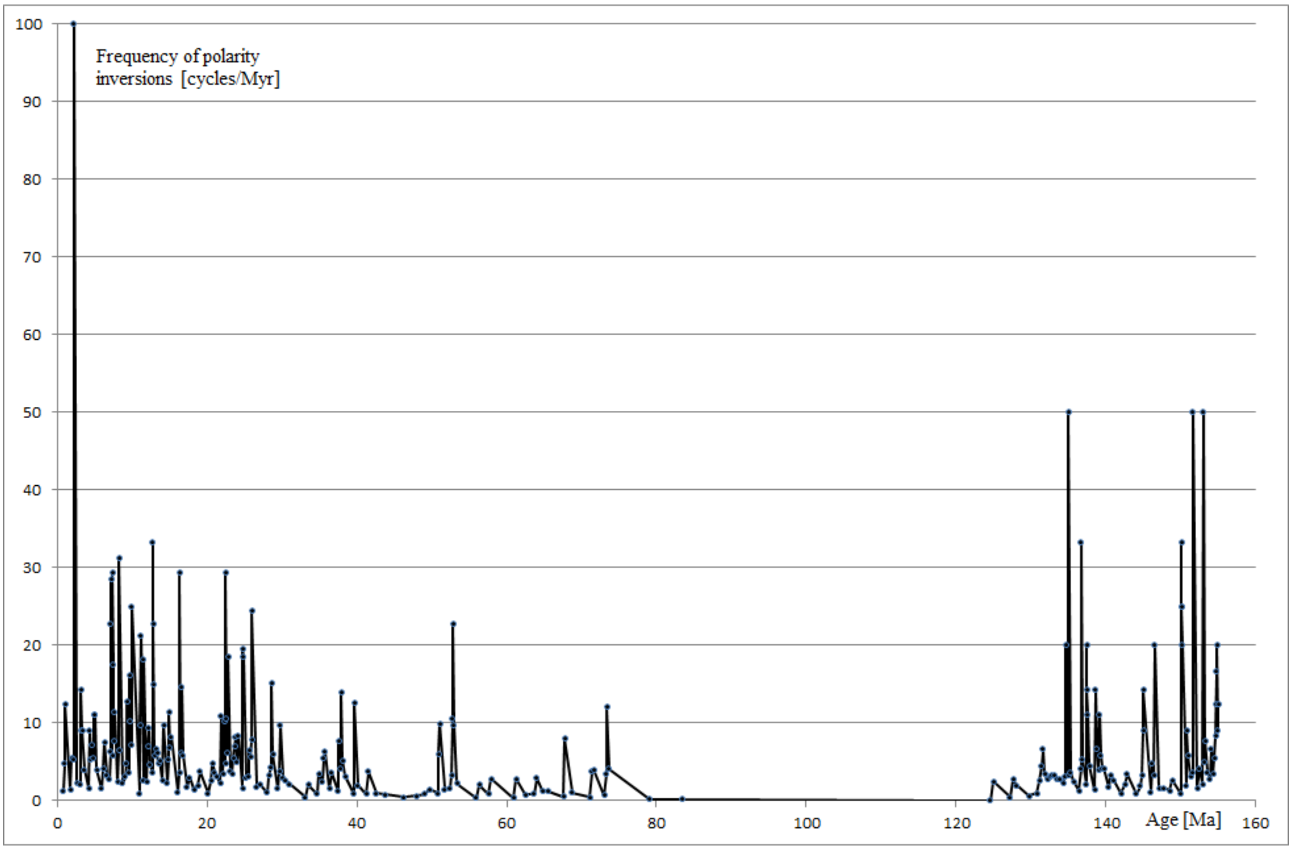


Note that other authors used different fitting PDFs. For example, Cox (1968) used an exponential distribution, whereas Lowrie & Kent (2004) used the gamma function.

4. Perform a Fourier analysis of the time scale CK-GTS2004.xls, plotting the power density as a function of the frequency of inversions;

Solution:

The first step is to plot the number of cycles per Myr (frequency of polarity inversions) vs time (see Excel file *Solution 4.4.xlsx*). This is shown below. It is obtained taking the reciprocal of the chron duration ΔT. Then, we perform a spectral analysis using any software tool that is capable to perform fast Fourier transform (FFT) analysis. Given a time series $x = x(t)$, the power spectral density (PSD) is a function that shows the frequency distribution of the signal power, that is, the strength of the signal variations as a function of the frequency. Therefore, it shows the frequencies at which variations are stronger. PSD is useful when you want to identify oscillatory signals in a time series and determine their amplitude.



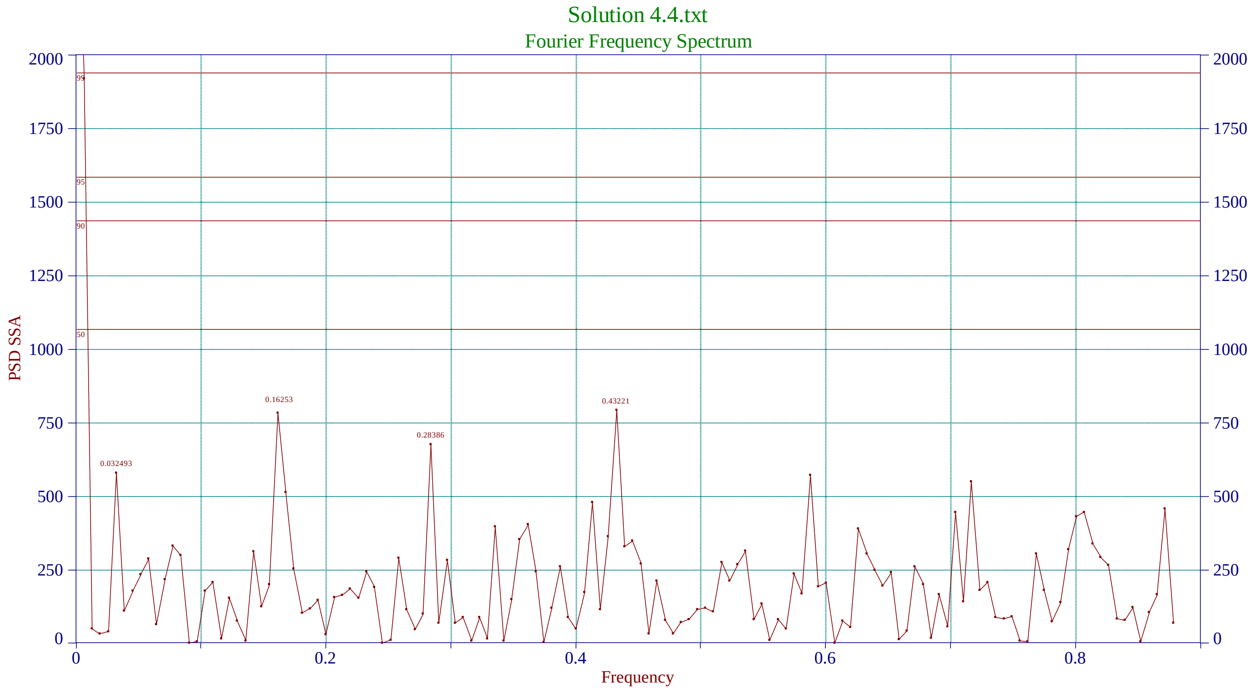
It is calculated taking first the autocorrelation function of the signal:

$$\phi(t) = \int_{-\infty}^{+\infty} x(\tau)x(t+\tau)d\tau$$

Then, we take the real part of the Fourier transform of this function:

$$\Phi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \phi(t)\cos\omega t dt$$

The plot below shows the PSD associated with the geomagnetic polarity inversions.



5. Describe quantitatively the motion of a charged particle in the auroral zone, where the magnetic field lines are close to being vertical and the field intensity increases as approaching the Earth;

Solution:

As shown in Chapter 3, the trajectory is helical, with frequency given by the cyclotron frequency (3.8). However, differently from the example in §3.1, the magnitude of ***B*** now depends from z, thereby the cyclotron frequency $\Omega = qB/m$ is now a decreasing function of the distance from the Earth's surface. Therefore, as a charged particle moves towards the Earth, it will increase its cyclotron frequency and generation of electromagnetic radiation.

6. How does the ring current influences the Earth's magnetic field, assuming that it is formed by protons concentrated in a thin band having 1 km^2 cross–section at distance $r = 4R_e$ with density $n_p = 4$ nA m^{-2}?

Solution:

A current density n_p = 4 nA m^{-2} distribute over 1 km^2 determines a total current I given by:

$$I = n_p S = 4\,\mathrm{mA}$$

This current of positive charges at distance $r = 4R_e$ from the Earth's center gives rise to a magnetic moment $\boldsymbol{m}$ having the direction and magnitude:

$$\boldsymbol{m} = \pi r^2 I\boldsymbol{k} = 8.16\times10^6\boldsymbol{k}\ \ \mathrm{Am}^2$$

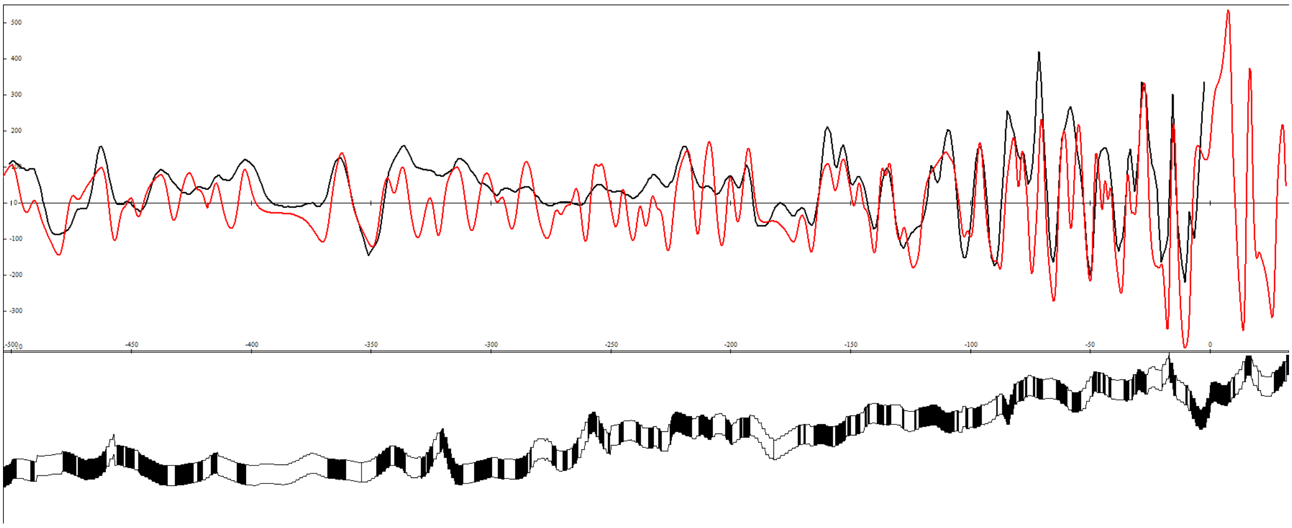
Therefore, the magnetic moment associated with the ring current opposes the Earth's magnetic moment (which has magnitude ~7.9×10^{22} Am^2). Furthermore, it precesses about the main field direction.

Chapter 5

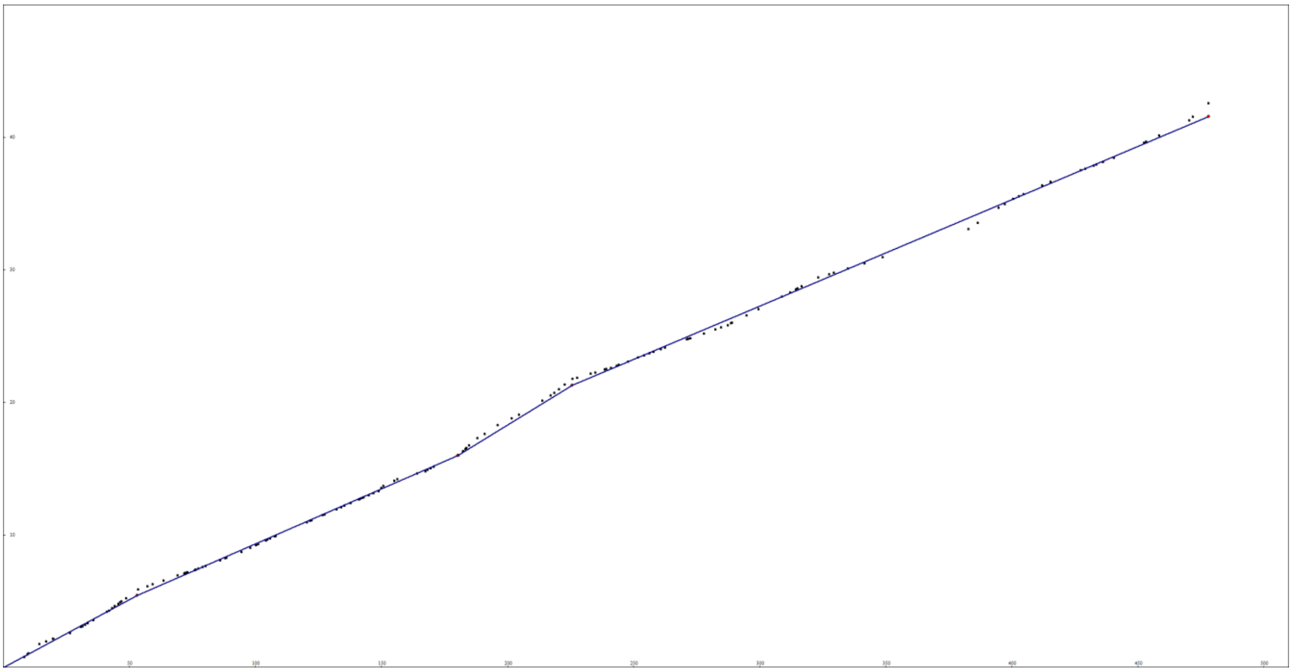
1. Use Magan to analyse the data in exercise_5.1.zip. Determine the stages, the average linear velocity for each stage, and the angular velocities assuming an angular distance θ = 50° from the Euler poles;

Solution:

The following plot shows a possible magnetization model, along with the corresponding magnetic anomaly signal (red line) and the observed anomalies (black line).



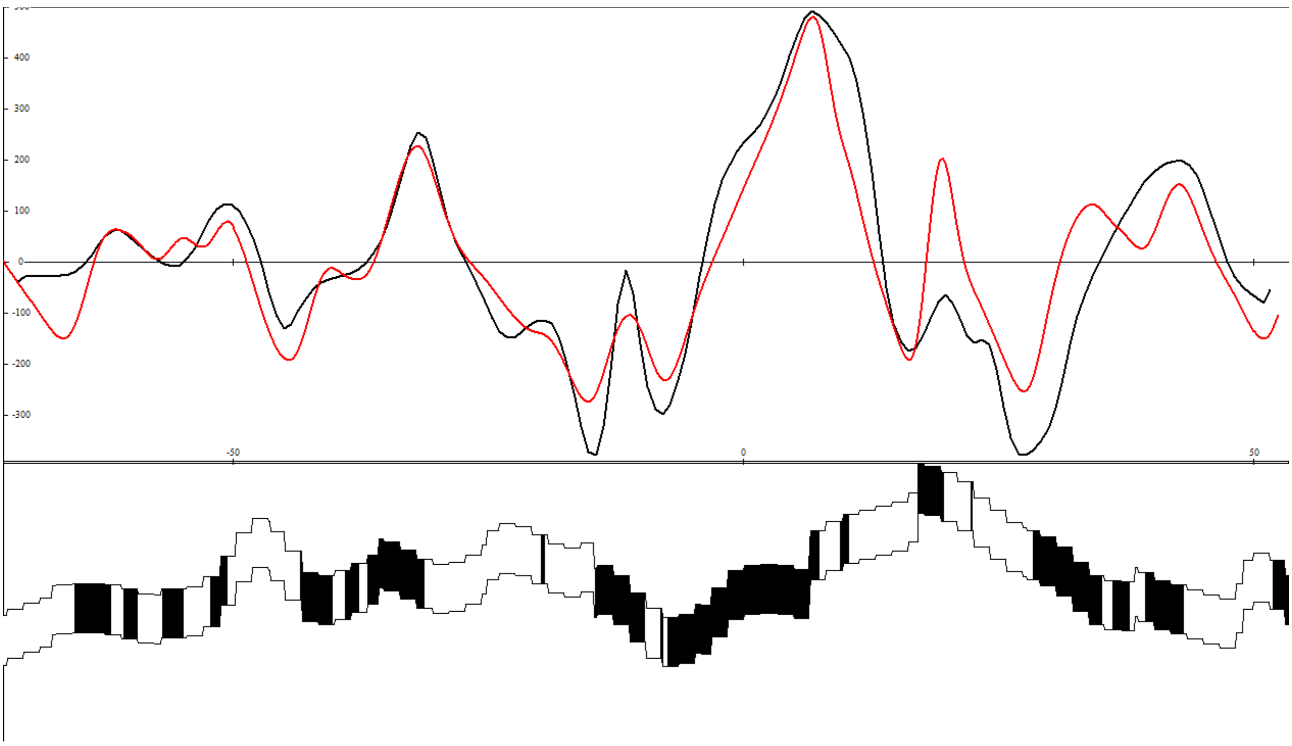
The distance–age plot can be fit by a four–stages spreading history, with a first stage from chron 19 to chron 6A, average velocity v = 24.92 mm/yr, a second stage that includes chrons C6–C5C, with average velocity v = 17.18 mm/yr, a third stage that includes chrons C5B–C3, with average velocity v = 24.09 mm/yr, and a final stage with linear velocity v = 19.48 mm/yr. These stages are shown in the (X–T) plot below:



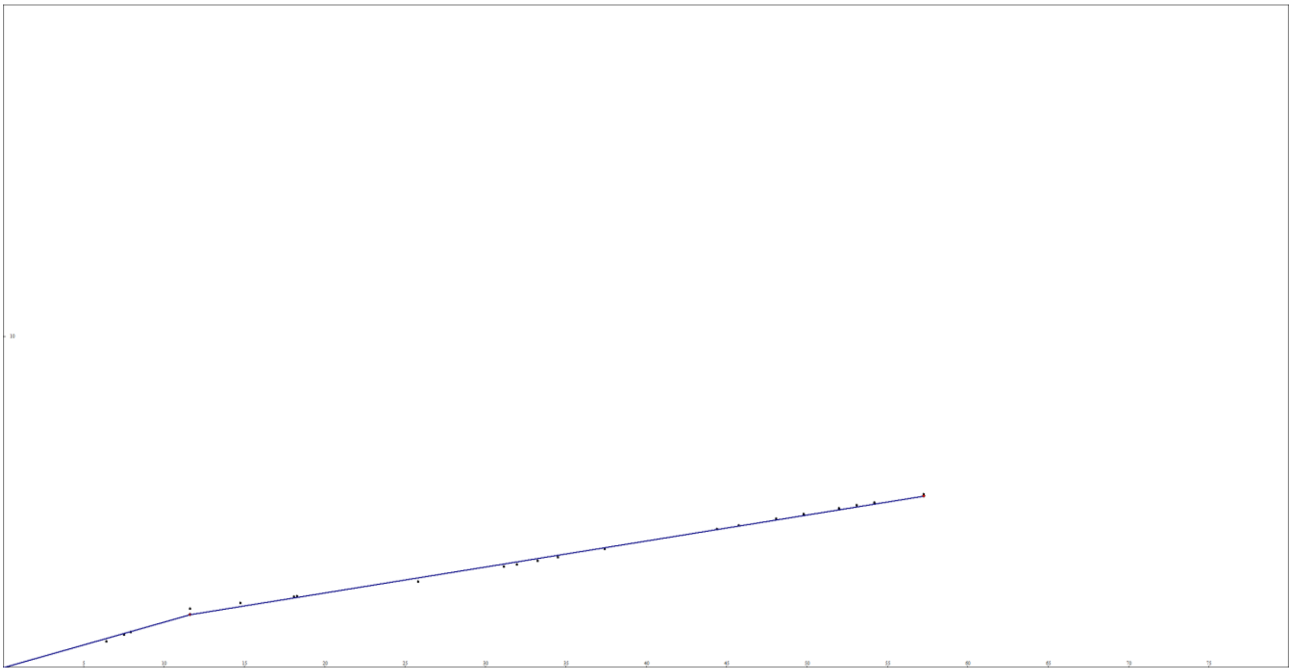
2. Use Magan to analyse the data in exercise_5.2.zip. This exercise requires to deal with spreading asymmetry. Determine the stages and the average linear velocity for each stage;

Solution:

The following plot shows a possible magnetization model, along with the corresponding magnetic anomaly signal (red line).



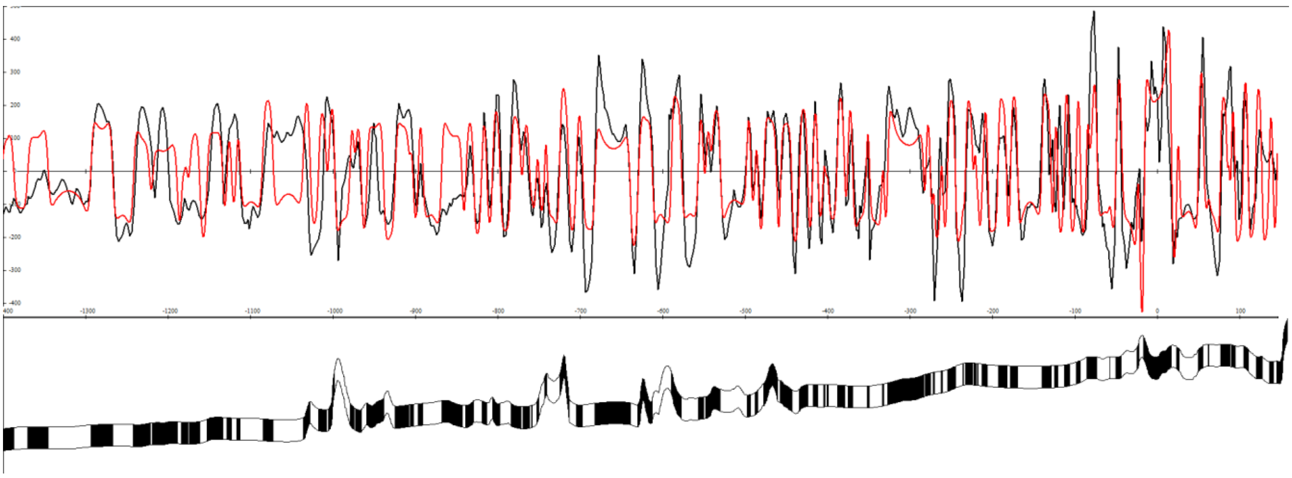
The distance–age plot can be fit by a two–stages spreading history, with a first stage from chron 3 to chron 2, average velocity v = 25.46 mm/yr, and a second stage that includes chron C1, with average velocity v = 14.57 mm/yr, as shown in the (X–T) plot below:



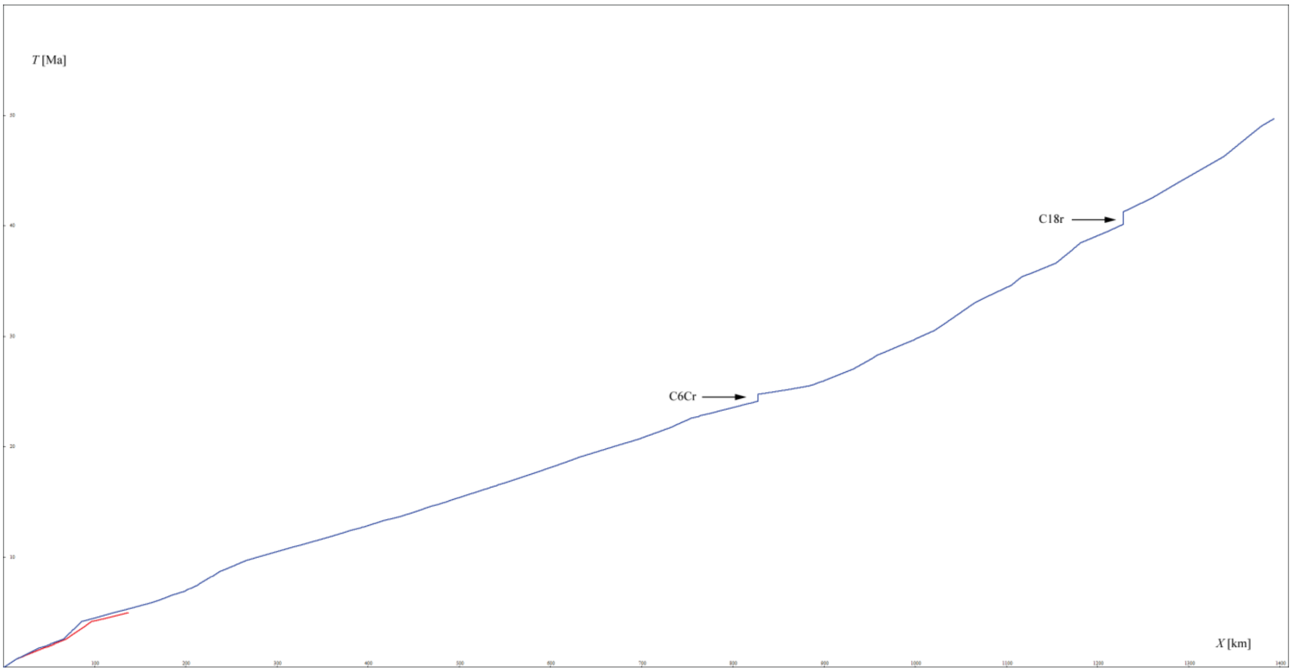
3. Use Magan to analyse the data in exercise_5.3.zip. This exercise includes a possible ridge jump. Determine the stages and the average linear velocity for each stage;

Solution:

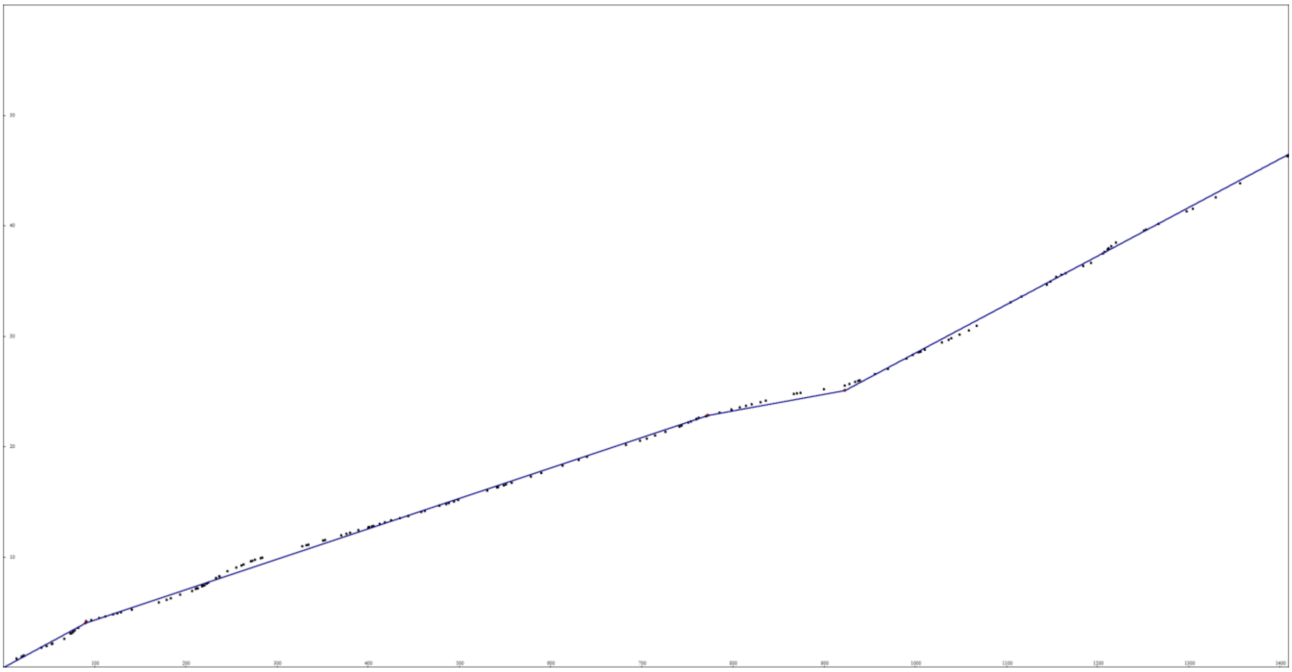
The magnetic profile is located in the northern Pacific, a region where several westward ridge jumps have been observed (see Shih & Molnar, 1975). A magnetization model and associated magnetic anomaly signal is shown below.



The model includes two westward ridge jumps, respectively at the end of C6Cr and C18r, as shown in the (X–T) plot below (blue line):



A best fitting linear spline of the age–distance curve is shown below:



It predicts four stages, with stage boundaries at: C2Ar, C6Br, C7r, and C21r. The corresponding average velocities are, respectively, 44.75 mm/yr, 72.79 mm/yr, 127.50 mm/yr, and 45.13 mm/yr, with an rms error of 0.25 mm/yr.

4. Use Magan to analyse the whole ship track data in exercise_5.3.zip. This track includes several magnetic profiles from the same area. Determine the stages and use a GIS to visually build isochrons. Then, use the procedure described in the book to build a kinematic model;

Solution:

To solve the problem, create a magnetic profile for each E–W trackline and use the same starting parameters of the previous exercise.

Chapter 6

1. Interpret the APW path of a continent as the trajectory of a moving point at the Earth's surface and explain what kind of information can be obtained about the kinematics at a reference site *S*;

Solution:

The velocity of the moving point influences both the velocity of latitude change of *S* and the velocity of vertical axis rotation of the continent about *S*.

2. Assuming that a tectonic plate is rotating clockwise about an Euler pole located at (0°N,90°E) with angular velocity ω = 1°/Myr, determine the northward and eastward components of velocity for a reference point at (30°N,45°E), and the rate of variation of its inclination;

Solution:

The components of the Euler vector for this plate are:

$$\boldsymbol{\omega} = (0, -\omega, 0) = (0, -\pi/180, 0)$$

The components of the reference point position vector are given by:

$$\begin{cases} x = R\sin\theta\cos\phi = 3901.4\ \text{km} \\ y = R\sin\theta\sin\phi = 3901.4\ \text{km} \\ z = R\cos\theta = 3185.5\ \text{km} \end{cases}$$

Therefore, the velocity is given by:

$$\begin{cases} v_x = -\omega z = -55.60\ \text{km/Myr} \\ v_y = 0 \\ v_z = \omega x = 68.09\ \text{km/Myr} \end{cases}$$

We can use (4.72) to convert the components of $\boldsymbol{v}$ to local coordinates. We have:

$$\begin{cases} \hat{\boldsymbol{r}} = \sin\theta\cos\phi\boldsymbol{i} + \sin\theta\sin\phi\boldsymbol{j} + \cos\theta\boldsymbol{k} = 0.6124\boldsymbol{i} + 0.6124\boldsymbol{j} + 0.5\boldsymbol{k} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\boldsymbol{i} + \cos\theta\sin\phi\boldsymbol{j} - \sin\theta\boldsymbol{k} = 0.3536\boldsymbol{i} + 0.3536\boldsymbol{j} - 0.8660\boldsymbol{k} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\boldsymbol{i} + \cos\phi\boldsymbol{j} = -0.7071\boldsymbol{i} + 0.7071\boldsymbol{j} \end{cases}$$

The components of $\boldsymbol{v}$ in this local reference frame are given by:

$$\begin{cases} v_r = \boldsymbol{v}\cdot\hat{\boldsymbol{r}} = -55.60\times 0.6124 + 68.09\times 0.5 = 0 \\ v_\theta = \boldsymbol{v}\cdot\hat{\boldsymbol{\theta}} = -55.60\times 0.3536 - 68.09\times 0.8660 = -78.63\ \text{mm/yr} \\ v_\phi = \boldsymbol{v}\cdot\hat{\boldsymbol{\phi}} = 55.60\times 0.7071 = 39.31\ \text{mm/yr} \end{cases}$$

Therefore, the site moves northward at 78.63 mm/yr and eastward at 39.31 mm/yr. The northward component of motion can be easily expressed in deg/Myr:

$$v_\theta\,[\text{deg/ Myr}] = -\frac{d\theta}{dt} = \frac{180}{\pi}\frac{v_\theta\,[\text{mm/yr}]}{R} = 0.71$$

where θ is the site colatitude. By the dipole formula (4.49) we have:

$$-\frac{\dot{\theta}}{\sin^2\theta} = \frac{\dot{I}}{2\cos^2 I}$$

Therefore,

$$\dot{I} = -\frac{2\dot{\theta}}{\sin^2\theta}\cos^2 I = -\frac{2\dot{\theta}}{1+3\sin^2\theta} = 0.44\ \mathrm{deg/Myr}$$

3. Given a mean paleopole for India at 100 Ma, describe a method to fit India to Madagascar in three steps;

Solution:

Let $P = (\theta_p,\phi_p)$ be the coordinates (colatitude,longitude) of a paleopole for India at 100 Ma. Let us select a reference site S close to the centroid of this continent, e.g., S = (24°N,77°E). From the paleopole P, it is easy to calculate the declination D and the paleolatitude λ of the reference site at 100 Ma. The paleolatitude can be determined calculating the Cartesian coordinates of the position vectors associated S and P and taking the scalar product of the two vectors. The declination can be calculated from (6.43) and (6.46). Now the first step is to rotate India about S by an angle D. This operation restores the correct orientation of the continent at 100 Ma with respect to the spin axis. The second step is to rotate India about the axis having pole $(0,\phi_S + 90°)$ by $\lambda_s - \lambda$. This operation restores the correctr paleolatitude for S at 100 Ma. Finally, we rotate the continent about the spin axis in so far as it fits to Madagascar.

4. Find an expression/algorithm for calculating the predicted paleolatitude and declination at a point P given the paleolatitude and declination at a reference site S.

Solution:

Given paleolatitude and declination at a reference site, you can easily determine the paleopole coordinates using (6.41) and (6.47). Now you invert these formulae using the new site coordinates.

Chapter 7

1. Find the principal axes for the 2–D stress tensor:

$$\boldsymbol{\tau} = \begin{bmatrix} 20 & \sqrt{125} \\ \sqrt{125} & 40 \end{bmatrix} \text{MPa}$$

and determine: 1) the components of traction on a vertical fault oriented E–W; 2) the plane of maximum shear stress and the shear and normal stresses along this plane;

Solution:

The eigenvalues equation (7.6) is satisfied iff:

$$\det(\boldsymbol{\tau} - \lambda \boldsymbol{I}) = \lambda^2 - 60\lambda + 675 = 0$$

The solutions to this equation are: $\lambda_1 = 45$ and $\lambda_2 = 15$. Equation (7.5) for $\lambda = \lambda_2$ assumes the form:

$$\begin{cases} 20n_1 + \sqrt{125}n_2 = 15n_1 \\ \sqrt{125}n_1 + 40n_2 = 15n_2 \end{cases}$$

The first of these two equations gives:

$$n_1 = -\sqrt{5}n_2$$

Now we take into account that $\boldsymbol{n}$ is a versor, thereby $n_1^2 + n_2^2 = 1$. Therefore: $n_2 = \pm 1/\sqrt{6} \cong \pm 0.4082$. Substituting this result into the equation for n_1 gives: $n_1 \cong \mp 0.9129$. Let us choose the following orientation for the first eigenvector $\boldsymbol{n}^{(2)}$:

$$\boldsymbol{n}^{(2)} = (+0.9129, -0.4082)$$

Now we repeat calculations for $\lambda = \lambda_1$. The result is:

$$\boldsymbol{n}^{(1)} = (+0.4082, +0.9129)$$

Now let us assume that the components of $\boldsymbol{\tau}$ have been determined in a local reference frame having orientation x = N, y = E. In this instance, to calculate the components of traction on a vertical fault oriented E–W we apply Cauchy's theorem (7.3) to the plane with versor $\boldsymbol{n}$ = (1,0). We have:

$$\begin{cases} T_1 = \tau_{11}n_1 + \tau_{12}n_2 = 20.0000\,\mathrm{MPa} \\ T_2 = \tau_{21}n_1 + \tau_{22}n_2 \cong 11.1803\,\mathrm{MPa} \end{cases}$$

The versor associated with the plane of maximum shear stress can be determined by the following formula:

$$\boldsymbol{n} = (1/\sqrt{2})(\boldsymbol{n}^1 + \boldsymbol{n}^2) = (0.9342, 0.3569)$$

The traction along this plane is given by:

$$\begin{cases} T_1(\boldsymbol{n}) = \tau_{11}n_1 + \tau_{12}n_2 \cong 22.6743\,\mathrm{MPa} \\ T_2(\boldsymbol{n}) = \tau_{21}n_1 + \tau_{22}n_2 \cong 24.7207\,\mathrm{MPa} \end{cases}$$

The normal stress along this plane is:

$$T_N(\boldsymbol{n}) = \boldsymbol{T} \cdot \boldsymbol{n} \cong 30.0051\,\mathrm{MPa}$$

Finally, the maximum shear stress will be given by:

$$T_S(\boldsymbol{n}) = \sqrt{T^2(\boldsymbol{n}) - T_N^2(\boldsymbol{n})} \cong 14.9976\,\mathrm{MPa}$$

2. Determine the body force field in conditions of static equilibrium for the following stress tensor:

$$\boldsymbol{\tau}(x,y,z)=\begin{bmatrix} -3x^2+5y-2z^3 & 4x+3xy & 3xz \\ 4x+3xy & 2x^3-4y & 0 \\ 3xz & 0 & -\frac{3}{2}z^2 \end{bmatrix}$$

and comment the result;

Solution:

The body force field in conditions of static equilibrium can be obtained from Cauchy's momentum equation (7.50) setting the acceleration to zero:

$$f_i=-\frac{\partial\tau_{ij}}{\partial x_j}$$

Therefore, we have:

$$\begin{aligned} f_x &= -\frac{\partial\tau_{xx}}{\partial x}-\frac{\partial\tau_{xy}}{\partial y}-\frac{\partial\tau_{xz}}{\partial z}=6x-3x-3x=0 \\ f_y &= -\frac{\partial\tau_{yx}}{\partial x}-\frac{\partial\tau_{yy}}{\partial y}-\frac{\partial\tau_{yz}}{\partial z}=-4-3y+4=-3y \\ f_z &= -\frac{\partial\tau_{zx}}{\partial x}-\frac{\partial\tau_{zy}}{\partial y}-\frac{\partial\tau_{zz}}{\partial z}=-3z+3z=0 \end{aligned}$$

Consequently, the body forces field has rectilinear field lines aligned with the *y* axis, and a magnitude linearly increasing with the distance from the origin.

3. Consider the following transformation of the stress tensor:

$$\tau_{ij}\rightarrow\tau_{ij}+a\delta_{ij}$$

where *a* is a constant. What is the effect on the principal axes?

Solution:

The transformation determines the following new form of the eigenvalue equation:

$$(\boldsymbol{\tau} - \lambda \boldsymbol{I} + a\boldsymbol{I})\boldsymbol{n} = \boldsymbol{0}$$

Therefore, the characteristic equation assumes the form:

$$\det[\boldsymbol{\tau} - (\lambda - a)\boldsymbol{I}] = 0$$

This implies that all the eigenvalues are translated by a:

$$\lambda \to \lambda' = \lambda + a$$

Now let $\boldsymbol{n}$ be an eigenvector of the transformed tensor $\boldsymbol{\tau}' = \boldsymbol{\tau} + a\boldsymbol{I}$. We have:

$$\tau'_{ij} n_j = \lambda' n_i$$

This equation can be rewritten as:

$$(\tau_{ij} + a\delta_{ij})n_j = (\lambda + a)n_i$$

which reduces to

$$\tau_{ij} n_j = \lambda n_i$$

Therefore, $\boldsymbol{n}$ is also an eigenvector for $\boldsymbol{\tau}$. Consequently, the two tensors have the same principal axes.

4. What is the relation between the principal axes of the stress tensor and those of the deviator?

Solution:

By the previous exercise, setting $a = p$ we see that the two tensors have the same principal axes.

5. Find a constitutive equation for the standard solid rheology and determine the creep and relaxation curves.

Solution:

The total stress, τ, is associated with a strain field ε that is partitioned between a Kelvin component and a spring Y_2. The total strain associated with the model of Fig. 7.13 is:

$$\varepsilon(t) = \varepsilon_K(t) + \varepsilon_S(t)$$

where,

$$\varepsilon_S(t) = \frac{1}{Y_2}\tau(t) \;;\; \dot{\varepsilon}_K(t) + \frac{Y_1}{2\eta_1}\varepsilon_K(t) = \frac{1}{2\eta_1}\tau(t)$$

For simplicity, we have assumed $p = 0$. We have:

$$\dot{\varepsilon} = \dot{\varepsilon}_K + \dot{\varepsilon}_S = \frac{1}{2\eta_1}\tau - \frac{Y_1}{2\eta_1}\varepsilon_K + \frac{1}{Y_2}\dot{\tau}$$

The strain of the Kelvin element can be written as:

$$\varepsilon_K = \varepsilon - \varepsilon_S = \varepsilon - \frac{1}{Y_2}\tau$$

Therefore, the general form of the constitutive equation is:

$$\dot{\varepsilon} + \frac{Y_1}{2\eta_1}\varepsilon = \frac{1}{Y_2}\dot{\tau} + \frac{1}{2\eta_1}\left(1 + \frac{Y_1}{Y_2}\right)\tau$$

To determine the creep curve of the standard solid, we set the initial strain conditions for a stress step such that $\tau(0) = \tau_0$:

$$\varepsilon(0) = \varepsilon_S(0) = \frac{\tau_0}{Y_2} \;;\; \varepsilon_K(0) = 0$$

Therefore, assuming that the stress drop occurs at $t = t_0$, we have that for $0 < t \le t_0$ the constitutive equation becomes:

$$\dot{\varepsilon} + \frac{Y_1}{2\eta_1}\varepsilon = \frac{1}{2\eta_1}\left(1 + \frac{Y_1}{Y_2}\right)\tau_0$$

The quantity at the right–hand side is a constant, thereby the equation is a linear first–order differential equation, whose general solution is:

$$\varepsilon(t) = \left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)\tau_0 + ce^{-(Y_1/2\eta_1)t}$$

Using the initial condition for ε we easily obtain: $c = -\tau_0/Y_1$. Therefore, the creep curve will be given by:

$$\varepsilon(t) = \frac{\tau_0}{Y_1}\left[1 - e^{-(Y_1/2\eta_1)t}\right] + \frac{\tau_0}{Y_2}$$

To determine the stress relaxation curve, let us assume that $\varepsilon(t) = \varepsilon_0$ for $t \ge 0$ and $\tau(0) = \tau_0$. Substituting into the constitutive equation gives, for $t \ge 0$:

$$\frac{Y_1}{2\eta_1}\varepsilon_0 = \frac{1}{Y_2}\dot{\tau} + \frac{1}{2\eta_1}\left(1 + \frac{Y_1}{Y_2}\right)\tau$$

This equation can be rewritten as follows:

$$\dot{\tau} + \frac{1}{2\eta_1}(Y_1 + Y_2)\tau = \frac{Y_1 Y_2}{2\eta_1}\varepsilon_0$$

Again, this is a linear first–order differential equation, whose general solution is:

$$\tau(t) = \left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)\varepsilon_0 + ce^{-(Y_1+Y_2)t/2\eta_1}$$

From the initial condition, we obtain:

$$c = \tau_0 - \left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)\varepsilon_0$$

Therefore, the relaxation law will be given by:

$$\tau(t) = \left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)\varepsilon_0\left(1 - e^{-(Y_1+Y_2)t/2\eta_1}\right) + \tau_0 e^{-(Y_1+Y_2)t/2\eta_1}$$

Chapter 8

1. Download the MS Excel file PREM.xlsx, containing the PREM model, from the supplemental material web site. Use this file to determine how long time takes a downward directed *P* wave generated at the Earth's surface to reach the antipodal point;

Solution:

To solve the problem, you simply insert a new column, where the travel time of vertical incidence is calculated for each layer. For example, at row 55 you should have the formula: "=1000*(B56-B55)/D55" (layer thickness divided by *P* wave velocity). The result is obtained summing the individual travel times and multiplying the sum by two. You should obtain T = 1203 s.

2. Determine an expression for the strain tensor associated with a monochromatic wave travelling in the x direction;

Solution:

For a monochromatic *P* wave travelling in the *x* direction, we have a displacement field of the form:

$$u_x = A\sin(\omega t - kx)\ ;\ u_y = u_z = 0$$

Therefore, the strain tensor will assume the expression:

$$\varepsilon = \begin{bmatrix} -kA\cos(\omega t - kx) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In the case of a monochromatic *S* wave travelling in the *x* direction, we have that the displacement field has the form:

$$u_x = 0 \ ; \ u_y = A_y \sin(\omega t - kx) \ ; \ u_z = A_z \sin(\omega t - kx)$$

Therefore, in this case the strain tensor will assume the expression:

$$\varepsilon = \begin{bmatrix} 0 & -(1/2)kA_y \cos(\omega t - kx) & -(1/2)kA_z \cos(\omega t - kx) \\ -(1/2)kA_y \cos(\omega t - kx) & 0 & 0 \\ -(1/2)kA_z \cos(\omega t - kx) & 0 & 0 \end{bmatrix}$$

3. Find a test to determine if a material is a Poisson solid from measurements of seismic velocity;

Solution:

A Poisson solid is a material such that $\lambda = \mu$. Therefore, in this instance the *P*–wave and *S*–wave velocities can be written as follows:

$$\alpha \equiv \sqrt{\frac{3\mu}{\rho}} \ ; \ \beta = \sqrt{\frac{\mu}{\rho}}$$

Consequently, for a Poisson solid we have that the ratio α/β assumes the value:

$$\frac{\alpha}{\beta} = \sqrt{3} \cong 1.73$$

4. Find an expression for determining the Poisson ratio from measurements of seismic velocity;

Solution:

By (8.4), (8.13), and (8.28) we have that the Poisson ratio can be written as follows:

$$\nu = \frac{\lambda}{2(\lambda+\mu)} = \frac{1}{2\left(1+\frac{\mu}{\lambda}\right)}$$

From (8.13) and (8.28) we see that the ratio α/β can be expressed as:

$$\frac{\alpha^2}{\beta^2} = 2 + \frac{\lambda}{\mu}$$

Consequently,

$$\frac{\mu}{\lambda} = \frac{\beta^2}{\alpha^2 - 2\beta^2}$$

Substituting into the expression for ν allows to express the Poisson ratio in terms of seismic velocities:

$$\nu = \frac{\alpha^2/2 - \beta^2}{\alpha^2 - \beta^2} = \frac{0.5(\alpha^2/\beta^2) - 1}{(\alpha^2/\beta^2) - 1}$$

5. Determine the relation between elastic moduli, Lamé parameters, and the velocities α and β when $\nu = -1$ and $\nu = 0$;

Solution:

For $\nu = -1$, from (8.4) we have that $\lambda = -2\mu/3$. Therefore, the seismic velocities will have a ratio: $\alpha/\beta = (4/3)^{1/2}$. Regarding the Young modulus, Y, by (8.3) we obtain: $Y = 0$, which implies that no extensional stress occurs when a cylinder is pulled by both ends (zero stiffness). When $\nu = 0$, it results $\lambda = 0$, thereby the velocity ratio is given by: $\alpha/\beta = \sqrt{2}$. From (8.3), we also have that in this instance $Y = 2\mu$.

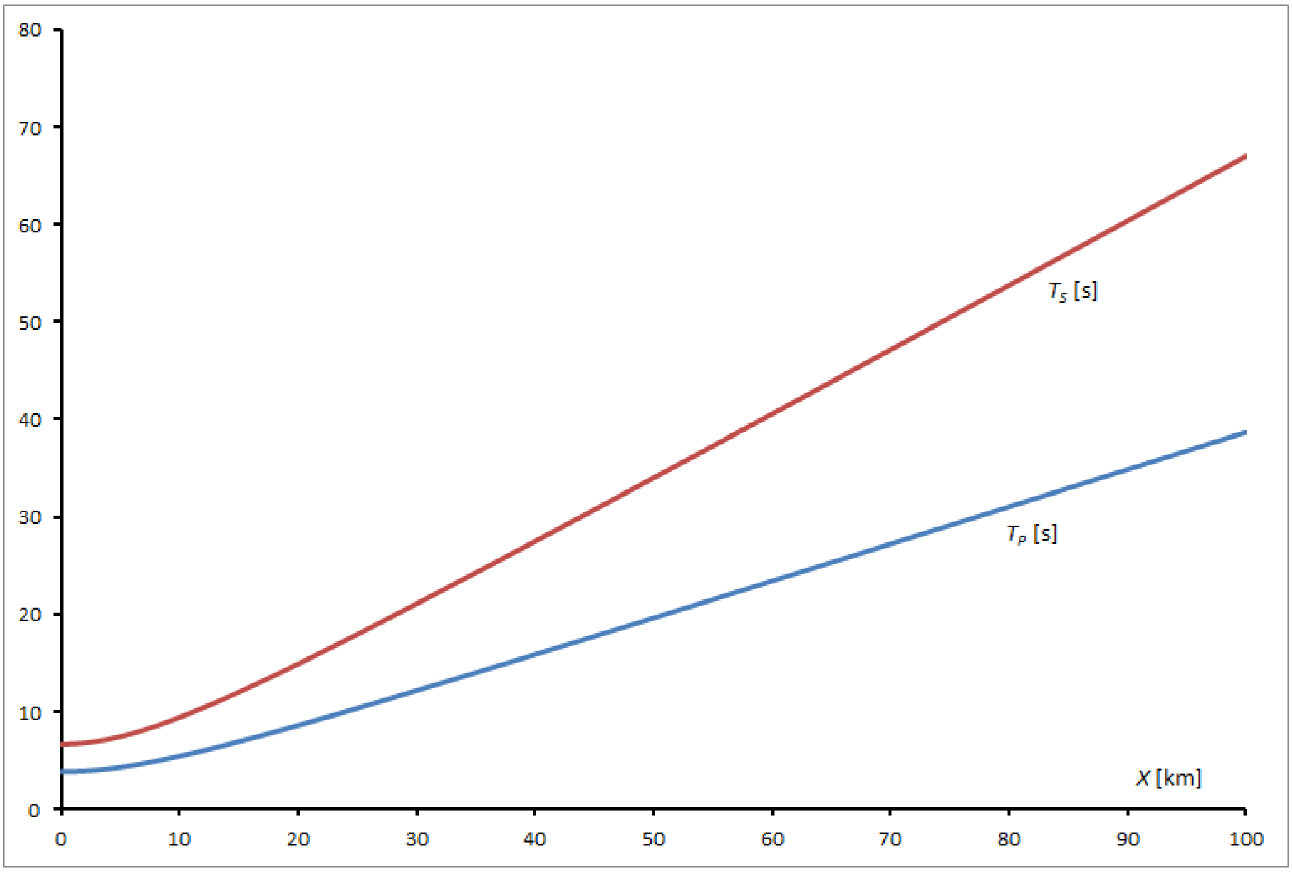
6. Estimate the α/β ratio from a set of arrival times of *P* and *S* waves at seismic stations.

Solution:

To solve this exercise, it is necessary to assume a flat Earth approximation, a homogeneous layer, and a small scale network of seismic stations (otherwise, it is not possible to assume a constant α/β ratio). In this instance, the time of first arrival to seismic stations is a simple hyperbola function of the distance *X* from the epicenter. Let T_P and T_S be the arrival time of the *P* and *S* wave, respectively. If *H* is the depth to the earthquake focus, we have:

$$\begin{cases} T_P(X) = \dfrac{1}{\alpha}\sqrt{X^2 + H^2} \\ T_S(X) = \dfrac{1}{\beta}\sqrt{X^2 + H^2} \end{cases}$$

Consequently, the arrival times of the *S* and *P* waves are aligned with hyperbolae, and the quantities 1/β and 1/α represent the asymptotic slopes of the curves for small *H*/*X* ratios, as shown in the following figure.



Therefore, the quantities $1/\beta$ and $1/\alpha$ can be obtained directly from a statistical regression of the arrival times with respect to the epicentral distances *X*.

Chapter 9

1. What is the geometry of a seismic ray with $\theta_0 = 0$?;

Solution:

When the take–off angle $\theta_0 = 0$, the ray parameter *p* is also zero (by Snell's law) and the seismic ray is a downgoing straight line.

2. Determine the travel–time curve $T = T(X)$ for waves reflected at a horizontal discontinuity, knowing that the two–way travel time (TTWT) for vertical incidence is T_0;

Solution:

There is a simple solution only in the case of a flat–Earth approximation. Assuming a homogeneous layer of thickness *H* and slowness *s*, by Pitagora's theorem we have that the arrival time will be given by:

$$T(X) = 2s\sqrt{X^2/4 + H^2} = \sqrt{T_0^2 + s^2 X^2}$$

3. What is the correction that must be applied on a set of travel times of reflected arrivals to obtain proportionality between arrival time and depth to the reflector (this is known as *normal moveout*)?

Solution:

Proportionality between arrival time of reflected waves and depth of the reflector only occurs in the case of vertical incidence. In this instance, we have: $T(0) \equiv T_0 = 2sH$. When the seismic source and the receiver are separated by a distance X, the arrival time is given by:

$$T(X) = \sqrt{T_0^2 - s^2 X^2} = T_0 \sqrt{1 + \left(\frac{sX}{T_0}\right)^2}$$

To remove the effect of the distance X, we can apply the following correction (normal moveout):

$$T(X) \rightarrow T(X) - \Delta T_{NMO}(X)$$

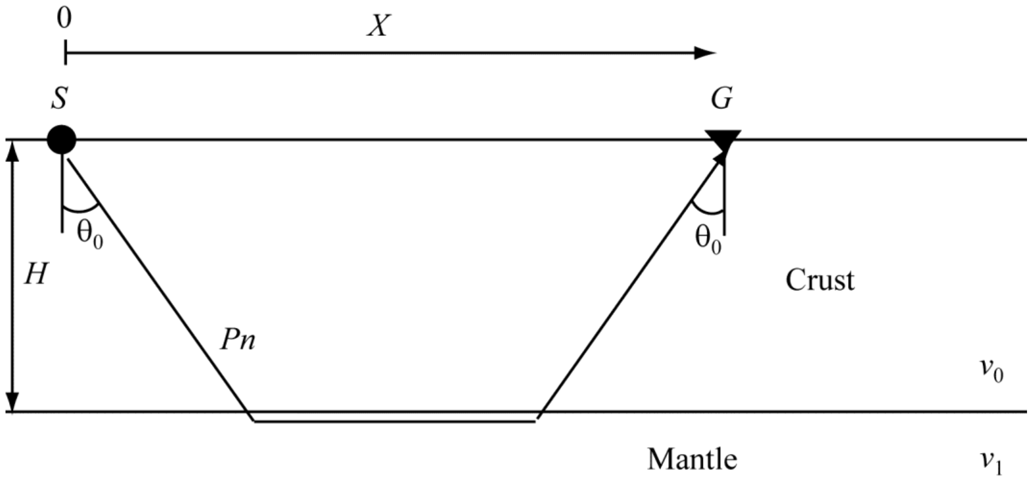
where,

$$\Delta T_{NMO}(X) = T(X) - T(0) = T_0 \left(\sqrt{1 + \left(\frac{sX}{T_0}\right)^2} - 1 \right) \cong \frac{s^2 X^2}{2T_0}$$

4. Determine the travel–time curve $T = T(X)$ of a Pn phase, given the crustal thickness and assuming a homogeneous crust and surficial source;

Solution:

Let H be the Moho depth, and let us assume a flat–Earth approximation. In this instance, the problem can be solved using the geometry illustrated in the following figure:



The travel time associated with the *Pn* phase can be calculated easily if we assume a homogeneous crust with velocity v_0. In this case we have:

$$T(X) = \frac{X - 2H\tan\theta_0}{v_1} + \frac{2H}{v_0\cos\theta_0} = \frac{X}{v_1} + 2H\left(\frac{1}{v_0\cos\theta_0} - \frac{\tan\theta_0}{v_1}\right)$$

Therefore, the travel time curve is a straight line with slope $1/v_1$. We note that in this instance the take–off angle θ_0 coincides with the critical incidence angle $\theta_c = \arcsin(v_0/v_1)$. Therefore,

$$T(X) = \frac{X}{v_1} + 2H\left(\frac{1}{v_0^2} - \frac{1}{v_1^2}\right)^{1/2} \equiv \frac{X}{v_1} + \tau$$

where the intercept τ is given by:

$$\tau = 2H\left(\frac{1}{v_0^2} - \frac{1}{v_1^2}\right)^{1/2}$$

5. Assuming a homogeneous crustal layer, the first arrival at short range would be a direct wave only below the crossover distance. Determine this distance;

Solution:

The arrival time of the direct wave from a source close to the Earth's surface through a homogeneous layer is $T_D(X) = X/v_0$. The first arrival of head waves occurs at times $T_H(X)$ given by (see previous exercise):

$$T_H(X) = \frac{X}{v_1} + 2H\left(\frac{1}{v_0^2} - \frac{1}{v_1^2}\right)^{1/2} \equiv \frac{X}{v_1} + \tau$$

Equating the two arrival times gives the crossover distance X_d:

$$X_d = 2H\left(\frac{v_1 + v_0}{v_1 - v_0}\right)^{1/2}$$

6. Determine the critical distance below which *Pn* phase arrivals are impossible, assuming a homogeneous crust;

Solution:

From the figure in the solution to Exercise 4 we see that the minimum distance (critical distance) for the arrival of *Pn* waves is:

$$X_c = 2H \tan\theta_0 = 2H \frac{v_0 / v_1}{\sqrt{1 - v_0^2 / v_1^2}}$$

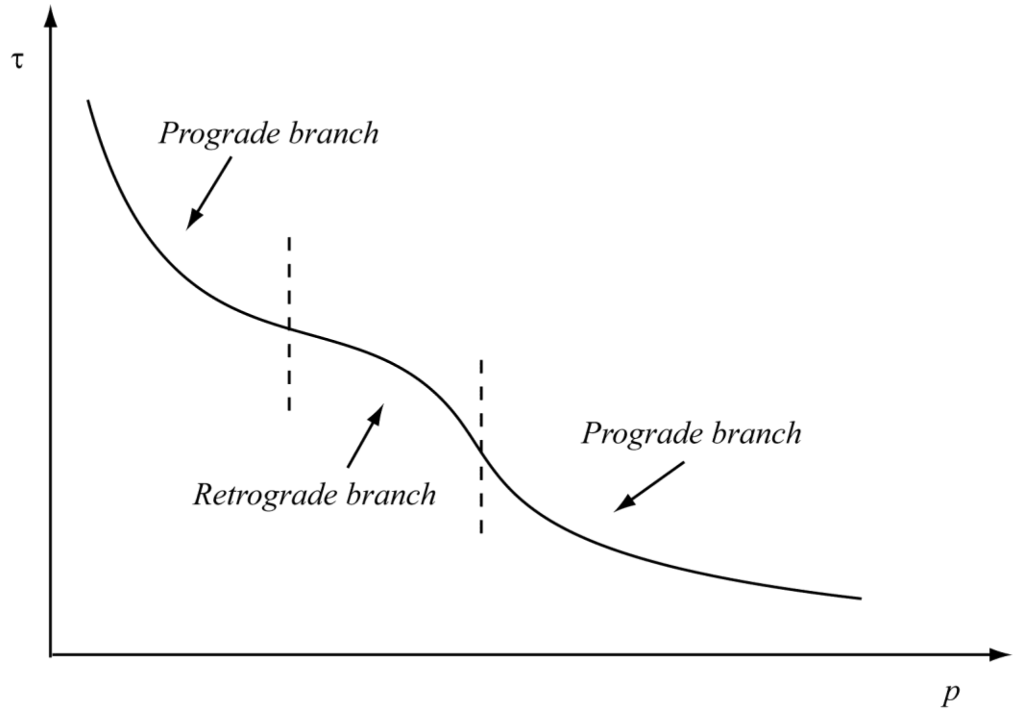
7. Draw the delay time curve associated with the travel time curve of Fig. 9.13;

Solution:

The delay time $\tau(p)$ is calculated taking the intercept of a tangent to the travel time curve:

$$\tau(p) = T(X) - pX(p)$$

In the retrograde branch of a travel time curve containing a triplication, the curve $\tau = \tau(p)$ is concave downward instead of being concave upward. Therefore, it has the trend illustrated in the following figure:



8. Download the MS Excel file *ESP215.txt*, containing a velocity model of oceanic crust in the Balearic Basin (Pascal et al., 1993, *Geophys. J. Int.*, **113**, 701–726.), from the supplemental material web site. This file contains a table that divides the oceanic crust into a series of flat layers with *P* wave velocity that increases linearly and variable thickness. Write a computer program that uses this ASCII table to build the local travel time table for *Pg* waves generated from a surficial source, assuming take–off angles from 85° to the minimum take–off angle for *Pg* waves, with increments of 5°, starting from (9.60) and (9.61).

Solution:

In the table ESP215.txt, we must deal with columns containing thickness, top velocity, and bottom velocity of the various crustal layers. Therefore, it is convenient to generate a new table containing only the required columns and without header line. The discrete solutions (9.60) and (9.61) are then used to build the travel–time algorithm used in the computer program *travel–time.cpp*. Given a take–off angle θ_0, the ray parameter is calculated by the following formula:

$$p = s_0 \sin\theta_0$$

At each step, the horizontal distance and the travel time associated with the *i*–th layer are given by:

$$X_i(p) = -\frac{1}{pm_i}\left(\sqrt{1-p^2v_{i+1}^2} - \sqrt{1-p^2v_i^2}\right);\ i = 0,1,\ldots,n-1$$

$$T_i(p) = -\frac{1}{m_i}\left\{\ln\left[\frac{1}{v_{i+1}}\left(1+\sqrt{1-p^2v_{i+1}^2}\right)\right] - \ln\left[\frac{1}{v_i}\left(1+\sqrt{1-p^2v_i^2}\right)\right]\right\}$$

From a quick inspection of the table, we note that there are velocity discontinuities at each layer boundary, thereby apparently the previous solutions could not be used. However, the discontinuities can be modelled as layers having very small thickness and high velocity gradient *m*. According to the previous formulae, these layers do not contribute significantly to the final result. Therefore, a draft of the procedure could be (in C language):

```
#define N 12                    //Number of table rows
void main(void)
{
```

```
char buffer[256];
double th = 85.0;
double m;
double p;
double v_top[N];
double v_bottom[N];
double thickness[N];
double X = 0.0;
double T = 0.0;
double tmp;
int k;
bool skip;
if (read_table(v_top,v_bottom,thickness) == false)  //Reads the velocity model from ESP215.txt
        {
        printf("Error while reading input file – press any key to exit");
        getch();
        return;
        }
th *= DegToRad;                         //Converts to radians
while (th > 0)
        {
        p = sin(th) / v_top[0];
        if (p < 1 / v_bottom[N – 1]);           //Not a Pg phase
                break;
        skip = false;
        for (k = 0;k < N;++k)
                {
                if (p > 1 / v_bottom[k] || p > 1 / v_top[k]);
                        {
                        skip = true;
                        break;
                        }
                m = (v_bottom[k] – v_top[k]) / thickness[k];
                X  –= (sqrt(1 – p * p * v_bottom[k] * v_bottom[k]) –
```

```
                  sqrt(1 – p * p * v_top[k] * v_top[k])) / (p * m);
            T –= (log((1 + sqrt(1 – p * p * v_bottom[k] * v_bottom[k])) / v_bottom[k] ) –
                  log((1 + sqrt(1 – p * p * v_top[k] * v_top[k])) / v_top[k] )) / m;
            }
      if (skip == false)
            printf("%f\t%f\n",2 * X,2 * T);
      th –= 5.0 * DegToRad;
      }
}
```

Chapter 10

1. An active fault has been mapped by geologists. After the last earthquake, they have found that its length is 10 km, the strike is ϕ = 30°N, the dip is δ = 45°, an average 10 cm downward dip–slip displacement of the hanging wall occurred, while seismic reflection studies suggest that the fault reaches a depth of 5 km. Estimate the moment tensor components for this earthquake;

Solution:

If H = 5 km is the maximum depth, then the downdip length of this fault is:

$$d = \frac{H}{\sin\delta} = \frac{5}{\sqrt{2}/2} \cong 7.07\ \text{km}$$

Therefore, if μ = 30 Gpa is the rigidity modulus of crustal rocks, then the scalar moment is given by:

$$M_0 = \mu\bar{u}S = \mu\bar{u}Ld = 2.12\times10^{18}\ \text{Nm}$$

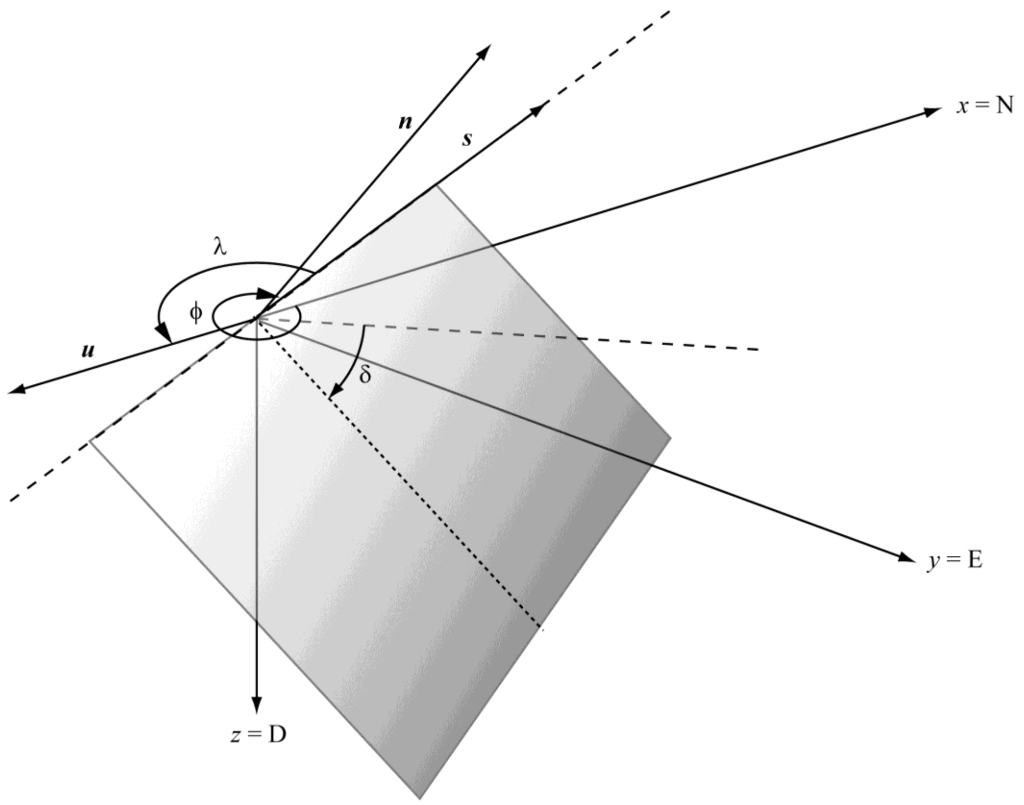
Finally, using (10.58) we find (in Nm):

M_{11}	M_{12}	M_{13}	M_{22}	M_{23}	M_{33}
-5.30×10^{17}	9.18×10^{17}	-144.459	-1.59×10^{18}	66.553	2.12×10^{18}

2. Prove the validity of expression (10.58) for the moment tensor components;

Solution:

From (10.56), we have that the components of the moment tensor can be calculated from the displacement versor, ***u***, and the versor, ***n***, normal to the fault plane. We first determine the components of the latter versor.



From the Figure above, we see that:

$$\begin{cases} n_x = -\sin\delta\sin\phi \\ n_y = +\sin\delta\cos\phi \\ n_z = -\cos\delta \end{cases}$$

Now we define a strike versor, ***s***, with components:

$$\begin{cases} s_x = \cos\phi \\ s_y = \sin\phi \\ s_z = 0 \end{cases}$$

The displacement versor can be calculated by rotating the strike versor, $\boldsymbol{s}$, about the axis of $\boldsymbol{n}$ by an angle λ:

$$\boldsymbol{u} = \boldsymbol{R}(\boldsymbol{n},\lambda)\boldsymbol{s}$$

Finally, using the transformation (2.18) we obtain (10.58).

3. Expression (10.58) for $\boldsymbol{M}$ can be rewritten as follows:

$$\boldsymbol{M} = \cos\delta\cos\lambda\boldsymbol{M}^{(1)} + \sin\delta\cos\lambda\boldsymbol{M}^{(2)} - \cos 2\delta\sin\lambda\boldsymbol{M}^{(3)} + \sin 2\delta\sin\lambda\boldsymbol{M}^{(4)}$$

where $\boldsymbol{M}^{(k)}$ are four elementary moment tensors. Find expressions for $\boldsymbol{M}^{(k)}$ and discuss the result.

Solution:

Let $\boldsymbol{M}(\delta,\lambda,\phi)$ be the moment tensor associated with the focal mechanism parameters (δ,λ,ϕ). The elementary moment tensors can be obtained using different combinations of dip, δ, and rake, λ. The first moment tensor describes slip along a horizontal fault (δ = 0°). In this instance, the rake is undefined and set to zero:

$$\boldsymbol{M}^{(1)} = \boldsymbol{M}(0,0,\phi) = M_0 \begin{bmatrix} 0 & 0 & -\cos\phi \\ 0 & 0 & -\sin\phi \\ -\cos\phi & -\sin\phi & 0 \end{bmatrix}$$

The second moment tensor is obtained setting δ = 90° (i.e., a vertical fault) and assuming horizontal slip (λ = 0°):

$$\boldsymbol{M}^{(2)} = \boldsymbol{M}(90,0,\phi) = M_0 \begin{bmatrix} -\sin 2\phi & \cos 2\phi & 0 \\ \cos 2\phi & +\sin 2\phi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The third moment tensor is obtained assuming again δ = 90° (i.e., a vertical fault), but this time we consider vertical slip (λ = 90°):

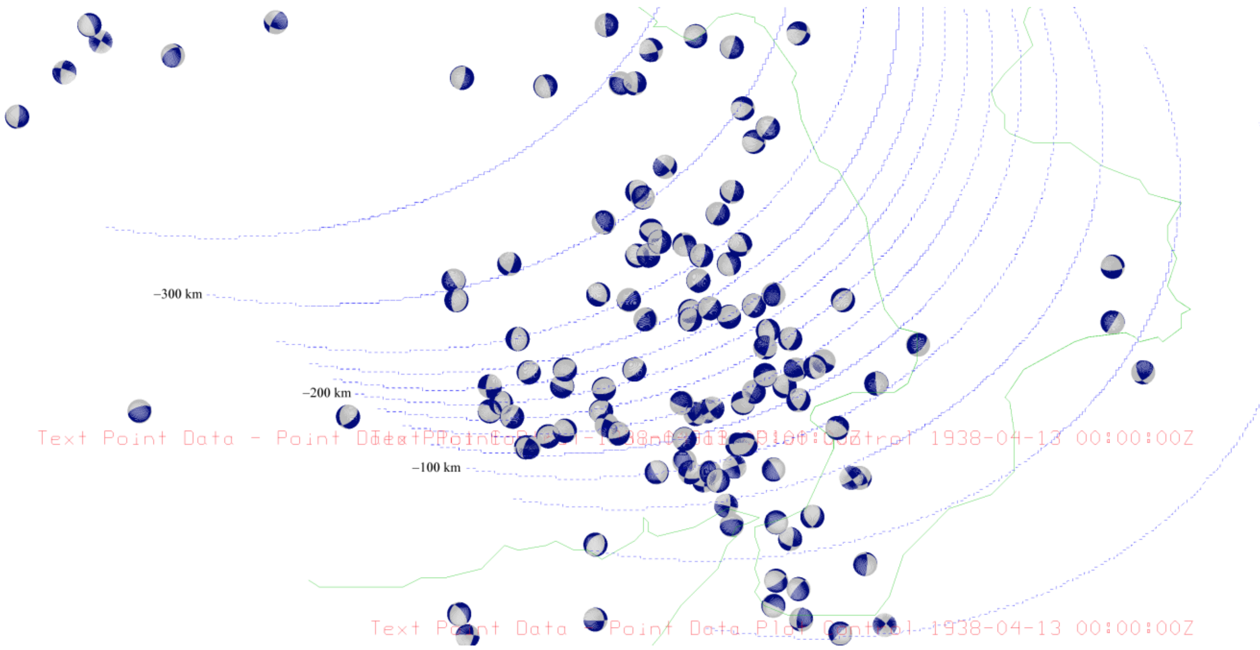
$$\boldsymbol{M}^{(3)} = \boldsymbol{M}(90,90,\phi) = M_0 \begin{bmatrix} 0 & 0 & \sin\phi \\ 0 & 0 & -\cos\phi \\ \sin\phi & -\cos\phi & 0 \end{bmatrix}$$

Finally, the fourth elementary tensor describes an inverse fault dipping 45°:

$$\boldsymbol{M}^{(4)} = \boldsymbol{M}(45,90,\phi) = M_0 \begin{bmatrix} -\sin^2\phi & \frac{1}{2}\sin 2\phi & 0 \\ \frac{1}{2}\sin 2\phi & -\cos^2\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

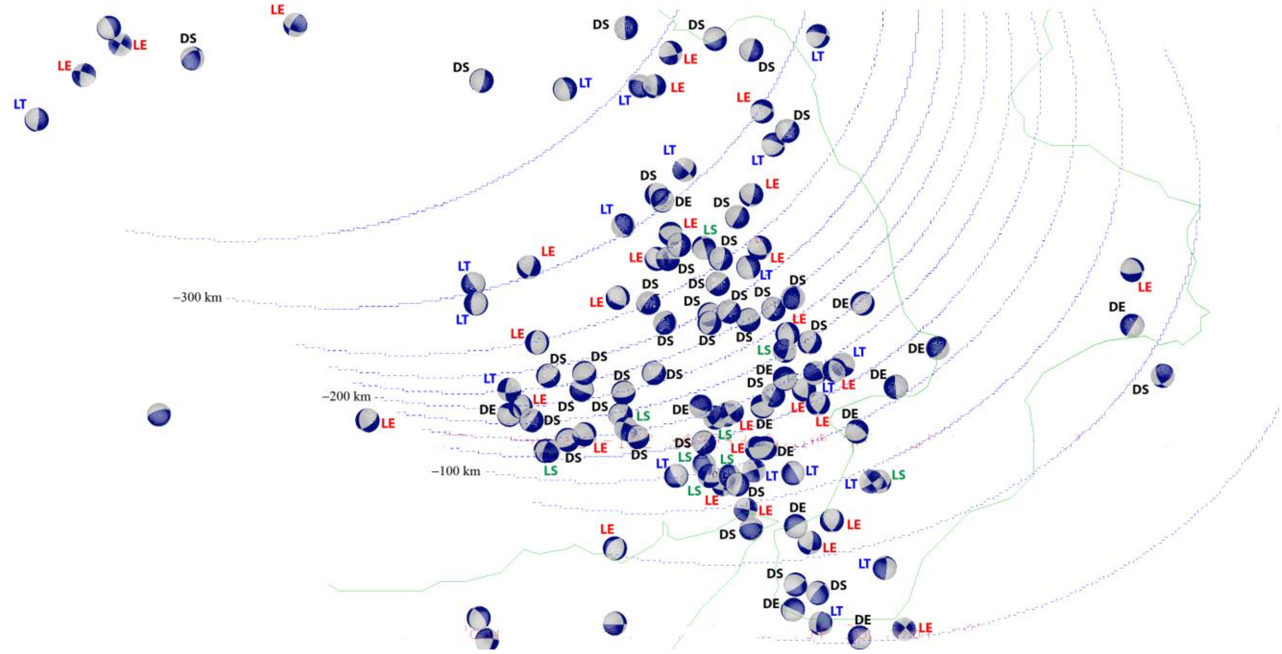
Chapter 11

1. The figure below shows focal mechanisms along the Wadati–Benioff zone of the Ionian slab in central Mediterranean. Curved lines represent isodepth lines of the slab. Classify each focal mechanism and discuss the style of deformation of this slab;



Solution:

Let us set: *LE* = Lateral extension, *LS* = Lateral shortening, *DE* = Downdip extension, *DS* = Downdip shortening, and *LT* = Lateral tearing. The style of slab deformation must be assigned taking into account of the slab dipping direction. The map below shows for each focal mechanism a probable deformation style:



This pattern of deformation can be interpreted as follows. There are at least 5–6 events associated with vertical tearing in a trench–normal direction, with increased subduction angles in the southwestern part of the slab. An elevated number of events is associated with lateral extension, mainly between 50 and 75 km and beyond 275 km. The focal mechanisms also indicate downdip shortening starting from 175 – 200 km depth, possibly associated with increased resistance of the asthenosphere to slab penetration.

2. Download the MS Excel file *CMT7609.xlsx*, containing the global CMT catalog 1976–2009. Select underthrusting earthquakes (depth ≤ 40 km) from the Hellenic Trench between (38°N,20°E) and (34.6°N,24°E). Determine the average velocity of convergence between Africa and Greece using Brune’s formula. Compare with the velocity of convergence Africa – Eurasia in the same area estimated by MORVEL (see Chapter 2);

Solution:

Apply a filter to CMT7609.xlsx to select earthquakes in the specified area and depth range. The file *Solution 11.2.xlsx* contains the 70 resulting CMT focal mechanisms. In order to use Brune's formula:

$$\bar{u} = \frac{1}{S}\sum_k \bar{u}^{(k)} S_k = \frac{1}{\mu S}\sum_k M_0^{(k)}$$

we need to determine the total area *S*. To this purpose, we observe that the length of the Hellenic trench in the selected area is $L \cong 525$ km. Assuming that the events in the range 0 – 40 km are associated with underthrusting and that the slab dip is $\alpha \cong 30°$, we obtain a downdip slab length: $d = 40 / \sin 30° = 80$ km. Therefore, $S = d \times L = 42000$ km^2 = 4.2×10^{10} m^2. From the table in *Solution 11.2.xlsx*, we see that the total release of seismic moment from 1976 to 2009 is: $M_0^{tot} = 7.10\times10^{19}$ Nm. Therefore, the annual rate is $\dot{M}_0^{tot} = 0.21\times10^{19}$ Nm/yr. Consequently, if T = 34 yrs, then:

$$\bar{v} \approx \bar{u}/T = \dot{M}_0^{tot}/(\mu S) = \frac{0.21\times10^{19}}{3\times10^{10}\times4.2\times10^{10}} = 1.67\text{mm/yr}$$

The estimated MORVEL velocity at (35.6°N,22.5°E) is 9.24 mm/yr. Therefore, the annual rate of seismic moment release is strongly underestimated dividing the total seismic moment release by the number of years in the catalog.

3. Write a computer program to calculate the strain rate tensor from a set of moment tensors using Kostrov's formula (11.4);

Solution:

The key code segment for this computer program is listed below:

```
m[0][0] = –sin(dip) * cos(rake) * sin(2 * strike) – sin(2 * dip) * sin(rake) * sin(strike) * sin(strike);
m[0][1] = sin(dip) * cos(rake) * cos(2 * strike) + 0.5 * sin(2 * dip) * sin(rake) * sin(2 * strike);
m[0][2] = –cos(dip) * cos(rake) * cos(strike) – cos(2 * dip) * sin(rake) * sin(strike);
m[1][0] = m[0][1];
m[1][1] = sin(dip) * cos(rake) * sin(2 * strike) – sin(2 * dip) * sin(rake) * cos(strike) * cos(strike);
```

```
m[1][2] = –cos(dip) * cos(rake) * sin(strike) + cos(2 * dip) * sin(rake) * cos(strike);
m[2][0] = m[0][2];
m[2][1] = m[1][2];
m[2][2] = sin(2 * dip) * sin(rake);
```

This calculates the components of the geometrical part of the moment tensor given a focal mechanism (dip, rake, strike). The average annual rate of seismic moment release in (11.4) is given by the ratio $\overline{M}_0 / T$. This quantity can be estimated using Molnar's formula (10.64). This approach requires a statistical fit of seismic moment release by the estimator:

$$N(M_0) = \alpha M_0^{-\beta}$$

where $\beta \cong 0.5$. To estimate α, we can use a standard least squares regression. Let us define the chi–squared statistic χ^2, which represents the goodness of the fit functional to be minimized, as follows:

$$\chi^2 = \sum_{k=1}^{n} \left[N_k - \alpha M_k^{-\beta} \right]^2$$

where $\{M_1, M_2, ..., M_n\}$ is an arbitrary set of scalar moments, $M_k < M_{k+1}$, and N_k is the *observed* annual number of events having scalar moment greater than or equal to M_k in the catalog. We have that χ^2 is minimum when:

$$0 = \frac{\partial \chi^2}{\partial \alpha} = -2 \sum_{k=1}^{n} M_k^{-\beta} \left[N_k - \alpha M_k^{-\beta} \right]$$

Therefore, we obtain:

$$\alpha = \frac{\sum_{k=1}^{n} N_k M_k^{-\beta}}{\sum_{k=1}^{n} M_k^{-2\beta}}$$

Finally, the annual rate of scalar moment release will be given by:

$$\dot{M}_0 = \frac{\alpha}{1-\beta} M_{0,\max}^{1-\beta}$$

4. Write a computer program to calculate the strain rate tensor from a set of asymmetric moment tensors using formula (11.17);

Solution:

In this computer program, the key code segment is listed below:

```
m[0][0] = –0.5 * (sin(dip) * cos(rake) * sin(2.0 * strike) + sin(2 * dip) * sin(rake) * sin(strike) * sin(strike));
m[0][1] = sin(dip) * cos(rake) * cos(strike) * cos(strike) + 0.25 * sin(2 * dip) * sin(rake) * sin(2 * strike);
m[0][2] = –cos(dip) * cos(rake) * cos(strike) – cos(dip) * cos(dip) * sin(rake) * sin(strike);
m[1][0] = –sin(dip) * cos(rake) * sin(strike) * sin(strike) + 0.25 * sin(2 * dip) * sin(rake) * sin(2 * strike);
 m[1][1] = 0.5 * (sin(dip) * cos(rake) * sin(2 * strike) - sin(2 * dip) * sin(rake) * cos(strike) * cos(strike));
 m[1][2] = –cos(dip) * cos(rake) * sin(strike) + cos(dip) * cos(dip) * sin(rake) * cos(strike);
m[2][0] = sin(dip) * sin(dip) * sin(rake) * sin(strike);
m[2][1] = –sin(dip) * sin(dip) * sin(rake) * cos(strike);
m[2][2] = 0.5 * sin(2 * dip) * sin(rake);
```

The code for determining the average annual rate of seismic moment release is the same of the previous exercise.

5. Perform a sudy of seismic deformation of Turkey using Kostrov’s formula and the asymmetric moment tensor approach. Discuss the differences between the two approaches.

Solution:

We can extract focal mechanisms from the global CMT catalog using the worksheet *CMT7609.xlsx* and a filter that selects the region: 42°N, 26°E, 36°N, 40°E. It is convenient to apply a further filter that selects crustal events between 0 and 40 km. The resulting data set contains 100 events and can be found in the Excel file *Turkey.xlsx*. A study of seismic deformation can be performed converting the table to ASCII and using the code illustrated in the previous exercises.

Chapter 12

1. Build a continental lithosphere geotherm using a layered crustal model, assuming 10 km upper crust, 10 km middle crust, and 15 km lower crust. Use reasonable values for the radiogenic heat rates of each layer and for the other parameters. In particular, use data from Hofmeister (1999) for the thermal conductivity;

Solution:

Following Furlong & Chapman (1987) (for the crust) and McKenzie et al. (2005) for the mantle we can assume the following parameters:

N	*Layer*	*Thickness* [km]	*Composition*	H'[μW m^{-3}]	k [Wm^{-1}K^{-1}]	ρ [kg m^{-3}]
1	Upper crust	10	Granite	2.75	2.5	2700
2	Middle crust	10	Amphibolites	0.70	2.4	2800
3	Lower crust	15	Granulites	0.37	2.1	2900
4	Upper Mantle	∞	Peridotites	0.02	3.1	3330

where $H' = H/\rho$ represents the radiogenic heat production density. We must solve Equation (12.18) for each layer, taking into account of the boundary conditions. Clearly, we must start from the uppermost layer. The solution (12.19) for the *i*–th layer can be written as follows:

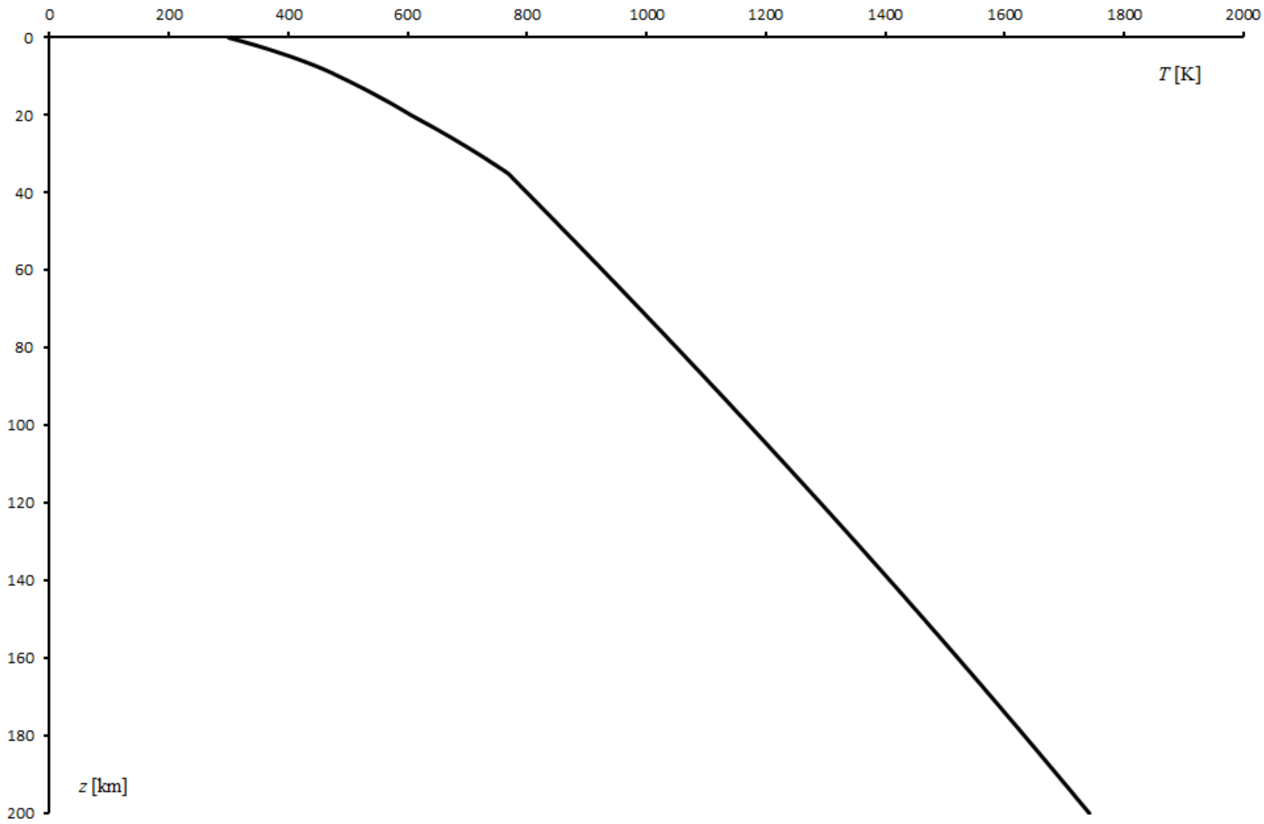
$$T(z) = a_i + b_i z - \frac{\rho_i H_i}{2k_i} z^2$$

where a_i and b_i must be determined using a continuity condition between layers. If z_i = 0,10,20,35 are layer boundary depths, then:

$$b_i = -\frac{1}{k_i}\left[q(z_{i-1}) - \rho_i H_i z_{i-1}\right]$$

$$a_i = T(z_{i-1}) - b_i z_{i-1} + \frac{\rho_i H_i}{2k_i} z_{i-1}^2$$

where q is the heat flux. The figure below shows a possible continental geotherm. The corresponding data can be found in *Solution 12.1.xlsx*.



2. Determine the thickness of the magnetic crustal layers in continental and oceanic regions;

Solution:

The thickness of the magnetic basement, z_c, coincides with the depth of the Curie geotherm for magnetite ($T_c \cong 585°C = 858$ K). In typical continental regions, the previous exercise suggests z_c = 49 km if the Moho is deeper than this height. In oceanic regions, the magnetic basement depth depends from the age, according to Fig. 12.9.

3. Determine the vertical slip rate along a fracture zone;

Solution:

Let Δt be the variation of age across a fracture zone. According to (12.68), the depth to the ocean floor increases with the square root of the distance from the ridge or, equivalently, with the square root of the age. If $t < 20$ Ma, then the thermal subsidence rate will be given by:

$$\frac{dw}{dt} = 182.5/\sqrt{t} \ \ [\mathrm{m/Myr}]$$

Therefore, the relative vertical slip between two oceanic blocks having age difference Δt will be:

$$s = 182.5\left(\frac{1}{\sqrt{t}} - \frac{1}{\sqrt{t+\Delta t}}\right)$$

For a maximum difference of 20 Ma, we obtain at t = 20 Ma: s = 0.065 m/Myr.

4. Write a computer program that converts an ocean floor age grid into a basement paleo–depth grid for any assigned time t in the geologic past. Any point having age $t' < t$ is removed from the output grid assigning a NODATA_value –99999.0;

Solution:

You can use the ocean floor agegrid of Müller et al. (1997) or the more recent version of Müller et al. (2008). Simply read each value of the input file sequentially. Then, apply (12.68) for each grid cell having $t' \geq t$. When $t' < t$, set the output grid value to –99999.0.

5. Determine the shear stress of a 100 Ma old and 100 km thick continental margin, which deforms by simple shear, assuming a vertical contact with the oceanic lithosphere. Repeat the exercise for a 20 Ma old continental margin and comment the differences;

Solution:

The shear stress across the COB arises from the fact that the oceanic lithosphere is subject to thermal subsidence, while the continental margin is buoyant. From (12.68) we have that the vertical slip rate for a 100 Ma old oceanic crust, is:

$$\frac{dw}{dt} = 68.75e^{-0.0278t} = 4.27 \ \ [\mathrm{m/Myr}]$$

Therefore, the predicted strain rate for a 100 km thick lithosphere would be:

$$\dot{\varepsilon} = 4.27\times10^{-5}\ \mathrm{Myr}^{-1} = 1.35\times10^{-18}\ \mathrm{s}^{-1}$$

Assuming Maxwell rheology and a viscosity $\eta = 10^{24}$ Pa s, we obtain a stress:

$$\tau = 2\eta\dot{\varepsilon} = 2.71\,\mathrm{MPa}$$

In the case of a 20 Ma old oceanic crust, we have that the vertical slip rate is given by:

$$\frac{dw}{dt} = 182.5/\sqrt{t} = 40.81\ [\mathrm{m/Myr}]$$

which is one order of magnitude greater. In this instance, the predicted strain rate for a 100 km thick lithosphere would be:

$$\dot{\varepsilon} = 4.08\times10^{-4}\ \mathrm{Myr}^{-1} = 1.30\times10^{-17}\ \mathrm{s}^{-1}$$

Assuming again Maxwell rheology and a viscosity $\eta = 10^{24}$ Pa s, we obtain a stress:

$$\tau = 2\eta\dot{\varepsilon} = 26\,\mathrm{MPa}$$

Therefore, the stress exerted on young margins is one order of magnitude greater than that exerted at old margins.

Chapter 13

1. Write and solve the equations of motion for a one–dimensional steady flow in the asthenosphere, considered as a 2–layers Newtonian incompressible fluid. It is assumed that the upper asthenosphere has viscosity $\eta_1 = 10^{20}$ Pa s and thickness $h_1 = 200$ km, while the lower layer has viscosity $\eta_2 = 10^{21}$ Pa s and thickness $h_2 = 200$ km. $v_0 = 100$ mm yr^{-1} is the velocity of the overlying lithosphere. Determine the depth of maximum velocity for a horizontal pressure gradient $\partial p/\partial x = -10$ kPa km^{-1};

Solution:

The simplest approach is to solve the following system of differential equations and boundary conditions:

$$\begin{cases} \dfrac{d^2 v_x}{dz^2} = \dfrac{1}{\eta_1}\dfrac{\partial p}{\partial x} \ ; \ v_x(h_1) = v_1 \ ; \ v_x(0) = v_0 \ ; \ 0 \le z \le h_1 \\ \dfrac{d^2 v_x}{dz^2} = \dfrac{1}{\eta_2}\dfrac{\partial p}{\partial x} \ ; \ v_x(h_1) = v_1 \ ; \ v_x(h_1 + h_2) = 0 \ ; \ h_1 \le z \le h_2 \end{cases}$$

Let us set:

$$k_1 \equiv \frac{1}{\eta_1}\frac{\partial p}{\partial x} = 10^{-19}\left[\mathrm{Pa^2 s\, m^{-1}}\right] ; \ k_2 \equiv \frac{1}{\eta_2}\frac{\partial p}{\partial x} = 10^{-20}\left[\mathrm{Pa^2 s\, m^{-1}}\right]$$

The first equation gives:

$$v_x(z) = \frac{1}{2}k_1 z^2 + a_1 z + b_1$$

Taking into account of the boundary conditions gives:

$$b_1 = v_0 \ ; \ v_1 = \frac{1}{2}k_1 h_1^2 + a_1 h_1 + v_0$$

From the second equation we get:

$$v_x(z) = \frac{1}{2}k_2 z^2 + a_2 z + b_2$$

Taking into account of the first boundary condition gives:

$$b_2 = \frac{1}{2}(k_1 - k_2)h_1^2 + (a_1 - a_2)h_1 + v_0$$

From the second boundary condition,

$$b_2 = -\frac{1}{2}k_2(h_1 + h_2)^2 - a_2(h_1 + h_2)$$

Equating the two expressions for b_2 gives:

$$\frac{1}{2}k_1h_1^2 + \frac{1}{2}k_2h_2^2 + a_1h_1 + a_2h_2 + v_0 + k_2h_1h_2 = 0$$

Now we take into account of the continuity of the stress component $\tau_{zx} = \tau_{zx}(z)$ along a vertical cross–section:

$$\tau_{xz}(z) = \eta\frac{\partial v_x}{\partial z} = \frac{1}{2}\frac{\partial p}{\partial x}z^2 + az$$

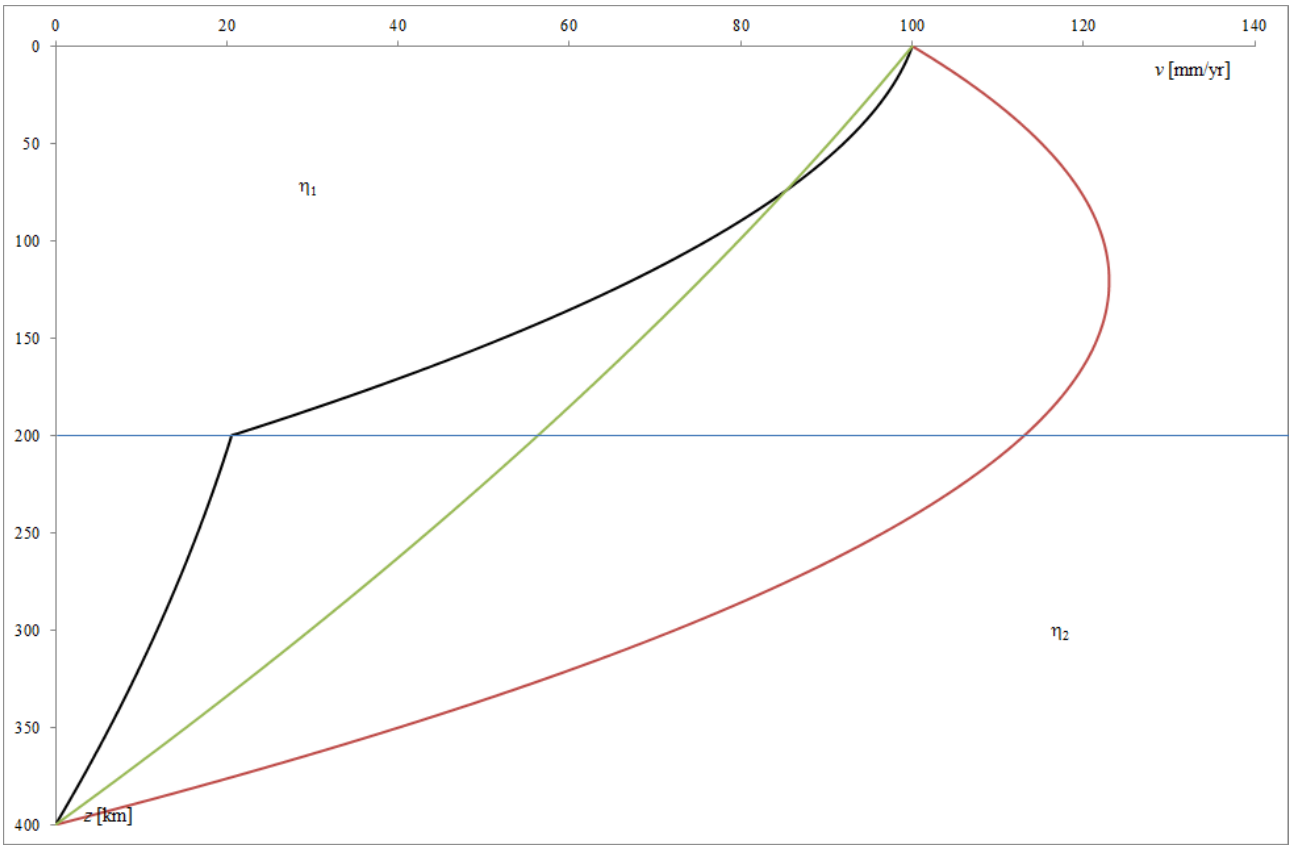
Note that this condition is less strong than a condition of continuity on the velocity derivative. Applying the condition at depth h_1 gives:

$$a_1\eta_1 = a_2\eta_2$$

Therefore,

$$a_2 = a_1\frac{k_2}{k_1}$$

Finally, we can easily solve for a_1, v_1, and b_2. The resulting velocity profile is shown in the figure below (see *Solution 13.1.xlsx*).



In this plot, the green and red lines represent velocity profiles for a homogeneous asthenosphere with viscosity η_1 and η_2, respectively, while the black line illustrates the profile in the case of two layers. In the latter case, the velocity has no maxima for $z \geq 0$.

2. Determine the horizontal pressure gradient that was necessary to accelerate India to v_0 = 180 mm yr^{-1} in the early Paleocene, assuming a 400 km thick asthenosphere and an average viscosity η = 10^{20} Pa s. Determine the time required to attain a velocity v = 100 mm yr^{-1} starting from v_0 = 0 mm yr^{-1};

Solution:

By (13.50), if v_0 = 180 mm yr^{-1} was close to the equilibrium velocity, then we would have a pressure gradient:

$$\frac{\partial p}{\partial x} = -\frac{2\eta v_0}{h^2} \cong 7.13\,\text{kPa km}^{-1}$$

To determine the time required to attain a velocity v_0 = 100 mm yr^{-1} starting from v_0 = 0, we consider that when the plate velocity is $v(t)$, the drag exerted by the asthenosphere along the LAB is (by (13.49)):

$$\tau_D(t) = -\frac{v(t)\eta}{h} - \frac{h}{2}\frac{\partial p}{\partial x}$$

Therefore, if A is the total LAB area, the total force exerted at time t is:

$$F_D(t) = \tau_D(t)A$$

The mass of India, m, could be calculated assuming an average lithosphere density ρ = 3290 kg m^{-3}, a LAB surface A = 0.147945 sr = 6,005,034 km^2, and an average thickness H = 200 km: $m = \rho V = \rho AH$. However, this calculation is not necessary, because the acceleration due to basal asthenospheric drag can be obtained directly from the stress. We have:

$$\dot{v}(t) = \frac{F_D(t)}{m} = \frac{\tau_D(t)A}{\rho AH} = \frac{\tau_D(t)}{\rho H}$$

Therefore, by (13.49) we have:

$$\dot{v}(t) = \frac{1}{\rho H}\left[-\frac{v(t)\eta}{h} - \frac{h}{2}\frac{\partial p}{\partial x}\right]$$

This differential equation can be rearranged as follows:

$$\dot{v}(t) + \frac{\eta}{\rho H h}v(t) = -\frac{h}{2\rho H}\frac{\partial p}{\partial x}$$

The solution of this first–order differential equation with the initial condition $v(0) = 0$ is:

$$v(t) = -\frac{h^2}{2\eta}\frac{\partial p}{\partial x}\left(1 - e^{-\frac{\eta}{\rho H h}t}\right)$$

To obtain the time at which $v(t) = 100$ mm yr^{-1} we simply substitute this velocity in the solution above and solve for t. The result is:

$$t = -\frac{\rho H h}{\eta}\ln\left(1 + \frac{2\eta v}{h^2 \partial p/\partial x}\right)$$

In the example of $v(t) = 100$ mm yr^{-1}, we obtain $t \cong 2.12$ μs *if* τ_D *is the unique stress exerted on India, or if* τ_D *is added to a torque balance*. Therefore, in this instance the continent would be pulled instantaneously to the final velocity $v_0 = 180$ mm/yr.

3. Let us assume that the lower boundary of a tectonic plate can be represented by a function $z = f(x)$. Assuming that no streamline has a cusp along this boundary, use the no–slip boundary condition to determine the boundary values of velocity for the asthenosphere along the LAB;

Solution:

4. Repeat the previous exercise assuming that the lower boundary of a tectonic plate is represented by a surface $z = f(x,y)$;

Solution:

5. Determine the corner flows of a subduction zone assuming that the subducting plate is at rest with respect to the transition zone and that the overriding plate moves with velocity v_0 in the positive x direction;

Solution:

You can first determine the velocity field in a reference frame fixed to the overriding plate. In this frame, the solution for ψ coincides with (13.97) and you calculate the components of velocity using (13.72). Then, you apply a classic Galilean transformation to move back to a reference frame fixed with the transition zone. The result is shown in Fig. 2.9b.

6. Prove the transformation equations (13.72);

Solution:

The transformation equations from/to polar and Cartesian coordinates are:

$$\begin{cases} x = r\cos\theta \\ z = r\sin\theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + z^2} \\ \theta = \arctan\left(\dfrac{z}{x}\right) \end{cases}$$

The derivatives of the stream function transform as follows:

$$\frac{\partial\psi}{\partial x}=\frac{\partial\psi}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial\psi}{\partial\theta}\frac{\partial\theta}{\partial x}=\cos\theta\frac{\partial\psi}{\partial r}-\frac{1}{r}\sin\theta\frac{\partial\psi}{\partial\theta}$$

$$\frac{\partial\psi}{\partial z}=\frac{\partial\psi}{\partial r}\frac{\partial r}{\partial z}+\frac{\partial\psi}{\partial\theta}\frac{\partial\theta}{\partial z}=\sin\theta\frac{\partial\psi}{\partial r}+\frac{1}{r}\cos\theta\frac{\partial\psi}{\partial\theta}$$

Therefore, by (13.63):

$$u_r=\frac{dr}{dt}=\frac{\partial r}{\partial x}u+\frac{\partial r}{\partial z}\upsilon=\cos\theta\frac{\partial\psi}{\partial z}-\sin\theta\frac{\partial\psi}{\partial x}=\frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

$$u_\theta=r\frac{d\theta}{dt}=r\frac{\partial\theta}{\partial x}u+r\frac{\partial\theta}{\partial z}\upsilon=-\sin\theta\frac{\partial\psi}{\partial z}-\cos\theta\frac{\partial\psi}{\partial x}=-\frac{\partial\psi}{\partial r}$$

7. Write the continuity equation for an incompressible fluid in polar coordinates;

Solution:

In Cartesian coordinates, we have:

$$\nabla\cdot\boldsymbol{v}=\frac{\partial u}{\partial x}+\frac{\partial\upsilon}{\partial y}=0$$

The transformation equations for the velocity are:

$$\begin{cases}u_r=u\cos\theta+\upsilon\sin\theta\\ u_\theta=-u\sin\theta+\upsilon\cos\theta\end{cases}$$

$$\begin{cases}u=u_r\cos\theta-u_\theta\sin\theta\\ \upsilon=u_r\sin\theta+u_\theta\cos\theta\end{cases}$$

Therefore, by the chain rule we obtain:

$$\frac{\partial u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{1}{r}u_r=0$$

8. Rewrite the equations of motion (13.69) in polar coordinates;

Solution:

In plane polar coordinates the Laplacian of a scalar field *f* can be written as follows:

$$\nabla^2 f=\frac{\partial^2 f}{\partial r^2}+\frac{1}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$$

Therefore, we have:

$$\begin{cases}-\cos\theta\dfrac{\partial P}{\partial r}+\dfrac{1}{r}\sin\theta\dfrac{\partial P}{\partial \theta}=\eta\left(\dfrac{\partial^2 u}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial u}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}\right)\\ -\sin\theta\dfrac{\partial P}{\partial r}-\dfrac{1}{r}\cos\theta\dfrac{\partial P}{\partial \theta}=\eta\left(\dfrac{\partial^2 \upsilon}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial \upsilon}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2 \upsilon}{\partial \theta^2}\right)\end{cases}$$

9. Find the velocity field (u,υ) for a fluid that is moving in the positive (downward) z direction relative to a spherical object fixed at the origin of the reference frame. This is a Stokes' flow. To solve the problem, you must solve the equations found in exercises (7) and (8). Assume that the velocity field is a uniform field in the z direction with magnitude v as $r \to \infty$.

Solution:

Chapter 14

1. Find the gravity anomaly generated by a spherical object at 1 km depth with density contrast $\Delta\rho = 200$ kg m^{-3};

Solution:

2. Find the gravity anomaly generated by an infinitely long horizontal dike at 1 km depth, with cylindrical cross–section of radius $R = 5$ m and density contrast $\Delta\rho = 100$ kg m^{-3};

Solution:

3. Find an expression for the thickness h_r of the crustal root generated by a mountain belt with average altitude h_m in excess of a normal continental crust, assuming that the lithospheric mantle has constant thickness;

Solution:

4. A gravimeter based on measurements of falling body trajectory has uncertainty $\pm 10^{-6}$ s on reading the arrival time at distance $d = 0.5 \pm 0.0001$ m. Estimate the uncertainty on gravity.

Solution: