

Supplementary Chapter A. 2-dimensional Basis Functions for General Unstructured Grids

In constructing our 2-dimensional basis functions for VDoHS rates we have considered five types of triangular elements: (i) the Lagrange second-order quadratic element with 6 parameters per element, (ii) a cubic element, which we denote as cubic(-), with 9 parameters per element instead of the 10 required for a general cubic function, (iii) another cubic element, which we denote as cubic(+), with 12 parameters per element and hence 2 independent quartic factors in the shape functions, (iv) a general quartic element with 15 parameters per element, and (v) the Argyris element which is a general quintic element with 21 parameters per element. The common feature of these elements is that each can be formulated with the independent parameters being function values or first or second derivatives, specified at either corner points or mid-points of sides of the elements, as indicated in Fig. A1. Consequently, at internal points within a grid the parameters are common to two or more triangles. This has the obvious advantage of reducing the number of independent parameters in the grid. In a typical triangular grid there are about half as many corner points and one and half times as many triangle sides as there are elements. It follows from the parameter types shown in Fig. A1 that the number of independent parameters per element for a scalar function is about 2 for the Lagrange element, 1.5 for the cubic(-) element, 3 for the cubic(+), and 4.5 for both the quartic and Argyris elements.

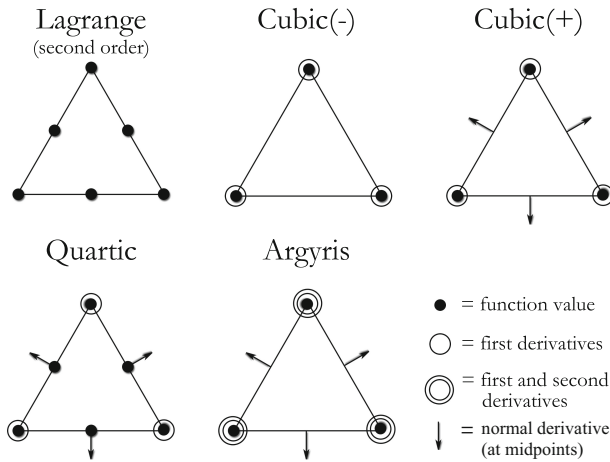


Fig. A1: The types of triangular elements considered in constructing 2-dimensional VDoHS rate basis functions.

The cubic(-) element, like the Lagrange second-order element, is fully accurate for general quadratic functions, while involving fewer independent parameters within a grid. As well, it accommodates all but one form of cubic function within each triangle. The form that cannot be accommodated is hat-shaped, with peaks at the midpoint and corners of the triangle, and is uniquely specified as being orthogonal within the triangle to every quadratic function and having equal coefficient values when triangle indices are interchanged. For the cubic(+) element, which is accurate for general cubic functions, the extra quartic terms are zero on the triangle edges and anti-symmetric in two of the triangle indices—though there are three such terms only two are independent, as the three terms sum to zero.

The big difference between the Argyris element and the other elements is that the Argyris element is the only one of the five elements that results in functions that have continuous derivatives everywhere along the sides where triangles meet. In this regard, the Argyris element is the simplest extension for triangular grids of the cubic splines used for the 1-dimensional basis functions in Figs 3 and 10. All the 2-dimensional elements result in continuous functions. With the Lagrange element, derivatives are discontinuous everywhere around the perimeters of the triangles. With the cubic(-) element derivatives are continuous only at corner points, while with the cubic(+) and quartic elements derivatives are continuous at midpoints of triangle sides as well. Given that the quartic element involves the same number of independent parameters per element as the Argyris element, the quartic element can be ruled out as the optimum element straight away.

The shape functions for the five elements are as follows. In these expressions, $\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3$ are the position vectors of the corner points of the triangle and ξ_1, ξ_2, ξ_3 are the spatially-linear triangle coordinates each with value 1 at the corresponding corner point and zero at the other corner points. These coordinates are such that $\xi_1 + \xi_2 + \xi_3 = 1$ and positions within the triangle are given by $\mathbf{x} = \xi_1 \mathbf{x}^1 + \xi_2 \mathbf{x}^2 + \xi_3 \mathbf{x}^3$. Additional quantities are $\chi_i = 1/|\nabla \xi_i|$, $\rho_{ji} = (\nabla \xi_i \cdot \nabla \xi_j)/|\nabla \xi_i|^2$ and $\kappa_i = (1 - 2\xi_i)\xi_1\xi_2\xi_3$ for $i, j = 1, 2, 3$. The shape functions listed below are those associated with function, first and second derivative values $f_1, (\nabla f)_1, (\nabla_i \nabla_j f)_1$ at the first corner point, and function and normal derivative values $f_{12}, (\mathbf{n} \cdot \nabla f)_{12}$ at the midpoint of the side between corner points 1 and 2. For the other corner points and midpoints the shape functions are obtained by straightforward interchange of the subscripts in the expressions.

Lagrange:

$$\begin{aligned}
& f_1 \left[2\xi_1^2 - \xi_1 \right] + \\
& f_{12} \left[4\xi_1 \xi_2 \right] + \dots
\end{aligned} \tag{A1}$$

Cubic(-):

$$\begin{aligned}
& f_1 \left[3\xi_1^2 - 2\xi_1^3 + 2\xi_1 \xi_2 \xi_3 \right] + \\
& \left(\mathbf{x}^2 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[\xi_1^2 \xi_2 + \frac{1}{2} \xi_1 \xi_2 \xi_3 \right] + \\
& \left(\mathbf{x}^3 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[\xi_1^2 \xi_3 + \frac{1}{2} \xi_1 \xi_2 \xi_3 \right] + \dots
\end{aligned} \tag{A2}$$

Cubic(+):

$$\begin{aligned}
& f_1 \left[3\xi_1^2 - 2\xi_1^3 - 6(\rho_{12}\kappa_2 + \rho_{13}\kappa_3) \right] + \\
& \left(\mathbf{x}^2 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[\xi_1^2 \xi_2 - \kappa_2 + (1 - \rho_{13}) \kappa_3 \right] + \\
& \left(\mathbf{x}^3 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[\xi_1^2 \xi_3 - \kappa_3 + (1 - \rho_{12}) \kappa_2 \right] + \\
& (\mathbf{n} \cdot \nabla f)_{12} \left[4\kappa_3 \chi_3 \right] + \dots
\end{aligned} \tag{A3}$$

Quartic:

$$\begin{aligned}
& f_1 \left[(2\xi_1 - 1)(5\xi_1^2 - 4\xi_1^3) - 6(\rho_{12}\kappa_2 + \rho_{13}\kappa_3) \right] + \\
& \left(\mathbf{x}^2 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[(2\xi_1 - 1)\xi_1^2 \xi_2 - \rho_{13}\kappa_3 \right] + \\
& \left(\mathbf{x}^3 - \mathbf{x}^1 \right) \cdot (\nabla f)_1 \left[(2\xi_1 - 1)\xi_1^2 \xi_3 - \rho_{12}\kappa_2 \right] + \\
& f_{12} \left[16\xi_1^2 \xi_2^2 + 16\kappa_3 \right] + \\
& (\mathbf{n} \cdot \nabla f)_{12} \left[4\kappa_3 \chi_3 \right] + \dots
\end{aligned} \tag{A4}$$

Argyris:

$$\begin{aligned}
& f_1 \left[6\xi_1^5 - 15\xi_1^4 + 10\xi_1^3 - 30(\rho_{12}\xi_3 + \rho_{13}\xi_2) \xi_1^2 \xi_2 \xi_3 \right] + \\
& (\mathbf{x}^2 - \mathbf{x}^1) \cdot (\nabla f)_1 \left[4\xi_1^3 \xi_2 - 3\xi_1^4 \xi_2 - (12\rho_{13} + 5\rho_{23}) \xi_1^2 \xi_2^2 \xi_3 - 5\xi_1^2 \xi_2 \xi_3^2 \right] + \\
& (\mathbf{x}^3 - \mathbf{x}^1) \cdot (\nabla f)_1 \left[4\xi_1^3 \xi_3 - 3\xi_1^4 \xi_3 - (12\rho_{12} + 5\rho_{32}) \xi_1^2 \xi_2 \xi_3^2 - 5\xi_1^2 \xi_2^2 \xi_3 \right] + \\
& \sum_i \sum_j (x_i^2 - x_i^1)(x_j^2 - x_j^1) (\nabla_i \nabla_j f)_1 \left[\frac{1}{2} \xi_1^3 \xi_2^2 - \left(\frac{3}{2} \rho_{13} + \rho_{23} \right) \xi_1^2 \xi_2^2 \xi_3 \right] + \\
& \sum_i \sum_j (x_i^2 - x_i^1)(x_j^3 - x_j^1) (\nabla_i \nabla_j f)_1 \left[\xi_1^3 \xi_2 \xi_3 - \xi_1^2 \xi_2^2 \xi_3 - \xi_1^2 \xi_2 \xi_3^2 \right] + \\
& \sum_i \sum_j (x_i^3 - x_i^1)(x_j^3 - x_j^1) (\nabla_i \nabla_j f)_1 \left[\frac{1}{2} \xi_1^3 \xi_3^2 - \left(\frac{3}{2} \rho_{12} + \rho_{32} \right) \xi_1^2 \xi_2 \xi_3^2 \right] + \\
& (\mathbf{n} \cdot \nabla f)_{12} \left[16\xi_1^2 \xi_2^2 \xi_3 \chi_3 \right] + \dots
\end{aligned} \tag{A5}$$

Schematically, for each type of element the function representation in a triangular grid is

$$f(\mathbf{x}) = \sum_k p_k \psi_k(\mathbf{x}). \tag{A6}$$

Here the parameters p_k are the independent function, first derivative and second derivative values at the corner points and midpoints of sides for all the triangles in the grid, and for each p_k the corresponding function $\psi_k(\mathbf{x})$ is comprised of the associated shape functions in all the triangles with p_k as one of the parameters. We now want to separate the parameters into distinct sets: S_0 is the set of function values at our basis function sites (see Fig. 5), S_f consists of function values at other corner points of triangles and all other parameters that are not second derivative values, and S_s (used only for the Argyris element) contains the second derivative values. In constructing our VDoHS rate basis functions, our aim is to reduce the set of free parameters to just the function values at our basis function sites in S_0 . For the set S_f we do this by finding the parameter values in S_f that minimise $\int |\nabla f(\mathbf{x})|^2 dS$ in terms of the parameter values in S_0 .

The presence of second derivative values among the parameters for the Argyris element means that we have to undertake a preliminary step, as minimising the spatial integral of the square of the magnitudes of the first derivatives of $f(\mathbf{x})$ is not a valid way to estimate second derivative values. On the other hand, minimising $\int \sum_i \sum_j |\nabla_i \nabla_j f(\mathbf{x})|^2 dS$ is appropriate. Writing

$$f(\mathbf{x}) = \sum_{k \in S_0 \cup S_f} p_k \psi_k(\mathbf{x}) + \sum_{k \in S_s} p_k^s \psi_k^s(\mathbf{x}), \quad (\text{A7})$$

and minimising this integral with respect to the parameter values p_k^s in S_s we obtain the following solution. With G^{ss} denoting the matrix with elements

$$G_{kq}^{ss} = \int \sum_i \sum_j (\nabla_i \nabla_j \psi_k^s(\mathbf{x})) (\nabla_i \nabla_j \psi_q^s(\mathbf{x})) dS \quad (\text{A8})$$

for p_k^s and p_q^s in S_s , G^s denoting the matrix with elements

$$G_{kq}^s = \int \sum_i \sum_j (\nabla_i \nabla_j \psi_k^s(\mathbf{x})) (\nabla_i \nabla_j \psi_q(\mathbf{x})) dS \quad (\text{A9})$$

for p_k^s in S_s and p_q in S_0 or S_f , and introducing $P^s = -(G^{ss})^{-1} G^s$, the parameter values p_k^s are given by $p_k^s = \sum_{q \in S_0 \cup S_f} P_{kq}^s p_q$.

By substituting in these values for the parameters p_k^s , we get an interim expression for $f(\mathbf{x})$ involving only parameter values in S_0 and S_f :

$$f(\mathbf{x}) = \sum_{k \in S_0 \cup S_f} p_k \left[\psi_k(\mathbf{x}) + \sum_{q \in S_s} P_{kq}^s \psi_q^s(\mathbf{x}) \right]. \quad (\text{A10})$$

This expression for the Argyris element is equivalent in form to the expressions for the other four types of element, which don't involve second derivatives as parameters. The only difference here is that for the Argyris element $\psi_k(\mathbf{x})$ is replaced by $\psi_k(\mathbf{x}) + \sum_{q \in S_s} P_{kq}^s \psi_q^s(\mathbf{x})$. To keep expressions below as simple as possible, we will use $\psi_k(\mathbf{x})$ to mean $\psi_k(\mathbf{x}) + \sum_{q \in S_s} P_{kq}^s \psi_q^s(\mathbf{x})$ in the Argyris case.

The minimisation of $\int |\nabla f(\mathbf{x})|^2 dS$ proceeds in the same way to obtain expressions for the corresponding parameter values p_k^f in S_f . Writing

$$f(\mathbf{x}) = \sum_{k \in S_0} p_k^0 \psi_k^0(\mathbf{x}) + \sum_{k \in S_f} p_k^f \psi_k^f(\mathbf{x}), \quad (\text{A11})$$

and minimising the integral with respect to the parameter values p_k^f in S_f , the solution is as follows. With G^{ff} denoting the matrix with elements

$$G_{kq}^{ff} = \int (\nabla \psi_k^f(\mathbf{x})) \cdot (\nabla \psi_q^f(\mathbf{x})) dS \quad (\text{A12})$$

for p_k^f and p_q^f in S_f , G^f denoting the matrix with elements

$$G_{kq}^f = \int (\nabla \psi_k^f(\mathbf{x})) \cdot (\nabla \psi_q(\mathbf{x})) dS \quad (\text{A13})$$

for p_k^f in S_f and p_q in S_0 , and introducing $P^f = -(G^{ff})^{-1} G^f$, the parameter values p_k^f are given by $p_k^f = \sum_{q \in S_0} P_{kq}^f p_q$.

Furthermore, we have reduced the expression for $f(\mathbf{x})$ to the form

$$f(\mathbf{x}) = \sum_{k \in S_0} p_k \left[\psi_k(\mathbf{x}) + \sum_{q \in S_f} P_{qk}^f \psi_q^f(\mathbf{x}) \right] = \sum_{k \in S_0} p_k \phi_k(\mathbf{x}), \quad (\text{A14})$$

in which the functions $\phi_k(\mathbf{x}) = \psi_k(\mathbf{x}) + \sum_{q \in S_f} P_{qk}^f \psi_q^f(\mathbf{x})$ are recognisable as being

our basis functions. A property that may not be obvious is that each $\phi_k(\mathbf{x})$ has value 1 at the corresponding basis function site and is zero at the other basis function sites. This follows from the original $\psi_k(\mathbf{x})$ for the function value at the basis function site having this property, plus all the shape functions that contribute to the forms $\psi_k^f(\mathbf{x})$ and $\psi_k^s(\mathbf{x})$ being zero at all the basis function sites.

Another property of the basis functions $\phi_k(\mathbf{x})$ is that for each element type the integral $\int |\nabla \phi_k(\mathbf{x})|^2 dS$ has a nearly uniform value that is effectively the same for all grids, independent of the spacing and arrangement of the basis function sites.

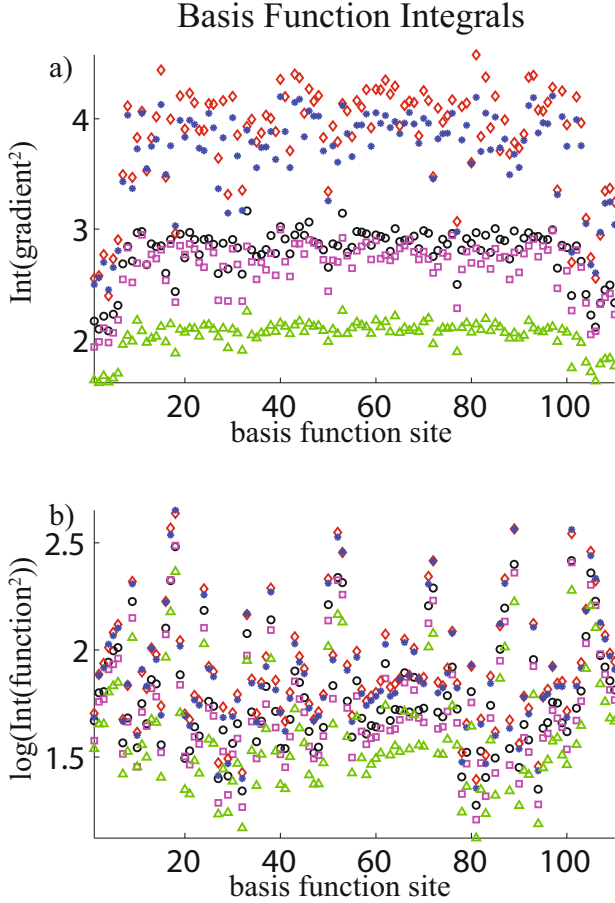


Fig. A2: The integrals $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ (top panel a) and $\int |\varphi_k(\mathbf{x})|^2 dS$ (bottom panel b) for basis functions constructed using Lagrange (green), Cubic(-) (blue), Cubic(+) (red), Quartic (magenta) and Argyris (black) elements for the grid in Fig. 5. Basis function numbers on the horizontal axes are ordered according to the x values of the positions in Fig. 5 of the basis function sites. The integral values in the top panel are unit-less, whereas the units in the bottom panel are km^2 and the scale is logarithmic, with the values of the integrals spanning over one and a half orders of magnitude.

The integral $\int |\varphi_k(\mathbf{x})|^2 dS$, on the other hand, is highly variable and is proportional to the area spanned by each basis function. The values of these two integrals for the grid in Fig. 5 are shown in Fig. A2, with the second integral $\int |\varphi_k(\mathbf{x})|^2 dS$ being shown on a logarithmic scale because of its wide range of values. The basis function sites are ordered from left to right according to their x positions in Fig. 5. To understand the

integral values, note that the four basis functions illustrated in Fig. 6 have their largest values in the regions surrounding the corresponding basis function sites $\mathbf{x}^0$. Note also that three in particular resemble Gaussians in those regions of the form

$$\varphi(\mathbf{x}) = \exp\left(-\frac{1}{2\sigma^2}|\mathbf{x} - \mathbf{x}^0|^2\right) \text{ with different values of } \sigma. \text{ Such Gaussian func-}$$

tions have integral values $\int |\nabla \varphi(\mathbf{x})|^2 dS = \pi$ and $\int |\varphi(\mathbf{x})|^2 dS = \pi\sigma^2$. The exception in Fig. 6a is the basis function associated with the function site close to the left-hand boundary. Zero-normal-derivative boundary conditions are applied at the left-hand and right-hand boundaries. This results in smaller values of $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ for basis functions near those boundaries, which have their integral values plotted at the left-hand and right-hand ends in Fig. A2.

The basis functions using the Argyris element are especially Gaussian-like, with values of $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ close to 3. Those using the quartic element are similar with slightly smaller values of both $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ and $\int |\varphi_k(\mathbf{x})|^2 dS$. The basis functions using the Lagrange element have comparatively spikey, less-rounded peaks, which result in values of $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ close to 2 and also smaller values of $\int |\varphi_k(\mathbf{x})|^2 dS$ than for the other elements. On the other hand, the basis functions using the cubic(-) and cubic(+) elements have comparatively flat peaks and steeper flanks. Their $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ values are less consistent and the range of values varies for different grids, with $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ and $\int |\varphi_k(\mathbf{x})|^2 dS$ being always larger than for the Argyris element. For the grid in Fig. 5 the average value of $\int |\nabla \varphi_k(\mathbf{x})|^2 dS$ is about 4 for the cubic(+) element and slightly less than 4 for the cubic(-) element.

The Argyris element is the best choice of the five types of element we have tested, based primarily on continuity of first derivatives. The quartic element follows closely behind, except regarding continuity of first derivatives. Given that the Argyris and quartic elements involve virtually the same number of independent parameters within a grid, the lack of complete continuity of first derivatives for the quartic element means it is inferior to the Argyris element. In the remaining figures in this Chapter we demonstrate that whichever element is chosen makes very little difference when applying the basis functions in our inversion methodology. Even so, there is a clear progression.

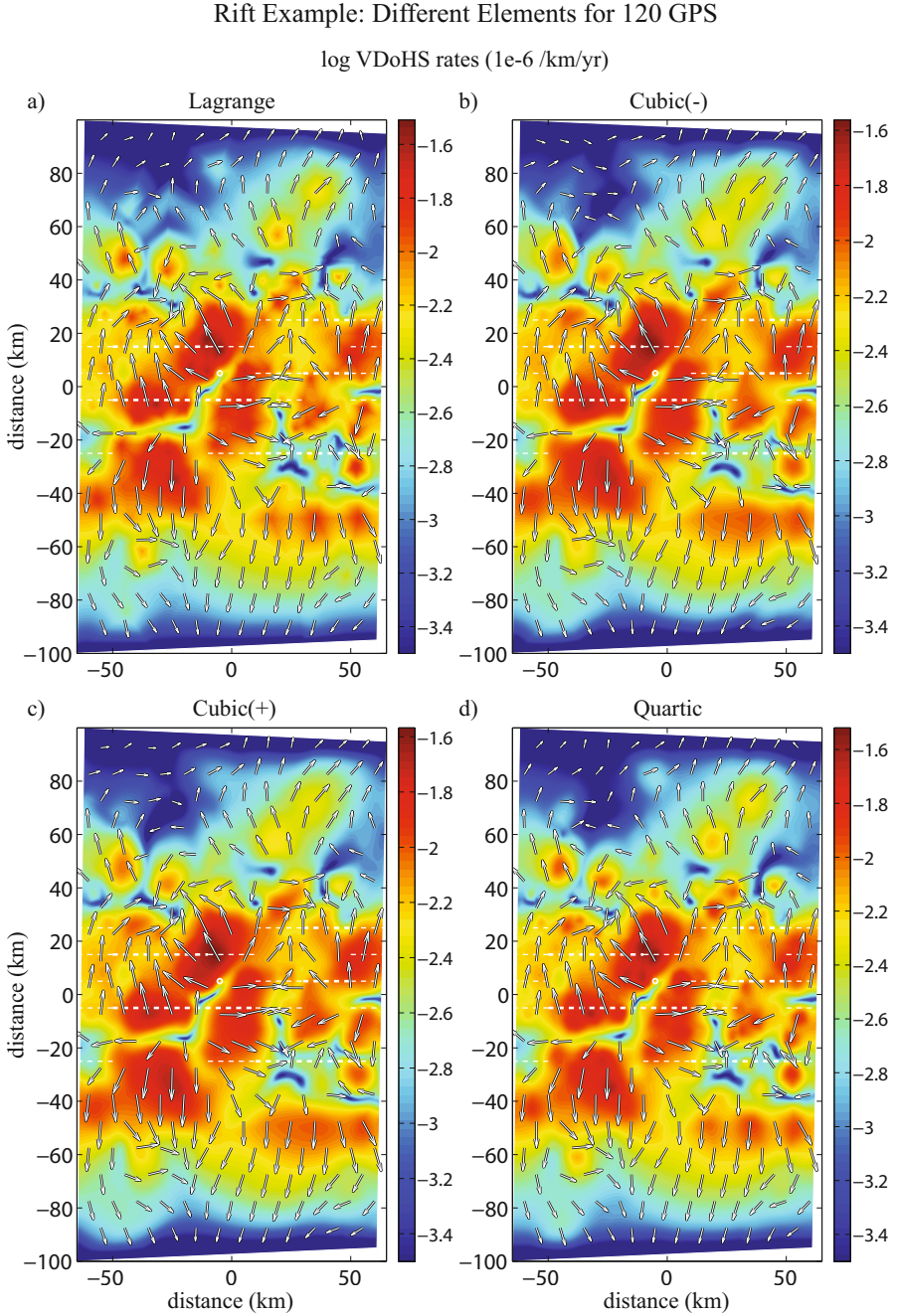


Fig. A3: VDoHS rates scaled by shear modulus from inversions for our rift example using different element types in constructing the VDoHS rate basis functions for the dataset with 120 GPS sites for the case of velocity standard error 0.2 mm/yr. The corresponding results using the Argyris element are in Fig. 27a.

Rift Example: Difference from Argyris for 120 GPS

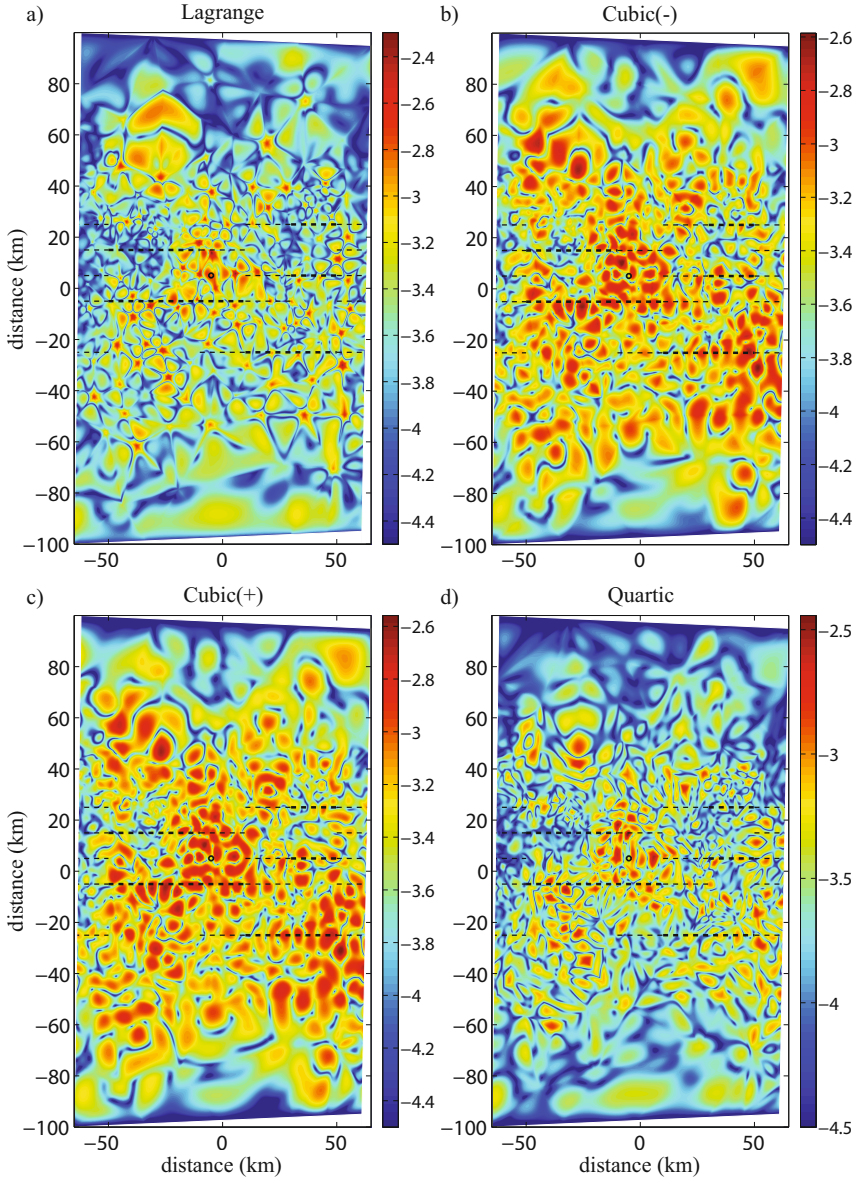
log VDoHS rate difference (10^{-6} /km/yr)

Fig. A4: Magnitudes of VDoHS rate differences scaled by shear modulus from inversions for our rift example using different element types in constructing the VDoHS rate basis functions for the dataset with 120 GPS sites for the case of velocity standard error 0.2 mm/yr. The differences plotted are relative to the VDoHS rates using the Argyris element in Fig. 27a, and the scales are logarithmic. Note that the differences are typically an order of magnitude smaller than the VDoHS rates.

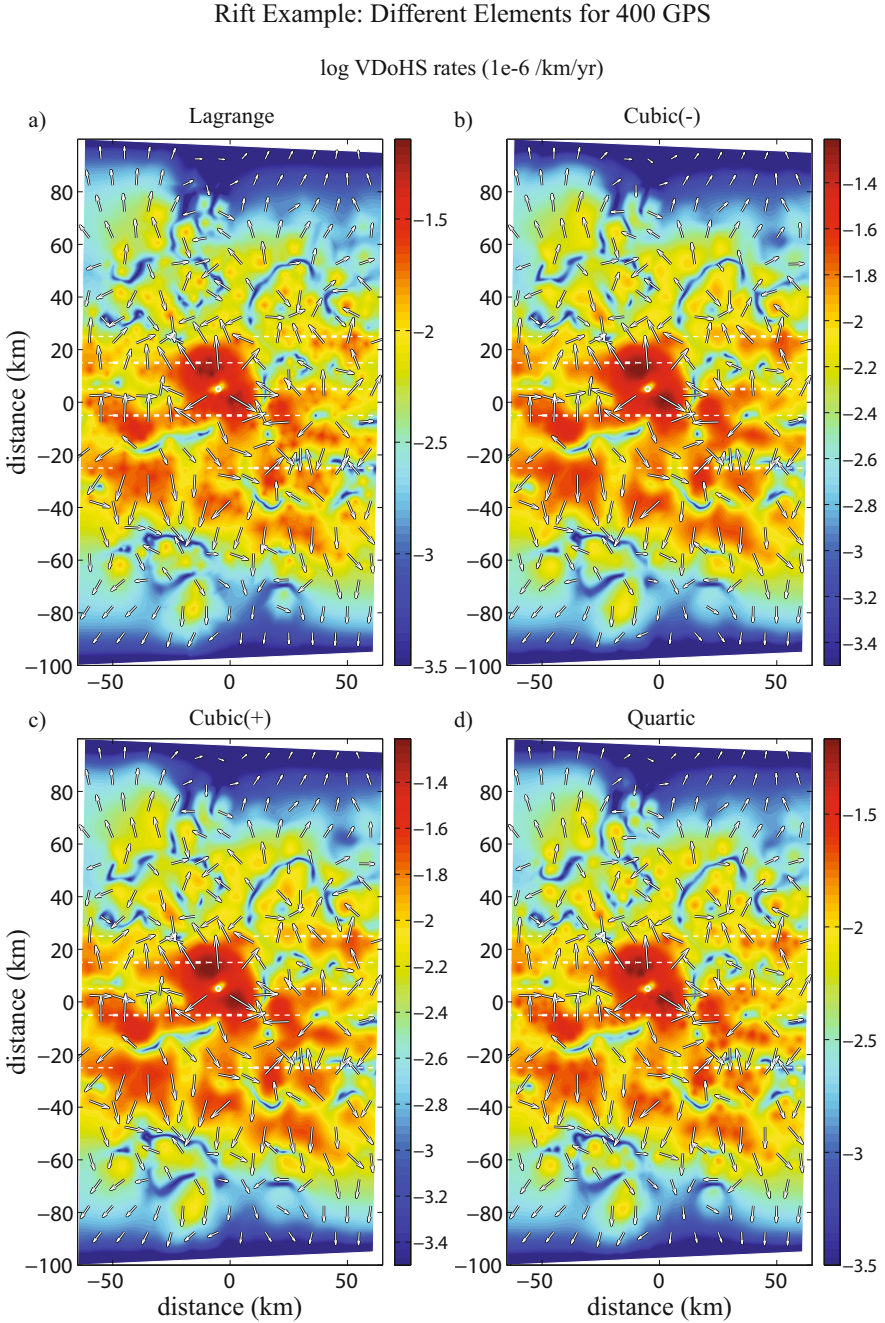


Fig. A5: VDoHS rates scaled by shear modulus from inversions for our rift example using different element types in constructing the VDoHS rate basis functions for the dataset with 400 GPS sites for the case of velocity standard error 0.2 mm/yr. The corresponding results using the Argyris element are in Fig. 27c.

Rift Example: Difference from Argyris for 400 GPS

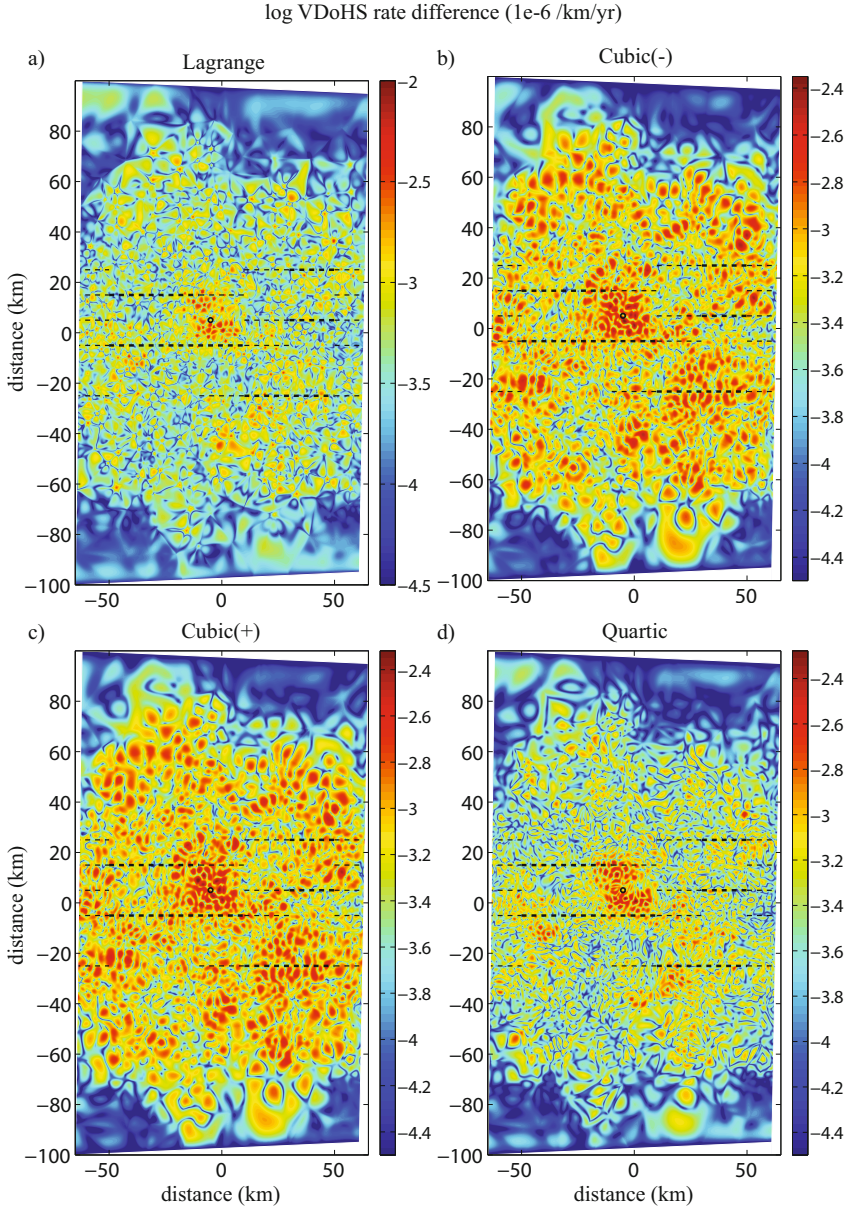


Fig. A6: Magnitudes of VDoHS rate differences scaled by shear modulus from inversions for our rift example using different element types in constructing the VDoHS rate basis functions for the dataset with 400 GPS sites for the case of velocity standard error 0.2 mm/yr. The differences plotted are relative to the VDoHS rates using the Argyris element in Fig. 27c, and the scales are logarithmic. Note that the differences are typically an order of magnitude smaller than the VDoHS rates.

The quartic element giving results that are closest to those with the Argyris element. Next come the cubic(+) and cubic(-) elements. The Lagrange element gives results that are least satisfactory and most obviously mesh dependent. Of the other elements, the Argyris is clearly the best at giving results that show little or no influence due to the location of triangle boundaries.

In Fig. A3 we show VDoHS rate results from inversions for our rift example in Fig. 25 for the GPS dataset with 120 sites and data standard error 0.2 mm/yr in Fig. 19. Here the Lagrange, cubic(-), cubic(+) and quartic elements are used, and the results are to be compared with the corresponding results using the Argyris element in the top left panel of Fig. 27. All five VDoHS rate solutions are very similar. This is emphasised in Fig. A4. There the magnitudes of the differences from the Argyris results are plotted. The corresponding results for the GPS dataset with 400 sites are shown in Figs A5 and A6. In this case, the reference results using the Argyris element are those in the bottom left panel of Fig. 27. Overall, the differences from the Argyris results are about an order of magnitude smaller than the magnitudes of the VDoHS rates. The largest differences are for the Lagrange element, with the cubic(-), cubic(+) and quartic elements having approximately the same biggest differences. The feature in Figs A4 and A6 that shows the quartic element gives the best agreement with Argyris results is that big differences for the cubic(-) and cubic(+) elements are spread over a much larger area.

Supplementary Chapter B. Supplementary Figures

There are 14 supplementary figures in addition to those in Supplementary Chapter A. They are referenced in the following sections of the book.

Fig. B1 is referenced in Section 4.4.

Figs B2-B8 are referenced in Section 5.4.

Figs B9-B12 are referenced in Section 6.3.

Figs B13-B14 are referenced in Section 3.2, and relate to the rift example in Chapter 6.

Random Additions to GPS Velocities

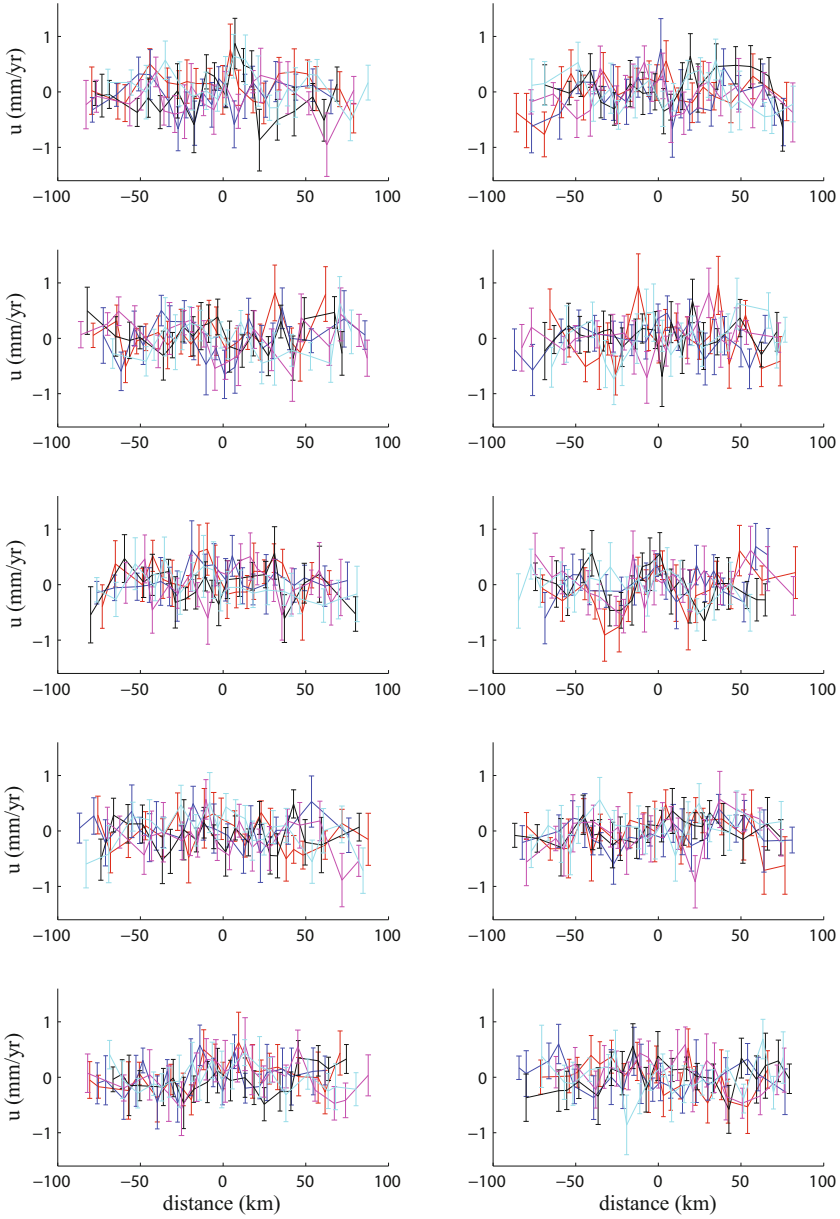


Fig. B1: The random additions for the 50 datasets of GPS velocities used in the inversions in Figs 10-12, shown as 10 groups of 5 datasets. Locations of the 20 GPS sites in each dataset are shown, and the error bars show the randomly chosen data standard errors, taken from a triangular probability distribution between 0.2 mm/yr and 0.6 mm/yr with average value 1/3 mm/yr. The random additions themselves are taken from Gaussian probability distributions with zero means and standard deviations equal to the standard error values.

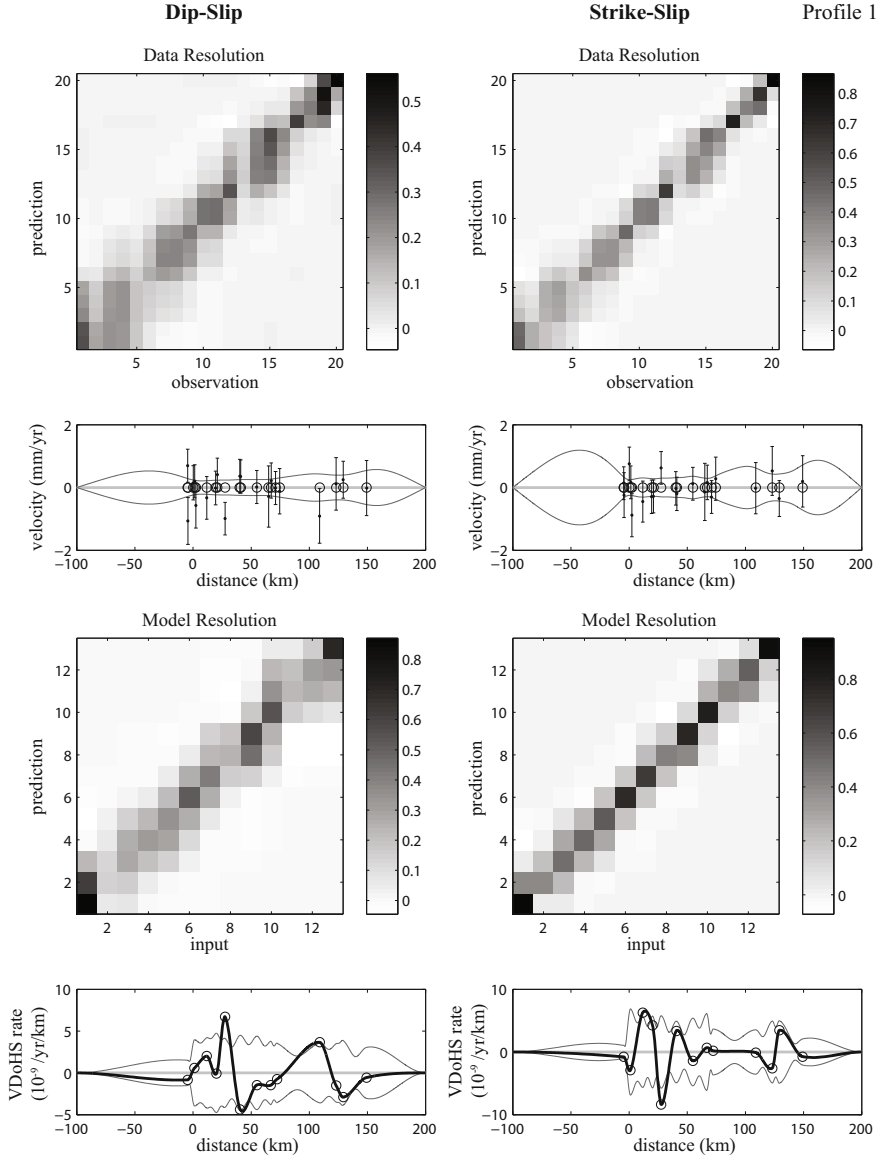


Fig. B2: Data and model resolution for profile 1, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

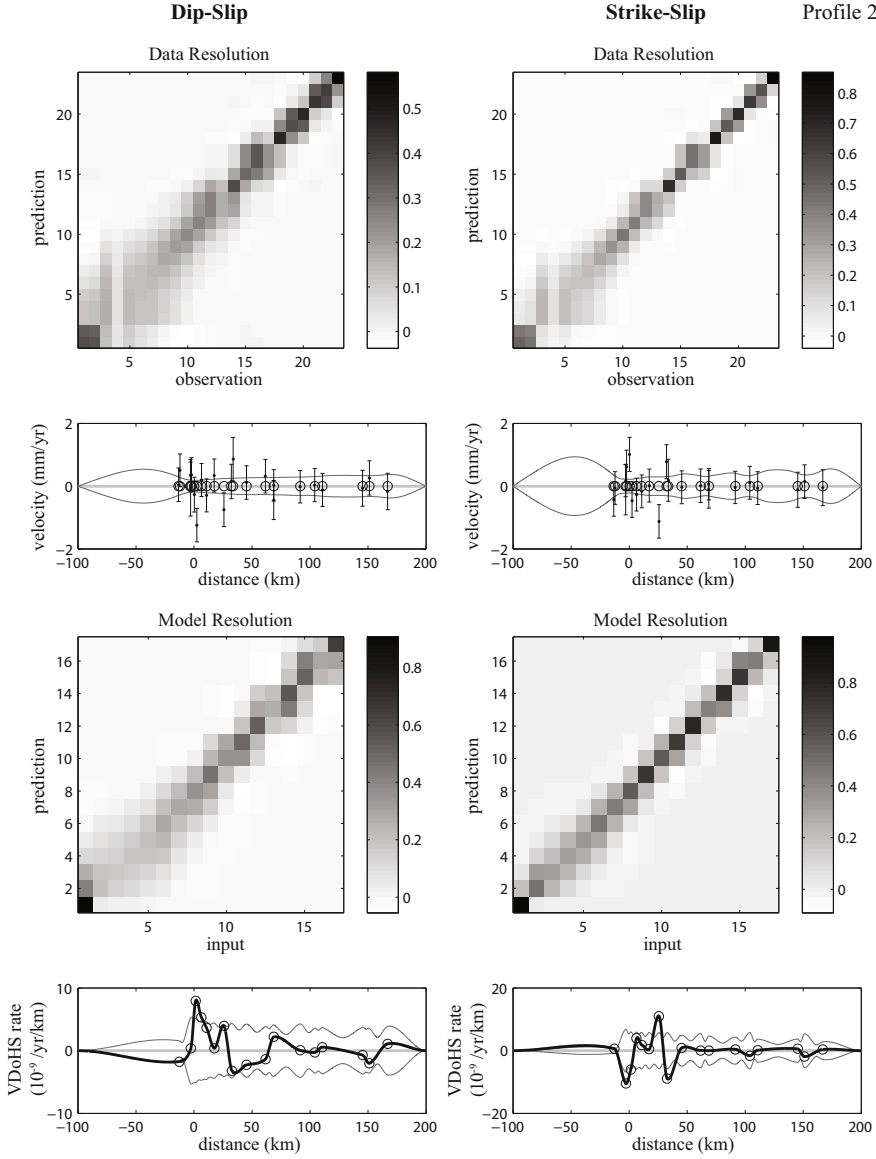


Fig. B3: Data and model resolution for profile 2, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

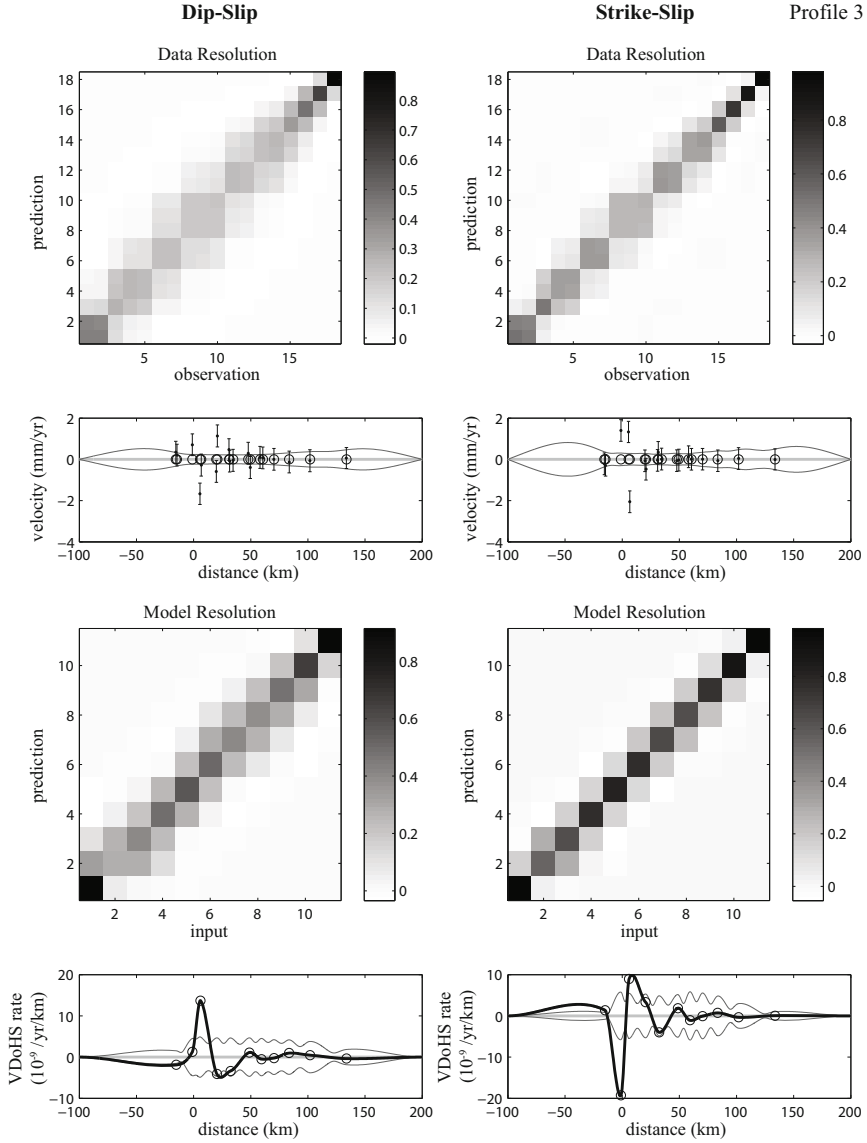


Fig. B4: Data and model resolution for profile 3, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

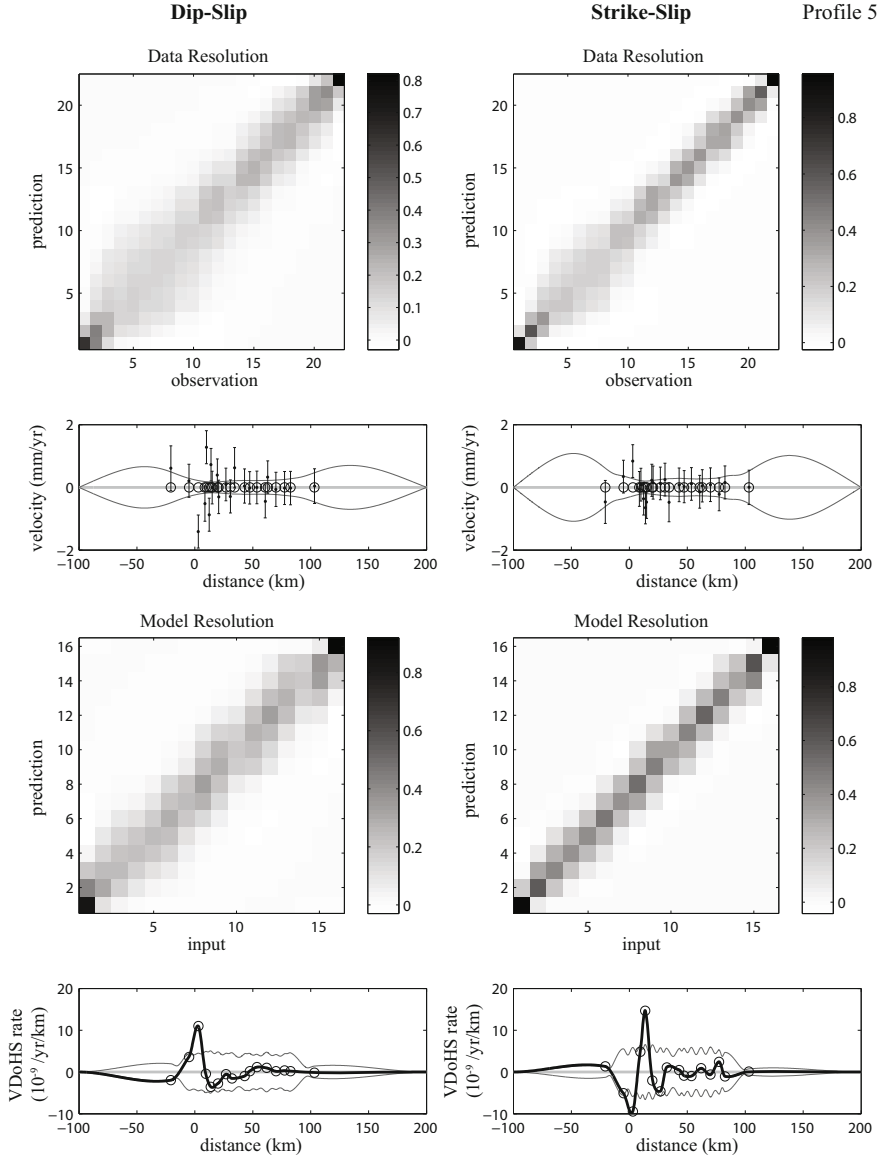


Fig. B5: Data and model resolution for profile 5, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

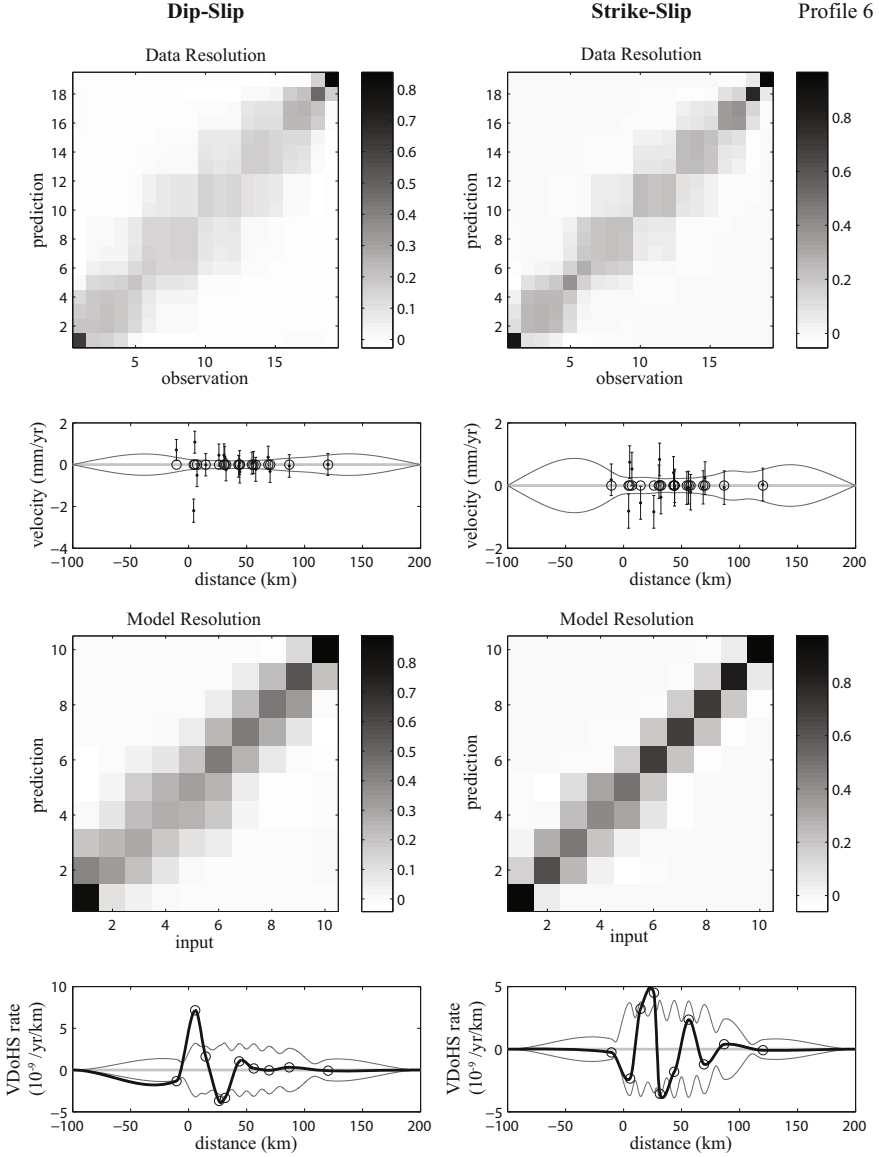


Fig. B6: Data and model resolution for profile 6, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $d - R_d d$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d d$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{m} - \tilde{m}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{m}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

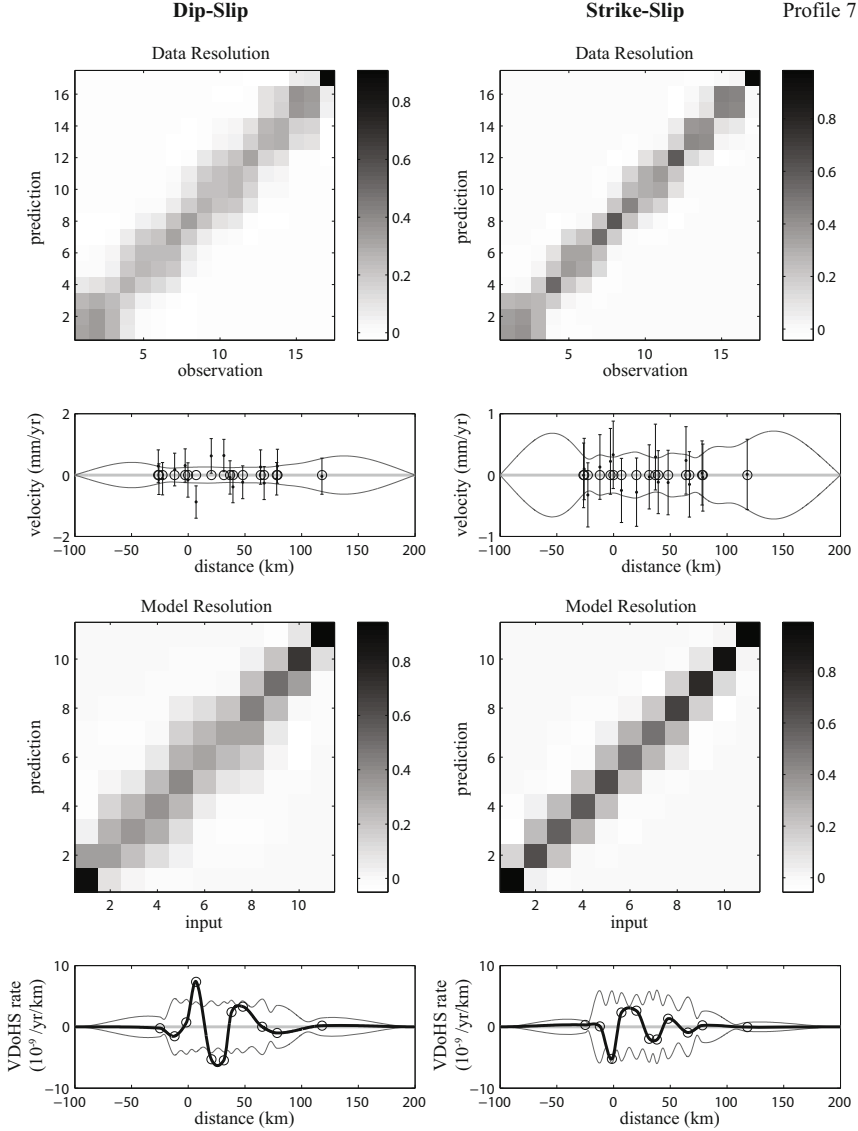


Fig. B7: Data and model resolution for profile 7, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

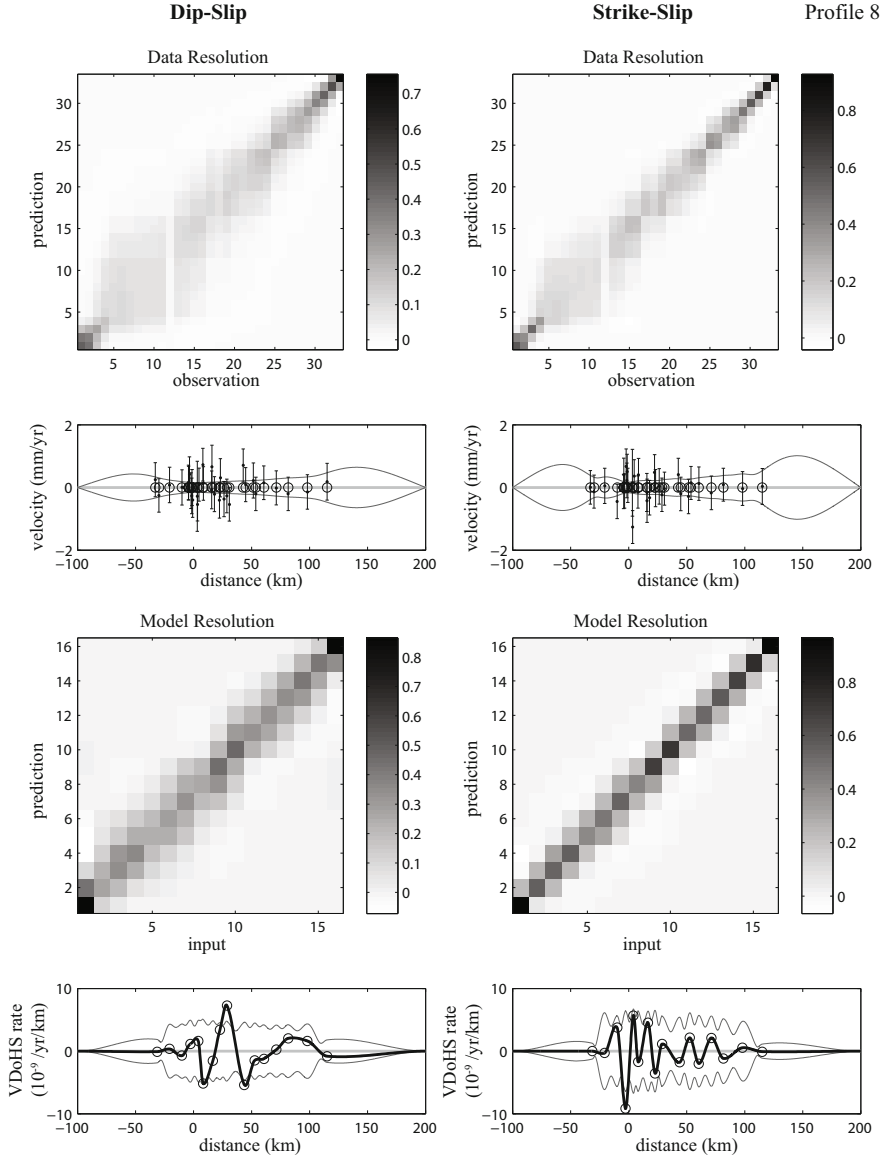


Fig. B8: Data and model resolution for profile 8, for dip-slip (left) and strike-slip (right). Top panels show the data resolution matrix R_d , second panels $\mathbf{d} - R_d \mathbf{d}$ with observational standard errors (error bars) and plus and minus one standard error for the fitted model $R_d \mathbf{d}$ (curves), third panels the model resolution matrix R_m , bottom panels $R_m \tilde{\mathbf{m}} - \tilde{\mathbf{m}}$ (thick black curve) with plus and minus one standard error for the fitted model $\tilde{\mathbf{m}}$ (grey curves). Locations of GPS sites (second panels) and basis function sites for VDoHS rate (bottom panels) are shown as circles.

Rift Example: Effect of Sampling

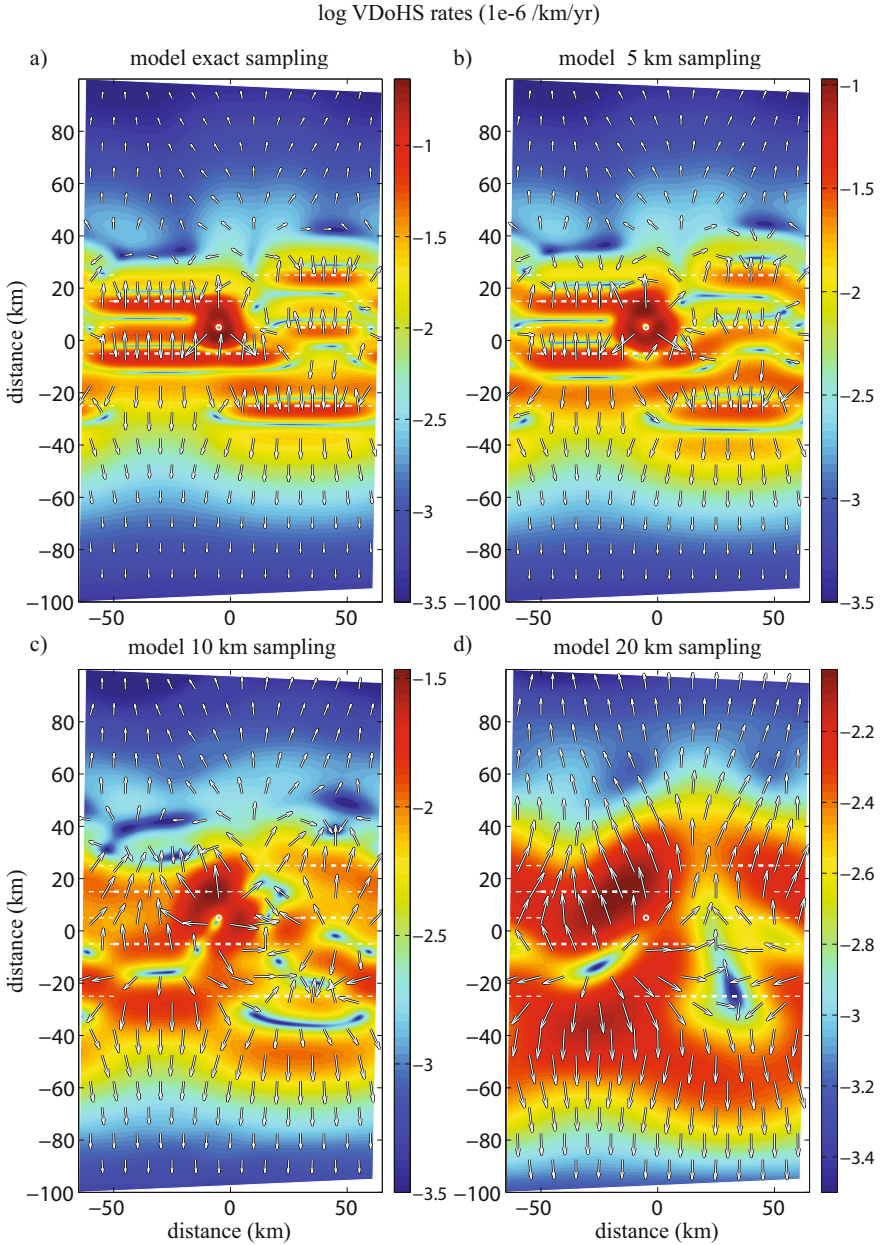


Fig. B9: VDoHS rates scaled by shear modulus for the rift example from regular sampling of the exact velocities on rectangular grids with sampling distances 5, 10 and 20 km (panels b-d), compared with the exact values (panel a). Plotting of the VDoHS rates is the same as in Fig. 25d, and results for intermediate sampling distances 7 and 14 km are shown in Figs 27d and b.

Rift Example: Strain Rates with 0.2mm/yr Standard Error

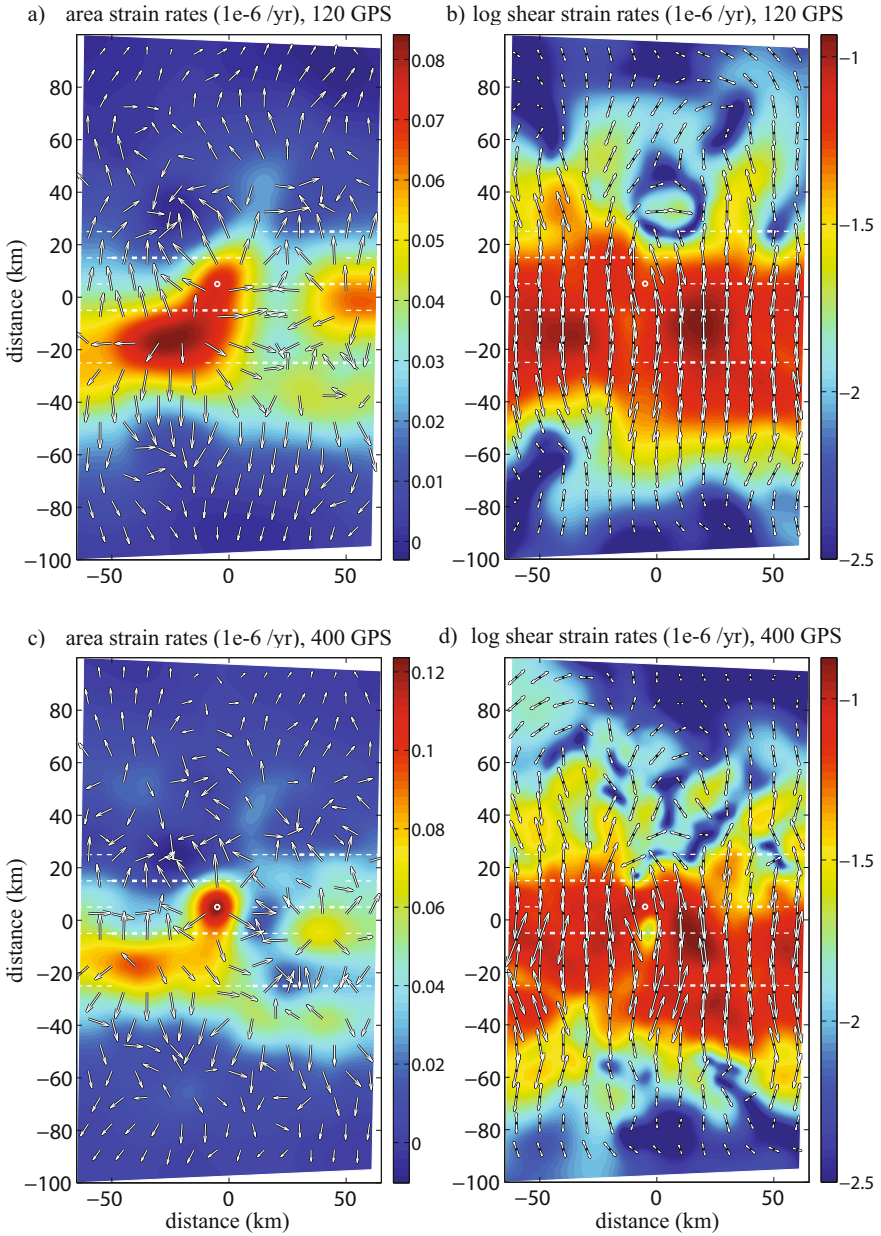


Fig. B10: Strain rates for the rift example from inversions for the cases of 0.2 mm/yr standard errors. Plotting of the area strain rates and shear strain rates is the same as in Figs 25b and c respectively. Note that the only source clearly imaged is the magma chamber, in the plot of area strain rate for the GPS dataset with 400 sites.

Rift Example: Strain Rates with 0.4mm/yr Standard Error

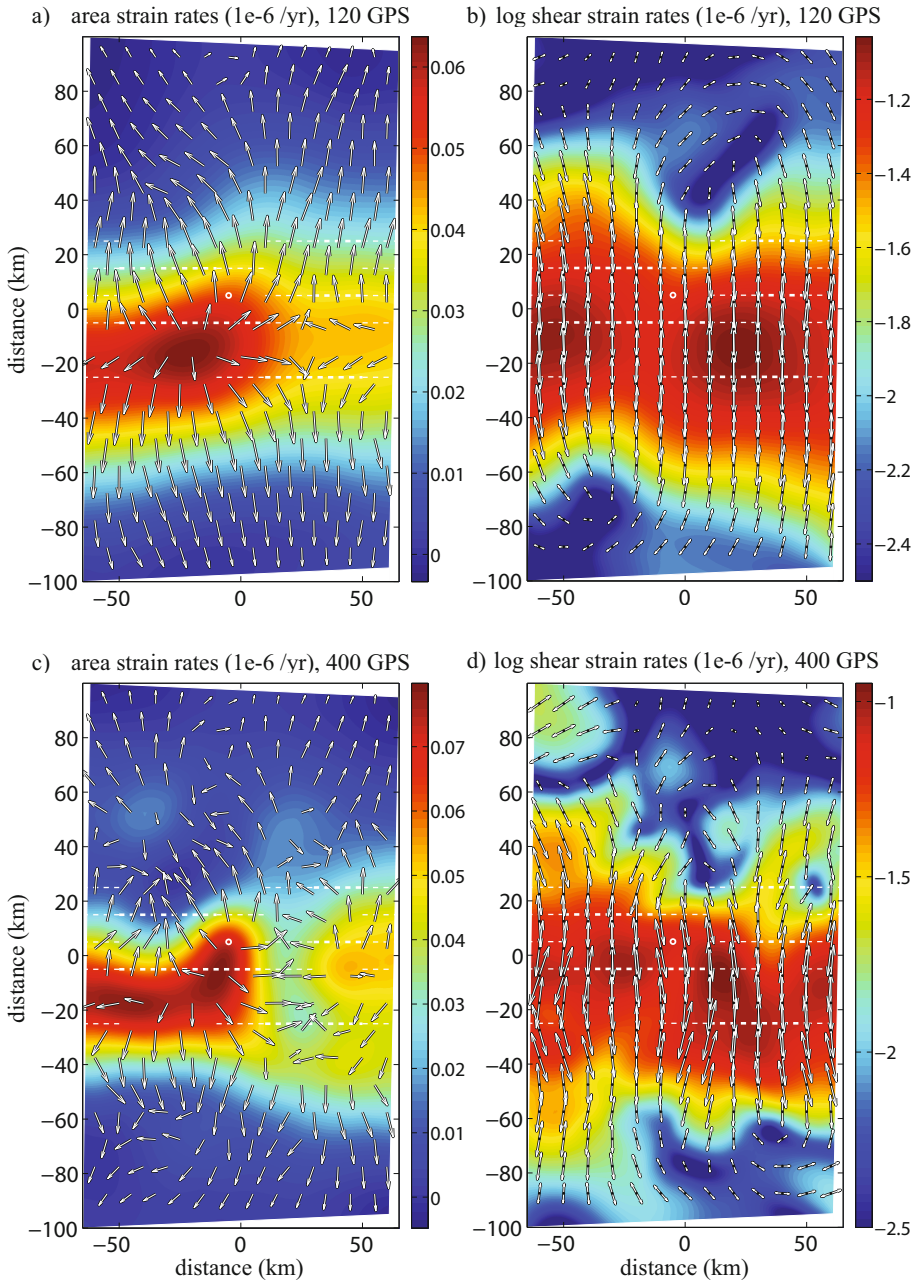


Fig. B11: Strain rates for the rift example from inversions for the cases of 0.4 mm/yr standard errors. Plotting of the area strain rates and shear strain rates is the same as in Figs 25b and c respectively.

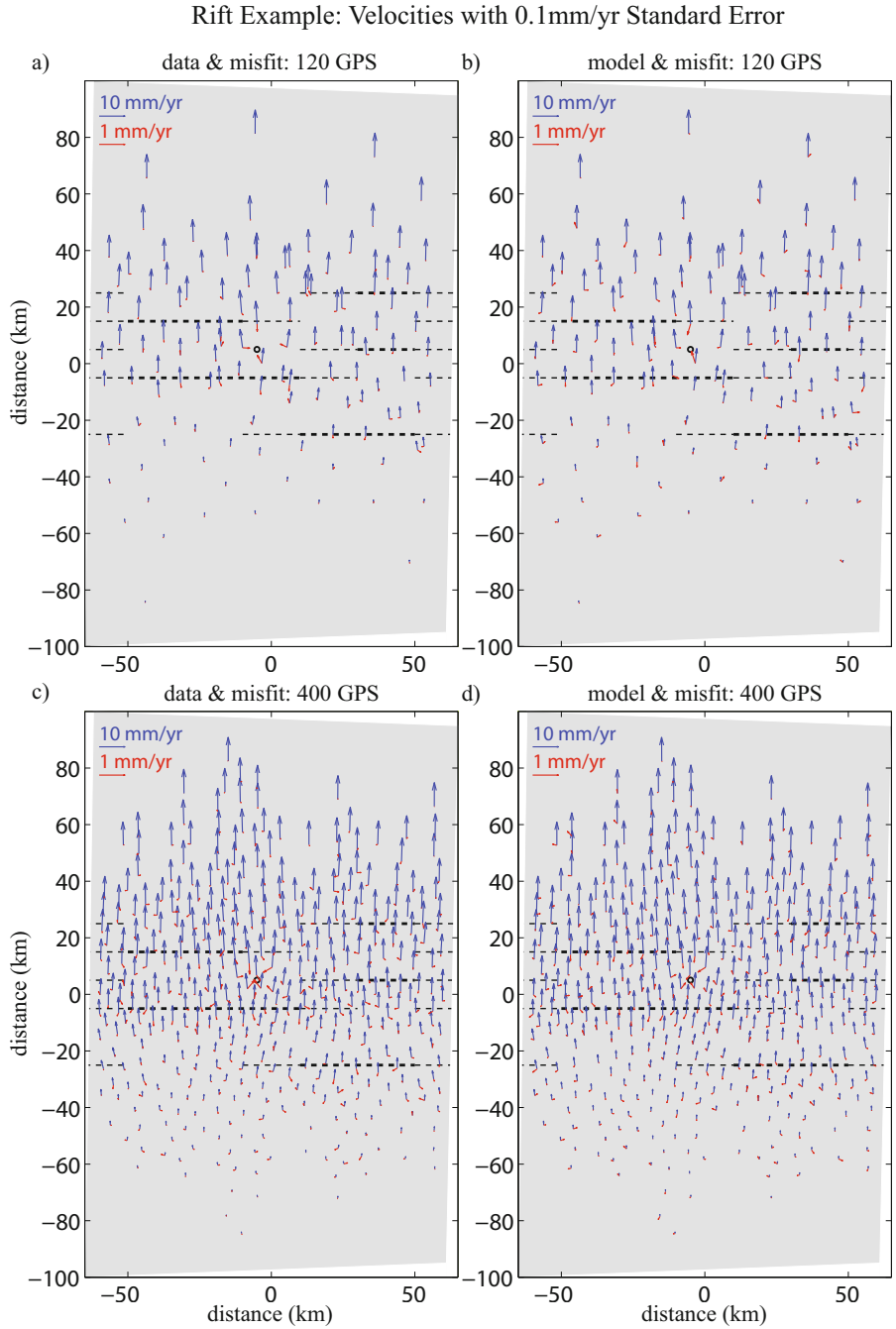


Fig. B12: Velocities (blue) and misfits (red) at GPS sites for the two random rift-example data-sets with 120 and 400 sites for the case of data standard error 0.1 mm/yr, plotted in the same way as the 0.2 mm/yr and 0.4 mm/yr results in Figs 26 and 28.

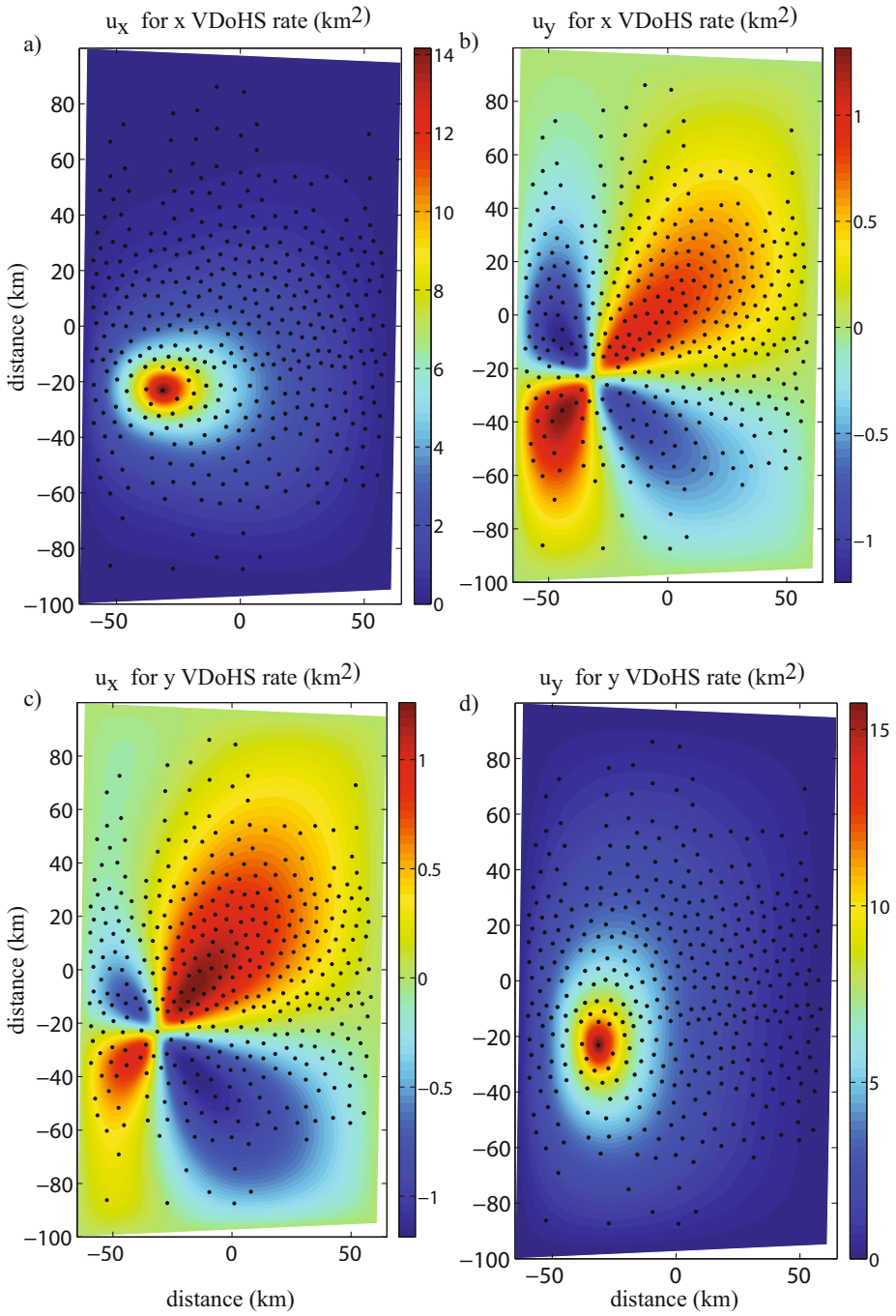


Fig. B13: (previous page): Example of $\mathbf{u}_i^x(\mathbf{x})$ (top panels) and $\mathbf{u}_i^y(\mathbf{x})$ (top panels) velocity functions determined from a 2-dimensional VDoHS rate basis function $\phi_i(\mathbf{x})$ for the 400 GPS dataset for the rift example in Chapter 5. The basis function site is at $(x, y) = (-31.3, -23.2)$ km. For the 400 GPS dataset, all the basis function sites shown as black dots coincide with GPS sites, as all the GPS sites are separated by more than the minimum spacing of 4 km. That is, in this case the GPS observations are also at the black dots. These velocity functions are solved for using finite elements with a much finer mesh than the mesh used in constructing the VDoHS rate basis functions. Zero-velocity boundary conditions are applied on the top and bottom boundaries and the left and right boundaries are reflecting boundaries. The $\mathbf{u}_i^x(\mathbf{x})$ values at the GPS sites form one column of the A-matrix relating GPS velocities to VDoHS rates for this 2-dimensional example, corresponding to the A-matrix for the 1-dimensional case in the top panel of Fig. 4,. The $\mathbf{u}_i^y(\mathbf{x})$ values at the GPS sites form another column of the A-matrix.

Fig. B14: (next page): Entries in the inverse of the A-matrix corresponding to the entries in the A-matrix shown in Fig. B13. The inverse of the A-matrix relates VDoHS rates at the basis function sites to velocities at the GPS sites for the hypothetical case of error-free data. The points shown as black dots are again the coinciding basis function and GPS sites for the 400 GPS dataset for the rift example in Chapter 5. The functions plotted here are constructed as follows. The VDoHS rate values at the basis function sites are those in the two columns of the inverse of the A-matrix for the GPS site at $(x, y) = (-31.3, -23.2)$ km. One column is for x component of velocity (top panels) and the other column is for the y component of velocity (bottom panels). These correspond to the columns of the inverse of the A-matrix for the 1-dimensional case in the bottom panel of Fig. 4. The VDoHS rate functions are completed by multiplying the value at each basis function site by the associated basis function $\phi_i(\mathbf{x})$ and summing the contributions at each point. Note how the VDoHS rate functions are much more localized around the GPS site than the velocity functions in Fig. B13 are about the coinciding basis function site. This corresponds in 2 dimensions to the 1-dimensional localization close to the diagonal of the inverse of the A-matrix in the bottom panel of Fig. 4.

