Passivity Certification of an LTI system

A linear system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t)\end{aligned}$$

with equal number of inputs and outputs, is passive if and only if there exists a matrix P such that

$$P = P^T \geq 0, \qquad A^TP + PA \leq 0, \qquad PB = C^T$$

The existence of such a P can be ascertained with semidefinite programming, demonstrated here using the CVX toolbox.

Contents

Requirements

This script requires the CVX toolbox: http://cvxr.com/cvx/

Example System

This example certifies passivity of the LTI system in Ex. 1.3, with data

$$A = \begin{bmatrix} 0 & 1 \\ -L & -k \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \gamma \end{bmatrix}, \qquad C = \begin{bmatrix} \mu & 1 \end{bmatrix}$$

It was shown that for positive L and γ, the system is passive if and only if $k \geq \mu \geq 0$

Define the system parameters and state space formulation

```
% System parameters
L = 1;
gamma = 1;
k = 1.5;
mu = 1;
```

```
% State Space Matrices
A = [0 1; -L -k];
B = [0; gamma];
C = [mu 1];
```

Passivity Certification

State dimension; input and (equal) output dimensions

```
nX = size(A,2);
nUY = size(B,2);
```

Formulate and solve SDP using CVX (only a feasibility problem)

```
cvx_begin sdp quiet
    variable P(nX,nX) semidefinite
    minimize 0
        A'*P+P*A <= 0;
        P*B == C';
cvx_end
```

Check CVX Result

```
cvx_status
```

```
cvx_status =

Solved
```

Verify Solution

Use a *Solution tolerance* of 10^{-8} for verifying semidefiniteness.

```
tol = 1e-8;
[all(eig(P) >= -tol),  all(eig(A'*P+P*A) <= tol),  all(P*B == C')]
```

```
ans =

     1     1     1
```

Output-Strict Passivity Certification

Suppose the output equation also contains a feedthrough term,

$y(t) = Cx(t) + Du(t)$

The system is output strict passive if and only if there exists $P = P^T \geq 0$ and $\epsilon \geq 0$ such that

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} \leq \begin{bmatrix} 0 & I \\ C & D \end{bmatrix}^T \begin{bmatrix} 0 & 1/2 \\ 1/2 & -\epsilon \end{bmatrix} \begin{bmatrix} 0 & I \\ C & D \end{bmatrix}$$

Define D and matrix from LMI

```
D = 0;
T = [zeros(nUY, nX) eye(nUY); C D];
```

Defining an objective beyond feasibility

Find the maximum epsilon for which the LMI certifies output strict passivity. For linear systems, the epsilon parameter in the output strict passivity condition gaurantees that the Nyquist plot of the system is contained inside a circle of radius $0.5\epsilon^{-1}$, centered at $0.5\epsilon^{-1}$, and hence ϵ^{-1} is an upper bound on the L_2 gain of the system.

```
cvx_begin sdp quiet
    variable P(nX,nX) semidefinite
    variable epsilon nonnegative
    maximize epsilon
        [A'*P+P*A P*B; B'*P 0] <= T'*[zeros(nUY) 1/2*eye(nUY); 1/2*eye(nUY) -epsilon*eye(nUY)]*T;
cvx_end
```

Check CVX Result

```
cvx_status
epsilon
```

```
cvx_status =

Solved


epsilon =

   5.0000e-01
```

Verify Solution

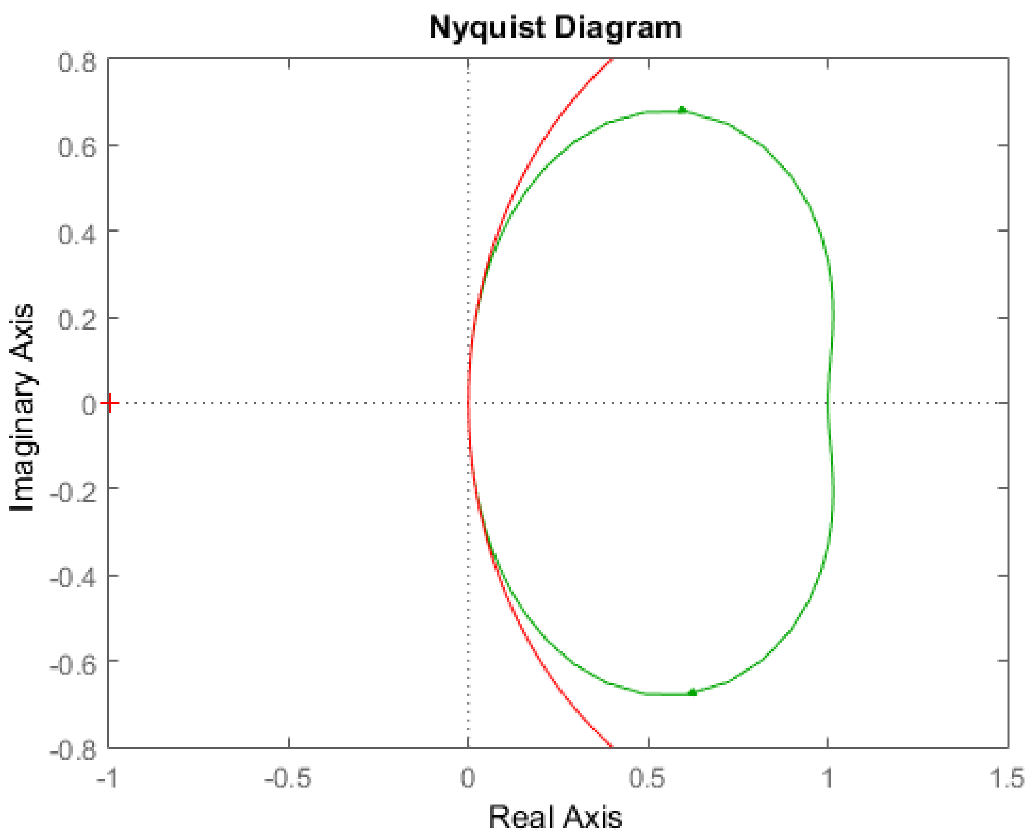
```
tol = 1e-8; % Solution tolerance
[all(eig(P) >= -tol),  all(eig([A'*P+P*A P*B; B'*P 0] - T'*[0 1/2; 1/2 -epsilon]*T) <= tol)]
```

```
ans =

     1     1
```

Verify Nyquist-plot claim

```
nyquist(ss(A,B,C,D), 'g'); hold on
theta = linspace(0,2*pi,360);
cr = 1/(2*epsilon);
plot(cr+cr*cos(theta), cr*sin(theta), 'r');
hold off
```



Attribution

This example supplements the book "Networks of Dissipative Systems: Compositional Certification of Stability, Performance, and Safety" by Murat Arcak, Chris Meissen, and Andrew Packard.