Solutions Manual to

Digital Image Processing
A Signal Processing and Algorithmic Approach

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Chapter 1

Introduction

1.1 Find the memory required, in bytes, to store the following images.

(i) 64×64 binary image.
(ii) 128×128 8-bit gray-level image.
(iii) 64×64 24-bit full-color image.
(iv) 512×512 binary image.
(v) 1024×1024 8-bit gray-level image.
(vi) 4096×4096 24-bit full-color image.

$$(64)(64)/8 = 512$$

$$(128)(128) = 16384$$

$$(64)(64)(3) = 12288$$

$$(512)(512)/8 = 32768$$

$$(1024)(1024) = 1048576$$

$$(4096)(4096)(3) = 50331648$$

1.2 Find the memory required, in bytes, to store the images given in Exercise (1.1) after (i) doubling the number of rows and columns and (ii) reducing the number of rows and columns by a factor of 2.

$$2048, 128$$

$$65536, 4096$$

$$49152, 3072$$

$$131072, 8192$$

$$4194304, 262144$$

$$201326592, 12582912$$

1.3 Find the pixel values of the 8×8 8-bit gray level image

$$\{x(m,n), m = 0, 1, 2, \ldots, 7 \quad \text{and} \quad n = 0, 1, 2, \ldots, 7\}$$

corresponding to the given 2-D function. (Round the real values of the image to the nearest integer after necessary scaling.)

(i)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}m + \frac{2\pi}{8}n - \frac{\pi}{4})$$

(ii)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}m + \frac{2\pi}{8}2n - \frac{\pi}{6})$$

(iii)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}0m + \frac{2\pi}{8}0n)$$

(iv)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}4m + \frac{2\pi}{8}4n)$$

(v)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}0m + \frac{2\pi}{8}n)$$

(vi)

$$x(m,n) = 1 + \cos(\frac{2\pi}{8}2m + \frac{2\pi}{8}0n)$$

(i)

$$\begin{bmatrix} 218 & 255 & 218 & 128 & 37 & 0 & 37 & 127 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 218 & 128 & 37 & 0 & 37 & 127 & 218 & 255 \\ 128 & 37 & 0 & 37 & 127 & 218 & 255 & 218 \\ 37 & 0 & 37 & 127 & 218 & 255 & 218 & 128 \\ 0 & 37 & 127 & 218 & 255 & 218 & 128 & 37 \\ 37 & 127 & 218 & 255 & 218 & 128 & 37 & 0 \\ 127 & 218 & 255 & 218 & 128 & 37 & 0 & 37 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 238 & 191 & 17 & 64 & 238 & 191 & 17 & 64 \\ 251 & 95 & 4 & 160 & 251 & 95 & 4 & 160 \\ 191 & 17 & 64 & 238 & 191 & 17 & 64 & 238 \\ 95 & 4 & 160 & 251 & 95 & 4 & 160 & 251 \\ 17 & 64 & 238 & 191 & 17 & 64 & 238 & 191 \\ 4 & 160 & 251 & 95 & 4 & 160 & 251 & 95 \\ 64 & 238 & 191 & 17 & 64 & 238 & 191 & 17 \\ 160 & 251 & 95 & 4 & 160 & 251 & 95 & 4 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 255 & 0 & 255 & 0 & 255 & 0 & 255 & 0 \\ 0 & 255 & 0 & 255 & 0 & 255 & 0 & 255 \\ 255 & 0 & 255 & 0 & 255 & 0 & 255 & 0 \\ 0 & 255 & 0 & 255 & 0 & 255 & 0 & 255 \\ 255 & 0 & 255 & 0 & 255 & 0 & 255 & 0 \\ 0 & 255 & 0 & 255 & 0 & 255 & 0 & 255 \\ 255 & 0 & 255 & 0 & 255 & 0 & 255 & 0 \\ 0 & 255 & 0 & 255 & 0 & 255 & 0 & 255 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \\ 255 & 218 & 128 & 37 & 0 & 37 & 127 & 218 \end{bmatrix}$$

(vi)

$$\begin{bmatrix} 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 127 & 127 & 127 & 127 & 127 & 127 & 127 & 127 \\ 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 127 & 127 & 127 & 127 & 127 & 127 & 127 & 127 \end{bmatrix}$$

1.4 Find the pixel values of the 8×8 binary image by setting the gray level values between 0-127 to 0 and 128-255 to 1 for each of the 8-bit gray level images

$$\{x(m, n), m = 0, 1, 2, \ldots, 7 \quad \text{and} \quad n = 0, 1, 2, \ldots, 7\}$$

obtained in Exercise (1.3).

(i)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(vi)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1.5 Find the bit-plane components of the image and verify that the image can be reconstructed from them.
(i)

$$\begin{bmatrix} 8 & 3 & 7 & 3 \\ 4 & 11 & 15 & 12 \\ 0 & 10 & 11 & 1 \\ 2 & 10 & 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 3 & 7 & 3 \\ 4 & 11 & 15 & 12 \\ 0 & 10 & 11 & 1 \\ 2 & 10 & 3 & 6 \end{bmatrix} = 2^3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 2 & 1 & 7 & 3 \\ 1 & 1 & 15 & 12 \\ 0 & 13 & 5 & 13 \\ 2 & 10 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 7 & 3 \\ 1 & 1 & 15 & 12 \\ 0 & 13 & 5 & 13 \\ 2 & 10 & 4 & 6 \end{bmatrix} = 2^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 8 & 1 & 7 & 8 \\ 5 & 11 & 15 & 12 \\ 0 & 6 & 7 & 13 \\ 2 & 10 & 7 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 7 & 8 \\ 5 & 11 & 15 & 12 \\ 0 & 6 & 7 & 13 \\ 2 & 10 & 7 & 6 \end{bmatrix} = 2^3 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(iv)

$$\begin{bmatrix} 7 & 1 & 7 & 3 \\ 1 & 6 & 15 & 12 \\ 0 & 11 & 1 & 13 \\ 2 & 10 & 9 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 1 & 7 & 3 \\ 1 & 6 & 15 & 12 \\ 0 & 11 & 1 & 13 \\ 2 & 10 & 9 & 9 \end{bmatrix} = 2^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} + 2^2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(v)

$$\begin{bmatrix} 8 & 1 & 7 & 7 \\ 1 & 11 & 6 & 12 \\ 0 & 8 & 7 & 13 \\ 9 & 10 & 9 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 7 & 7 \\ 1 & 11 & 6 & 12 \\ 0 & 8 & 7 & 13 \\ 9 & 10 & 9 & 6 \end{bmatrix} = 2^3 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} + 2 \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

1.6 (i) Find the spatial resolution of an image if the scene of size 4 meters by 4 meters is represented by a 256×256 image.
$256/4 = 64$ pixels per meter.
(ii) Find the spatial resolution of an image if the scene of size 10 kilometers by 10 kilometers is represented by a 4096×4096 image.
$4096/10 = 409.6$ pixels per kilometer.
(iii) Find the spatial resolution of an image if the scene of size 7 millimeters by 7 millimeters is represented by a 1024×1024 image.
$1024/7 = 146.2857$ pixels per millimeter.

1.7 Let the sampling frequencies in horizontal and vertical directions be 32 cycles per sample. Is there aliasing or not? If so, what are the impersonated frequencies.
(i)

$$x(k,l) = \cos(\frac{2\pi}{32}28k + \frac{2\pi}{32}30l - \frac{\pi}{6})$$

Aliasing

$$x(k,l) = \cos(\frac{2\pi}{32}4k + \frac{2\pi}{32}2l + \frac{\pi}{6})$$

(ii)

$$x(k,l) = \cos(\frac{2\pi}{32}15k + \frac{2\pi}{32}14l + \frac{\pi}{2})$$

No aliasing
(iii)

$$x(k,l) = \cos(\frac{2\pi}{32}27k + \frac{2\pi}{32}22l - \frac{\pi}{3})$$

Aliasing

$$x(k,l) = \cos(\frac{2\pi}{32}5k + \frac{2\pi}{32}10l + \frac{\pi}{3})$$

(iv)

$$x(k,l) = \cos(\frac{2\pi}{32}3k + \frac{2\pi}{32}3l + \frac{\pi}{2})$$

No aliasing
(v)

$$x(k,l) = \cos(\frac{2\pi}{32}32k + \frac{2\pi}{32}32l + \frac{\pi}{4})$$

Aliasing

$$x(k,l) = \cos(\frac{2\pi}{32}0k + \frac{2\pi}{32}0l + \frac{\pi}{4})$$

(vi)

$$x(k,l) = \cos(\frac{2\pi}{32}17k + \frac{2\pi}{32}11l - \frac{\pi}{3})$$

Aliasing

$$x(k,l) = \cos(\frac{2\pi}{32}15k - \frac{2\pi}{32}11l + \frac{\pi}{3})$$

Chapter 2

Enhance_sp

2.1 Find the complement of the 4×4 8-bit gray level image and verify that the image can be restored by complementing the complemented image.
(i)

$$\begin{bmatrix} 112 & 148 & 72 & 153 \\ 120 & 125 & 30 & 99 \\ 95 & 120 & 89 & 33 \\ 170 & 99 & 109 & 40 \end{bmatrix}$$

$$\begin{bmatrix} 143 & 107 & 183 & 102 \\ 135 & 130 & 225 & 156 \\ 160 & 135 & 166 & 222 \\ 85 & 156 & 146 & 215 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 164 & 127 & 117 & 59 \\ 154 & 122 & 104 & 83 \\ 129 & 136 & 100 & 60 \\ 117 & 128 & 80 & 48 \end{bmatrix}$$

$$\begin{bmatrix} 91 & 128 & 138 & 196 \\ 101 & 133 & 151 & 172 \\ 126 & 119 & 155 & 195 \\ 138 & 127 & 175 & 207 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 46 & 48 & 46 & 45 \\ 42 & 49 & 46 & 45 \\ 64 & 73 & 60 & 43 \\ 94 & 69 & 63 & 37 \end{bmatrix}$$

$$\begin{bmatrix} 209 & 207 & 209 & 210 \\ 213 & 206 & 209 & 210 \\ 191 & 182 & 195 & 212 \\ 161 & 186 & 192 & 218 \end{bmatrix}$$

2.2 Find the complement of the 4×4 binary image and verify that the image can be restored by coplementing the complemented image.

(i)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

2.3 For the list of gray levels, apply gamma correction and find the corresponding new gray levels. Apply the inverse transformation to the new gray levels and verify that the given gray levels are obtained.

$$\{0, 25, 50, 100, 150, 200, 250, 255\}$$

(i) $\gamma = 0.8$.

$$\{0, 40, 69, 121, 167, 210, 251, 255\}$$

(ii) $\gamma = 1.1$.

$$\{0, 20, 42, 91, 142, 195, 250, 255\}$$

(i) (iii) $\gamma = 1.8$.

$$\{0, 4, 14, 47, 98, 165, 246, 255\}$$

2.4 Given a 4×4 4-bit image, find the histogram equalized version of it.
(i)

$$\begin{bmatrix} 4 & 4 & 3 & 3 \\ 4 & 4 & 4 & 3 \\ 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

The cumulative distribution is

$$0, 0, 0, 0.1875, 0.9375, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$$

The new pixel values are

$$0, 0, 0, 3, 14, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15$$

The histogram-equalized image is

$$\begin{bmatrix} 14 & 14 & 3 & 3 \\ 14 & 14 & 14 & 3 \\ 15 & 14 & 14 & 14 \\ 14 & 14 & 14 & 14 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 3 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

The cumulative distribution is

$$0.3750, 0.8125, 0.9375, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$$

The new pixel values are

$$6, 12, 14, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15$$

The histogram-equalized image is

$$\begin{bmatrix} 12 & 12 & 6 & 12 \\ 12 & 12 & 6 & 15 \\ 12 & 6 & 6 & 14 \\ 12 & 6 & 6 & 14 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 3 & 5 & 5 & 3 \\ 4 & 4 & 4 & 3 \\ 4 & 2 & 3 & 4 \\ 4 & 2 & 2 & 3 \end{bmatrix}$$

The cumulative distribution is

$$0, 0, 0.1875, 0.5, 0.8750, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$$

The new pixel values are

$$0, 0, 3, 8, 13, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, 15$$

The histogram-equalized image is

$$\begin{bmatrix} 8 & 15 & 15 & 8 \\ 13 & 13 & 13 & 8 \\ 13 & 3 & 8 & 13 \\ 13 & 3 & 3 & 8 \end{bmatrix}$$

2.5 Given 4×4 4-bit reference and input images, use histogram matching to restore the input image.
(i) The reference, input and output images, respectively, are

$$\begin{bmatrix} 4 & 4 & 3 & 3 \\ 4 & 4 & 4 & 3 \\ 5 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix} \quad \begin{bmatrix} 15 & 15 & 0 & 0 \\ 15 & 15 & 15 & 0 \\ 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 \end{bmatrix} \quad \begin{bmatrix} 5 & 5 & 3 & 3 \\ 5 & 5 & 5 & 3 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \end{bmatrix}$$

The cumulative distributions of the reference and input images, respectively, are

$$\{0, 0, 0, 0.1875, 0.9375, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$$

$$\{0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.1875,$$
$$0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 0.1875, 1\}$$

The pixels in the input image 0-15 are mapped to

$$\{3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 5\}$$

(ii) The reference, input and output images, respectively, are

$$\begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 2 & 2 & 3 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} 15 & 15 & 15 & 15 \\ 15 & 15 & 15 & 15 \\ 15 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 \\ 3 & 2 & 2 & 3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$$

The cumulative distributions of the reference and input images, respectively, are

$$\{0, 0, 0.3750, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$$

$$\{0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 0.3750, 1\}$$

The pixels in the input image 0-15 are mapped to

$$\{2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3\}$$

(iii) The reference, input and output images, respectively, are

$$\begin{bmatrix} 3 & 5 & 5 & 3 \\ 4 & 4 & 4 & 3 \\ 4 & 2 & 3 & 4 \\ 4 & 2 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 0 & 15 & 15 & 0 \\ 15 & 15 & 15 & 0 \\ 15 & 0 & 0 & 15 \\ 15 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 3 & 5 & 5 & 3 \\ 5 & 5 & 5 & 3 \\ 5 & 3 & 3 & 5 \\ 5 & 3 & 3 & 3 \end{bmatrix}$$

The cumulative distributions of the reference and input images, respectively, are

$$\{0, 0, 0.1875, 0.5, 0.8750, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, \}$$

$$\{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1\}$$

The pixels in the input image 0-15 are mapped to

$$\{3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 5\}$$

2.6 Given a 8×8 8-bit image, find binary, hard and soft thresholded versions with the threshold $T = 160$.

$$\begin{bmatrix} 255 & 255 & 255 & 117 & 50 & 39 & 50 & 56 \\ 255 & 255 & 255 & 194 & 45 & 26 & 48 & 54 \\ 255 & 255 & 255 & 241 & 61 & 25 & 53 & 57 \\ 255 & 255 & 255 & 255 & 104 & 32 & 64 & 64 \\ 255 & 255 & 255 & 255 & 154 & 37 & 59 & 61 \\ 255 & 255 & 255 & 255 & 199 & 54 & 55 & 61 \\ 255 & 255 & 255 & 255 & 230 & 71 & 59 & 64 \\ 255 & 255 & 255 & 255 & 250 & 95 & 60 & 68 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 255 & 255 & 255 & 0 & 0 & 0 & 0 & 0 \\ 255 & 255 & 255 & 194 & 0 & 0 & 0 & 0 \\ 255 & 255 & 255 & 241 & 0 & 0 & 0 & 0 \\ 255 & 255 & 255 & 255 & 0 & 0 & 0 & 0 \\ 255 & 255 & 255 & 255 & 0 & 0 & 0 & 0 \\ 255 & 255 & 255 & 255 & 199 & 0 & 0 & 0 \\ 255 & 255 & 255 & 255 & 230 & 0 & 0 & 0 \\ 255 & 255 & 255 & 255 & 250 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 95 & 95 & 95 & 0 & 0 & 0 & 0 & 0 \\ 95 & 95 & 95 & 34 & 0 & 0 & 0 & 0 \\ 95 & 95 & 95 & 81 & 0 & 0 & 0 & 0 \\ 95 & 95 & 95 & 95 & 0 & 0 & 0 & 0 \\ 95 & 95 & 95 & 95 & 0 & 0 & 0 & 0 \\ 95 & 95 & 95 & 95 & 39 & 0 & 0 & 0 \\ 95 & 95 & 95 & 95 & 70 & 0 & 0 & 0 \\ 95 & 95 & 95 & 95 & 90 & 0 & 0 & 0 \end{bmatrix}$$

2.7 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 averaging filter with the borders zero-padded.

$$x(m,n) = \begin{bmatrix} 70 & 62 & 51 & 45 \\ 71 & 62 & 57 & 55 \\ 73 & 65 & 56 & 60 \\ 68 & 69 & 63 & 66 \end{bmatrix}$$

$$y(m,n) = \frac{1}{9} \begin{bmatrix} 265 & 373 & 332 & 208 \\ 403 & 567 & 513 & 324 \\ 408 & 584 & 553 & 357 \\ 275 & 394 & 379 & 245 \end{bmatrix}$$

2.8 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 averaging filter with the borders replicated.

$$x(m,n) = \begin{bmatrix} 41 & 43 & 45 & 43 \\ 40 & 41 & 42 & 41 \\ 42 & 38 & 39 & 42 \\ 39 & 33 & 37 & 36 \end{bmatrix}$$

$$y(m,n) = \frac{1}{9} \begin{bmatrix} 371 & 381 & 386 & 386 \\ 368 & 371 & 374 & 378 \\ 354 & 351 & 349 & 356 \\ 344 & 337 & 331 & 341 \end{bmatrix}$$

2.9 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 averaging filter with the borders periodically extended.

$$x(m,n) = \begin{bmatrix} 45 & 78 & 87 & 51 \\ 59 & 56 & 62 & 49 \\ 59 & 39 & 44 & 57 \\ 56 & 36 & 35 & 51 \end{bmatrix}$$

$$y(m,n) = \frac{1}{9} \begin{bmatrix} 481 & 514 & 505 & 495 \\ 493 & 529 & 523 & 513 \\ 462 & 446 & 429 & 472 \\ 472 & 479 & 478 & 485 \end{bmatrix}$$

2.10 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 Gaussian filter with $\sigma = 0.5$ and the borders symmetrically extended.

$$x(m,n) = \begin{bmatrix} 202 & 195 & 192 & 191 \\ 216 & 211 & 200 & 209 \\ 224 & 212 & 215 & 227 \\ 224 & 205 & 227 & 230 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 202.7682 & 197.0167 & 193.2693 & 192.9102 \\ 214.7263 & 209.1150 & 202.5467 & 208.1208 \\ 221.8699 & 212.8023 & 215.4462 & 224.2604 \\ 222.0558 & 209.8174 & 224.0159 & 229.2589 \end{bmatrix}$$

2.11 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 Gaussian filter with $\sigma = 0.5$ and the borders periodically extended.

$$x(m,n) = \begin{bmatrix} 202 & 195 & 192 & 191 \\ 216 & 211 & 200 & 209 \\ 224 & 212 & 215 & 227 \\ 224 & 205 & 227 & 230 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 204.0419 & 198.5015 & 196.7588 & 197.9520 \\ 214.0488 & 209.1150 & 202.5467 & 208.7983 \\ 222.1100 & 212.8023 & 215.4462 & 224.0203 \\ 220.2609 & 208.3326 & 220.5264 & 224.7383 \end{bmatrix}$$

2.12 Given a 4×4 image, find the 4×4 lowpass filtered output using the 3×3 Gaussian filter with $\sigma = 0.5$ and the borders zero-padded.

$$x(m,n) = \begin{bmatrix} 95 & 82 & 54 & 33 \\ 84 & 78 & 56 & 64 \\ 73 & 71 & 53 & 60 \\ 73 & 73 & 54 & 36 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 73.6368 & 71.4016 & 49.3887 & 30.9644 \\ 74.3803 & 75.9877 & 58.3450 & 53.3411 \\ 66.0361 & 70.2204 & 55.8732 & 51.2330 \\ 58.2554 & 63.2379 & 48.5095 & 32.4531 \end{bmatrix}$$

2.13 Given a 4×4 image, find the 4×4 highpass filtered output using the 3×3 Laplacian filter with the borders zero-padded.

$$h(m,n) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{2.1}$$

$$x(m,n) = \begin{bmatrix} 45 & 52 & 56 & 52 \\ 49 & 60 & 55 & 55 \\ 47 & 55 & 53 & 46 \\ 45 & 48 & 51 & 40 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} -79 & -47 & -65 & -97 \\ -44 & -29 & 4 & -67 \\ -39 & -12 & -5 & -36 \\ -85 & -41 & -63 & -63 \end{bmatrix}$$

2.14 Given a 4×4 image, find the 4×4 highpass filtered output using the 3×3 Laplacian filter with the borders symmetrically extended.

$$x(m,n) = \begin{bmatrix} 64 & 62 & 62 & 68 \\ 68 & 66 & 58 & 64 \\ 75 & 70 & 60 & 58 \\ 72 & 69 & 59 & 60 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 2 & 6 & 2 & -10 \\ 1 & -6 & 20 & -8 \\ -15 & -10 & 5 & 10 \\ 0 & -6 & 12 & -3 \end{bmatrix}$$

2.15 Given a 4×4 image, find the 4×4 highpass filtered output using the 3×3 Laplacian filter with the borders replicated.

$$x(m,n) = \begin{bmatrix} 39 & 40 & 35 & 33 \\ 31 & 40 & 39 & 37 \\ 34 & 38 & 41 & 43 \\ 37 & 39 & 42 & 43 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} -7 & -6 & 7 & 6 \\ 20 & -12 & -3 & 4 \\ 4 & 2 & -2 & -8 \\ -1 & 0 & -3 & -1 \end{bmatrix}$$

2.16 Given a 4×4 image, find the 4×4 enhanced output using the 3×3 Laplacian filter with the borders replicated.

$$h(m,n) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{2.2}$$

$$x(m,n) = \begin{bmatrix} 190 & 206 & 228 & 238 \\ 180 & 205 & 227 & 219 \\ 182 & 203 & 211 & 159 \\ 184 & 212 & 206 & 177 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 184 & 201 & 241 & 267 \\ 143 & 209 & 272 & 252 \\ 161 & 205 & 260 & 29 \\ 158 & 255 & 224 & 166 \end{bmatrix}$$

2.17 Given a 4×4 image, find the 4×4 enhanced output using the 3×3 Laplacian filter with the borders periodically extended.

$$x(m,n) = \begin{bmatrix} 138 & 163 & 162 & 177 \\ 148 & 157 & 167 & 175 \\ 153 & 165 & 160 & 178 \\ 157 & 162 & 164 & 188 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 45 & 196 & 139 & 222 \\ 117 & 142 & 181 & 205 \\ 117 & 193 & 126 & 214 \\ 144 & 161 & 148 & 264 \end{bmatrix}$$

2.18 Given a 4×4 image, find the 4×4 enhanced output using the 3×3 Laplacian filter with the borders zero-padded.

$$x(m,n) = \begin{bmatrix} 201 & 195 & 191 & 169 \\ 210 & 201 & 181 & 157 \\ 213 & 207 & 190 & 166 \\ 204 & 204 & 197 & 159 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 600 & 382 & 410 & 497 \\ 435 & 212 & 166 & 269 \\ 444 & 227 & 199 & 324 \\ 603 & 412 & 432 & 432 \end{bmatrix}$$

2.19 Given a 4×4 image, find the 4×4 median filtered output using the 3×3 window with the borders zero-padded.
(i)

$$x(m,n) = \begin{bmatrix} 201 & 195 & 191 & 169 \\ 210 & 201 & 181 & 157 \\ 213 & 207 & 190 & 166 \\ 204 & 204 & 197 & 159 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 0 & 191 & 169 & 0 \\ 201 & 201 & 190 & 166 \\ 204 & 204 & 190 & 159 \\ 0 & 197 & 166 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 138 & 163 & 162 & 177 \\ 148 & 157 & 167 & 175 \\ 153 & 165 & 160 & 178 \\ 157 & 162 & 164 & 188 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 0 & 148 & 162 & 0 \\ 148 & 160 & 165 & 162 \\ 153 & 160 & 165 & 164 \\ 0 & 157 & 162 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 190 & 206 & 228 & 238 \\ 180 & 205 & 227 & 219 \\ 182 & 203 & 211 & 159 \\ 184 & 212 & 206 & 177 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 0 & 190 & 206 & 0 \\ 182 & 205 & 211 & 211 \\ 182 & 205 & 206 & 177 \\ 0 & 184 & 177 & 0 \end{bmatrix}$$

Chapter 3

DFT

3.1 The discrete periodic waveform $x(n)$ is periodic with period 4 samples. Express the waveform in terms of complex exponentials and, thereby, find its 1-D DFT coefficients. Find the 4 samples from both the expressions and check that they are the same. Find the least-squares errors, if $x(n)$ is represented by its dc component alone with the values of the dc component $X(0)$, $0.9X(0)$ and $1.1X(0)$.

(i)
$$x(n) = 1 + 3\cos(\frac{2\pi}{4}n + \frac{\pi}{3}) + 2\cos(2\frac{2\pi}{4}n)$$

(ii)
$$x(n) = -2 + \cos(\frac{2\pi}{4}n - \frac{\pi}{3}) + \cos(2\frac{2\pi}{4}n)$$

(iii)
$$x(n) = 2 + \cos(\frac{2\pi}{4}n + \frac{\pi}{6}) + \cos(2\frac{2\pi}{4}n)$$

(i)

$$\begin{aligned} x(n) &= 1 + 3\cos(\frac{2\pi}{4}n + \frac{\pi}{3}) + 2\cos(2\frac{2\pi}{4}n) \\ &= (e^{j0\frac{2\pi}{4}n} + (0.75 + j0.75\sqrt{3})e^{j\frac{2\pi}{4}n} + 2e^{j2\frac{2\pi}{4}n} + (0.75 - j0.75\sqrt{3})e^{j3\frac{2\pi}{4}n}) \end{aligned}$$

The samples are
$$\{4.5000, -3.5981, 1.5000, 1.5981\}$$

The DFT coefficients are
$$4\{1, 0.75 + j0.75\sqrt{3}, 2, 0.75 - j0.75\sqrt{3}\}$$

With $X(0)$, the error is

$$(1 - 4.5000)^2 + (1 - (-3.5981))^2 + (1 - 1.5000)^2 + (1 - 1.5981)^2 = 34$$

least squares errors are $34, 34.0400, 34.0400$.

(ii)

$$\begin{aligned} x(n) &= -2 + \cos(\frac{2\pi}{4}n - \frac{\pi}{3}) + \cos(2\frac{2\pi}{4}n) \\ &= -2(e^{j0\frac{2\pi}{4}n} + (0.25 - j0.25\sqrt{3})e^{j\frac{2\pi}{4}n} + e^{j2\frac{2\pi}{4}n} + (0.25 + j0.25\sqrt{3})e^{j3\frac{2\pi}{4}n}) \end{aligned}$$

The samples are
$$\{-0.5000, -2.1340, -1.5000, -3.8660\}$$

The DFT coefficients are

$$4\{-2, 0.25 - j0.25\sqrt{3}, 1, 0.25 + j0.25\sqrt{3}\}$$

With $X(0)$, the error is

$$(-2 + 0.5000)^2 + (-2 + (2.134))^2 + (-2 + 1.5000)^2 + (-2 + 3.866)^2 = 6$$

least squares errors are $6, 6.16, 6.16$.

(iii)

$$\begin{aligned} x(n) &= 1 + \cos(\frac{2\pi}{4}n + \frac{\pi}{6}) + \cos(2\frac{2\pi}{4}n) \\ &= 1(e^{j0\frac{2\pi}{4}n} + (0.25\sqrt{3} + j0.25)e^{j\frac{2\pi}{4}n} + e^{j2\frac{2\pi}{4}n} + (0.25\sqrt{3} - j0.25)e^{j3\frac{2\pi}{4}n}) \end{aligned}$$

The samples are

$$\{2.8660, -0.5000, 1.1340, 0.5000\}$$

The DFT coefficients are

$$4\{1, 0.25\sqrt{3} + j0.25, 1, 0.25\sqrt{3} - j0.25\}$$

With $X(0)$, the error is

$$(1 - 2.866)^2 + (1 - (-0.5))^2 + (1 - 1.134)^2 + (1 - 0.5)^2 = 6$$

least squares errors are $6, 6.04, 6.04$.

3.2 Find the 1-D DFT of the 4 samples using the matrix form of the DFT definition. Reconstruct the input from the DFT coefficients using the IDFT and verify that they are the same as the input. Verify Parseval's theorem.

(i)

$$\{x(0) = 2, x(1) = 1, x(2) = 3, x(3) = 2\}$$

(ii)

$$\{x(0) = 1, x(1) = 1, x(2) = 2, x(3) = -3\}$$

(iii)

$$\{x(0) = -1, x(1) = 0, x(2) = -3, x(3) = 2\}$$

(i) The DFT of

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ -1+j \\ 2 \\ -1-j \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 8 \\ -1+j \\ 2 \\ -1-j \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

Signal power is 18.

(ii) The DFT of

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1-j4 \\ 5 \\ -1+j4 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 \\ -1-j4 \\ 5 \\ -1+j4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -3 \end{bmatrix}$$

Signal power is 15.

(iii) The DFT of

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2+j2 \\ -6 \\ 2-j2 \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} -2 \\ 2+j2 \\ -6 \\ 2-j2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -3 \\ 2 \end{bmatrix}$$

Signal power is 14.

3.3 The discrete periodic image $x(m,n)$ is periodic with period 4 samples in both the directions. Express the waveform in terms of complex exponentials and, thereby, find its 2-D DFT coefficients. Find the 4×4 samples from both the expressions and check that they are the same. Find the least-squares errors, if $x(m,n)$ is represented by its dc component alone with the values of the dc component $X(0,0)$, $0.9X(0,0)$ and $1.1X(0,0)$.

(i)

$$x(m,n) = 1 + 2\cos(\frac{2\pi}{4}(m+n)) - \frac{\pi}{3}) + \cos(2\frac{2\pi}{4}(m+n))$$

(ii)

$$x(m,n) = 2 + 2\cos(\frac{2\pi}{4}(m+2n)) + \frac{\pi}{3}) - \cos(2\frac{2\pi}{4}(m+n))$$

(iii)

$$x(m,n) = -1 + 2\cos(\frac{2\pi}{4}(2m+n)) - \frac{\pi}{6}) + 2\cos(2\frac{2\pi}{4}(m+n))$$

(i)

$$\begin{aligned} x(m,n) &= 1 + 2\cos(\frac{2\pi}{4}(m+n)) - \frac{\pi}{3}) + \cos(2\frac{2\pi}{4}(m+n)) \\ &= (e^{j\frac{2\pi}{4}(0m+0n)} + (0.5 - j0.5\sqrt{3})e^{j\frac{2\pi}{4}(m+n)} + e^{j2\frac{2\pi}{4}(m+n)} + (0.5 + j0.5\sqrt{3})e^{j3\frac{2\pi}{4}(m+n)}) \end{aligned}$$

The image is

$$\begin{bmatrix} 3.0000 & 1.7321 & 1.0000 & -1.7321 \\ 1.7321 & 1.0000 & -1.7321 & 3.0000 \\ 1.0000 & -1.7321 & 3.0000 & 1.7321 \\ -1.7321 & 3.0000 & 1.7321 & 1.0000 \end{bmatrix}$$

The DFT coefficients are

$$\begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 8-j13.8564 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 8+j13.8564 \end{bmatrix}$$

least squares errors are $48, 48.16, 48.16$.
(ii)

$$\begin{aligned} x(m,n) &= 2 + 2\cos(\frac{2\pi}{4}(m+2n)) + \frac{\pi}{3}) - \cos(2\frac{2\pi}{4}(m+n)) \\ &= 2(e^{j\frac{2\pi}{4}(0m+0n)} + (0.5+j0.5\sqrt{3})e^{j\frac{2\pi}{4}(m+2n)} - e^{j2\frac{2\pi}{4}(m+n)} + (0.5-j0.5\sqrt{3})e^{j3\frac{2\pi}{4}(m+2n)}) \end{aligned}$$

The image is

$$\begin{bmatrix} 2.0000 & 2.0000 & 2.0000 & 2.0000 \\ 1.2679 & 2.7321 & 1.2679 & 2.7321 \\ -0.0000 & 4.0000 & 0.0000 & 4.0000 \\ 4.7321 & -0.7321 & 4.7321 & -0.7321 \end{bmatrix}$$

The DFT coefficients are

$$\begin{bmatrix} 32 & 0 & 0 & 0 \\ 0 & 0 & 8+j13.8564 & 0 \\ 0 & 0 & -16 & 0 \\ 0 & 0 & 8-j13.8564 & 0 \end{bmatrix}$$

least squares errors are $48, 48.64, 48.64$.
(iii)

$$\begin{aligned} x(m,n) &= -1 + 2\cos(\frac{2\pi}{4}(2m+n)) - \frac{\pi}{6}) + 2\cos(2\frac{2\pi}{4}(m+n)) \\ &= -1(e^{j\frac{2\pi}{4}(0m+0n)} + (0.5\sqrt{3}-j0.5)e^{j\frac{2\pi}{4}(2m+n)} + 2e^{j2\frac{2\pi}{4}(m+n)} + (0.5\sqrt{3}+j0.5)e^{j3\frac{2\pi}{4}(2m+n)}) \end{aligned}$$

The image is

$$\begin{bmatrix} 2.7321 & -2.0000 & -0.7321 & -4.0000 \\ -4.7321 & -0.0000 & -1.2679 & 2.0000 \\ 2.7321 & -2.0000 & -0.7321 & -4.0000 \\ -4.7321 & 0.0000 & -1.2679 & 2.0000 \end{bmatrix}$$

The DFT coefficients are

$$\begin{bmatrix} -16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 13.8564-j8 & 32 & 13.8564+j8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

least squares errors are $96, 96.16, 96.16$.

3.4 Find the 2-D DFT of the image using the matrix form of the DFT definition. Reconstruct the input from the DFT coefficients using the IDFT and verify that they are the same as the input. Verify Parseval's theorem. Express the magnitude of the DFT coefficients in the center-zero format using the log scale, $\log_{10}(1+|X(k,l)|)$. The origin is at the top left corner.
(i)

$$\begin{bmatrix} 112 & 148 & 72 & 153 \\ 120 & 125 & 30 & 99 \\ 95 & 120 & 89 & 33 \\ 170 & 99 & 109 & 40 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 143 & 107 & 183 & 102 \\ 135 & 130 & 225 & 156 \\ 160 & 135 & 166 & 222 \\ 85 & 156 & 146 & 215 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 164 & 127 & 117 & 59 \\ 154 & 122 & 104 & 83 \\ 129 & 136 & 100 & 60 \\ 117 & 128 & 80 & 48 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 112 & 148 & 72 & 153 \\ 120 & 125 & 30 & 99 \\ 95 & 120 & 89 & 33 \\ 170 & 99 & 109 & 40 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= 1000 \begin{bmatrix} 1.6140 & 0.1970 - j0.1670 & -0.0200 & 0.1970 + j0.1670 \\ 0.1480 + j0.0440 & 0.0670 + j0.0630 & -0.1480 + j0.2140 & 0.0010 - j0.1210 \\ 0.0300 & -0.1050 + j0.0030 & -0.1520 & -0.1050 - j0.0030 \\ 0.1480 - j0.0440 & 0.0010 + j0.1210 & -0.1480 - j0.2140 & 0.0670 - j0.0630 \end{bmatrix}$$

The signal power is 188384. The magnitude in log scale and center-zero format is

$$\begin{bmatrix} 2.1847 & 2.0255 & 1.4914 & 2.0255 \\ 2.4170 & 1.9683 & 2.1915 & 2.0864 \\ 1.3222 & 2.4137 & 3.2082 & 2.4137 \\ 2.4170 & 2.0864 & 2.1915 & 1.9683 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 143 & 107 & 183 & 102 \\ 135 & 130 & 225 & 156 \\ 160 & 135 & 166 & 222 \\ 85 & 156 & 146 & 215 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= 1000 \begin{bmatrix} 2.4660 & -0.1970 + j0.1670 & 0.0200 & -0.1970 - j0.1670 \\ -0.1480 - j0.0440 & -0.0670 - j0.0630 & 0.1480 - j0.2140 & -0.0010 + j0.1210 \\ -0.0300 + j0.0000 & 0.1050 - j0.0030 & 0.1520 & 0.1050 + j0.0030 \\ -0.1480 + j0.0440 & -0.0010 - j0.1210 & 0.1480 + j0.2140 & -0.0670 + j0.0630 \end{bmatrix}$$

The signal power is 405644. The magnitude in log scale and center-zero format is

$$\begin{bmatrix} 2.1847 & 2.0255 & 1.4914 & 2.0255 \\ 2.4170 & 1.9683 & 2.1915 & 2.0864 \\ 1.3222 & 2.4137 & 3.3922 & 2.4137 \\ 2.4170 & 2.0864 & 2.1915 & 1.9683 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 164 & 127 & 117 & 59 \\ 154 & 122 & 104 & 83 \\ 129 & 136 & 100 & 60 \\ 117 & 128 & 80 & 48 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= 1000 \begin{bmatrix} 1.7280 & 0.1630 - j0.2630 & 0.2020 & 0.1630 + j0.2630 \\ 0.0420 - j0.0900 & 0.0590 - j0.0050 & 0.0620 - j0.0320 & -0.0230 - j0.0210 \\ 0.0560 & -0.0110 - j0.0250 & 0.0540 & -0.0110 + j0.0250 \\ 0.0420 + j0.0900 & -0.0230 + j0.0210 & 0.0620 + j0.0320 & 0.0590 + j0.0050 \end{bmatrix}$$

The signal power is 204014. The magnitude in log scale and center-zero format is

$$\begin{bmatrix} 1.7404 & 1.4520 & 1.7559 & 1.4520 \\ 1.8499 & 1.7797 & 2.0014 & 1.5071 \\ 2.3075 & 2.4919 & 3.2378 & 2.4919 \\ 1.8499 & 1.5071 & 2.0014 & 1.7797 \end{bmatrix}$$

3.5 Find the 2-D DFT of the 4×4 impulse image $x(m,n)$ using the (i) row-column method and (ii) using the shift theorem.
(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(i)

$$X(k,l) = e^{-j0.5\pi(k+2l)} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -j1 & j1 & -j1 & j1 \\ -1 & 1 & -1 & 1 \\ j1 & -j1 & j1 & -j1 \end{bmatrix}$$

(ii)

$$X(k,l) = e^{-j0.5\pi(2k+2l)} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

(iii)

$$X(k,l) = e^{-j0.5\pi(3k+3l)} = \begin{bmatrix} 1 & j1 & -1 & -j1 \\ j & -1 & -j1 & 1 \\ -1 & -j1 & 1 & j1 \\ -j1 & 1 & j1 & -1 \end{bmatrix}$$

***3.6** Using the DFT and IDFT, find: (a) the periodic convolution of $x(m,n)$ and $h(m,n)$, (b) the periodic correlation of $x(m,n)$ and $h(m,n)$, and $h(m,n)$ and $x(m,n)$, (c) the autocorrelation of $x(m,n)$.
(i)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 3 & 3 \\ 1 & 0 & 1 & 2 \\ 4 & 1 & 0 & 1 \\ 2 & 0 & 1 & 2 \end{bmatrix} \quad h(m,n) = \begin{bmatrix} -2 & 1 & 3 & 2 \\ 1 & 1 & -1 & -2 \\ 4 & 0 & 0 & -1 \\ 1 & 0 & 2 & 2 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 2 & -2 & 1 \\ -2 & 0 & 1 & 4 \\ 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 4 \end{bmatrix} \quad h(m,n) = \begin{bmatrix} 0 & -1 & 2 & 2 \\ -3 & 1 & 1 & -1 \\ 1 & 1 & -3 & 0 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 3 & 4 \\ -2 & 0 & 1 & 4 \\ 1 & 1 & 3 & 2 \\ 3 & 1 & 0 & 4 \end{bmatrix} \quad h(m,n) = \begin{bmatrix} 3 & 1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & -2 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

(i) The DFT of $x(m,n)$ is

$$X(k,l) = \begin{bmatrix} 24.00 + j0.00 & 4.00 + j6.00 & 4.00 + j0.00 & 4.00 - j6.00 \\ 3.00 + j1.00 & -5.00 + j3.00 & -1.00 + j1.00 & -5.00 - j1.00 \\ 6.00 + j0.00 & 2.00 - j2.00 & 2.00 + j0.00 & 2.00 + j2.00 \\ 3.00 - j1.00 & -5.00 + j1.00 & -1.00 - j1.00 & -5.00 - j3.00 \end{bmatrix}$$

The DFT of $h(m,n)$ is

$$H(k,l) = \begin{bmatrix} 11.00 + j0.00 & 0.00 - j1.00 & 5.00 + j0.00 & 0.00 + j1.00 \\ 1.00 + j6.00 & -14.00 - j1.00 & -7.00 + j0.00 & -4.00 - j5.00 \\ 3.00 + j0.00 & -2.00 + j1.00 & 1.00 + j0.00 & -2.00 - j1.00 \\ 1.00 - j6.00 & -4.00 + j5.00 & -7.00 + j0.00 & -14.00 + j1.00 \end{bmatrix}$$

The pointwise product $Y(k,l) = X(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 264.00 + j0.00 & 6.00 - j4.00 & 20.00 + j0.00 & 6.00 + j4.00 \\ -3.00 + j19.00 & 73.00 - j37.00 & 7.00 - j7.00 & 15.00 + j29.00 \\ 18.00 + j0.00 & -2.00 + j6.00 & 2.00 + j0.00 & -2.00 - j6.00 \\ -3.00 - j19.00 & 15.00 - j29.00 & 7.00 + j7.00 & 73.00 + j37.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the convolution output in the spatial domain.

$$y(m,n) = \begin{bmatrix} 31 & 23 & 8 & 7 \\ 17 & 5 & 13 & 17 \\ 8 & 9 & 29 & 26 \\ 18 & 26 & 18 & 9 \end{bmatrix}$$

The pointwise product $Y(k,l) = X^*(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 264.00 + j0.00 & -6.00 - j4.00 & 20.00 + j0.00 & -6.00 + j4.00 \\ 9.00 + j17.00 & 67.00 + j47.00 & 7.00 + j7.00 & 25.00 + j21.00 \\ 18.00 + j0.00 & -6.00 - j2.00 & 2.00 + j0.00 & -6.00 + j2.00 \\ 9.00 - j17.00 & 25.00 - j21.00 & 7.00 - j7.00 & 67.00 - j47.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the correlation output in the spatial domain.

$$r_{hx}(m,n) = \begin{bmatrix} 31 & 14 & 11 & 19 \\ 5 & 8 & 22 & 18 \\ 4 & 20 & 30 & 12 \\ 28 & 21 & 11 & 10 \end{bmatrix}$$

The correlation of $h(m,n)$ and $x(m,n)$ is the reversal of $r_{hx}(m,n)$ in the two directions.

$$r_{xh}(m,n) = \begin{bmatrix} 31 & 19 & 11 & 14 \\ 28 & 10 & 11 & 21 \\ 4 & 12 & 30 & 20 \\ 5 & 18 & 22 & 8 \end{bmatrix}$$

The IDFT of $X(k,l)X^*(k,l)$ is the autocorrelation output of $x(m,n)$ in the spatial domain.

$$r_{xx}(m,n) = \begin{bmatrix} 56 & 38 & 26 & 38 \\ 40 & 32 & 29 & 34 \\ 38 & 36 & 38 & 36 \\ 40 & 34 & 29 & 32 \end{bmatrix}$$

(ii) The DFT of $x(m,n)$ is

$$X(k,l) = \begin{bmatrix} 15.00 + j0.00 & 0.00 + j7.00 & -15.00 + j0.00 & 0.00 - j7.00 \\ -1.00 + j4.00 & 2.00 - j1.00 & -1.00 + j2.00 & 0.00 + j3.00 \\ -5.00 + j0.00 & 10.00 - j7.00 & 1.00 + j0.00 & 10.00 + j7.00 \\ -1.00 - j4.00 & 0.00 - j3.00 & -1.00 - j2.00 & 2.00 + j1.00 \end{bmatrix}$$

The DFT of $h(m,n)$ is

$$H(k,l) = \begin{bmatrix} 4.00 + j0.00 & -3.00 + j1.00 & -6.00 + j0.00 & -3.00 - j1.00 \\ 4.00 + j6.00 & -9.00 + j7.00 & 4.00 + j0.00 & -3.00 - j1.00 \\ 0.00 + j0.00 & 7.00 + j3.00 & 2.00 + j0.00 & 7.00 - j3.00 \\ 4.00 - j6.00 & -3.00 + j1.00 & 4.00 + j0.00 & -9.00 - j7.00 \end{bmatrix}$$

The pointwise product $Y(k,l) = X(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 60.00 + j0.00 & -7.00 - j21.00 & 90.00 + j0.00 & -7.00 + j21.00 \\ -28.00 + j10.00 & -11.00 + j23.00 & -4.00 + j8.00 & 3.00 - j9.00 \\ -0.00 + j0.00 & 91.00 - j19.00 & 2.00 + j0.00 & 91.00 + j19.00 \\ -28.00 - j10.00 & 3.00 + j9.00 & -4.00 - j8.00 & -11.00 - j23.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the convolution output in the spatial domain.

$$y(m,n) = \begin{bmatrix} 15 & -4 & -4 & -6 \\ -7 & 0 & 21 & -4 \\ 25 & 10 & 2 & -8 \\ 1 & -3 & 22 & 0 \end{bmatrix}$$

The pointwise product $Y(k,l) = X^*(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 60.00 + j0.00 & 7.00 + j21.00 & 90.00 + j0.00 & 7.00 - j21.00 \\ 20.00 - j22.00 & -25.00 + j5.00 & -4.00 - j8.00 & -3.00 + j9.00 \\ -0.00 + j0.00 & 49.00 + j79.00 & 2.00 + j0.00 & 49.00 - j79.00 \\ 20.00 + j22.00 & -3.00 - j9.00 & -4.00 + j8.00 & -25.00 - j5.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the correlation output in the spatial domain.

$$r_{hx}(m,n) = \begin{bmatrix} 15 & -11 & 8 & 13 \\ 6 & 10 & 20 & -10 \\ 18 & -18 & -3 & 8 \\ 2 & 1 & 9 & -8 \end{bmatrix}$$

The correlation of $h(m,n)$ and $x(m,n)$ is the reversal of $r_{hx}(m,n)$ in the two directions.

$$r_{xh}(m,n) = \begin{bmatrix} 15 & 13 & 8 & -11 \\ 2 & -8 & 9 & 1 \\ 18 & 8 & -3 & -18 \\ 6 & -10 & 20 & 10 \end{bmatrix}$$

The IDFT of $X(k,l)X^*(k,l)$ is the autocorrelation output of $x(m,n)$ in the spatial domain.

$$r_{xx}(m,n) = \begin{bmatrix} 59 & 3 & 6 & 3 \\ 14 & -1 & 39 & -2 \\ 50 & 0 & 4 & 0 \\ 14 & -2 & 39 & -1 \end{bmatrix}$$

(iii) The DFT of $x(m,n)$ is

$$X(k,l) = \begin{bmatrix} 28.00 + j0.00 & -3.00 + j11.00 & -6.00 + j0.00 & -3.00 - j11.00 \\ 3.00 + j5.00 & 2.00 + j8.00 & -1.00 + j3.00 & 0.00 + j4.00 \\ 6.00 + j0.00 & -3.00 - j3.00 & 8.00 + j0.00 & -3.00 + j3.00 \\ 3.00 - j5.00 & 0.00 - j4.00 & -1.00 - j3.00 & 2.00 - j8.00 \end{bmatrix}$$

The DFT of $h(m,n)$ is

$$H(k,l) = \begin{bmatrix} 18.00 + j0.00 & 3.00 + j5.00 & -8.00 + j0.00 & 3.00 - j5.00 \\ 8.00 + j2.00 & -1.00 - j1.00 & 4.00 + j4.00 & -3.00 - j5.00 \\ 6.00 + j0.00 & 5.00 + j3.00 & 0.00 + j0.00 & 5.00 - j3.00 \\ 8.00 - j2.00 & -3.00 + j5.00 & 4.00 - j4.00 & -1.00 + j1.00 \end{bmatrix}$$

The pointwise product $Y(k,l) = X(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 504.00 + j0.00 & -64.00 + j18.00 & 48.00 + j0.00 & -64.00 - j18.00 \\ 14.00 + j46.00 & 6.00 - j10.00 & -16.00 + j8.00 & 20.00 - j12.00 \\ 36.00 + j0.00 & -6.00 - j24.00 & 0.00 + j0.00 & -6.00 + j24.00 \\ 14.00 - j46.00 & 20.00 + j12.00 & -16.00 - j8.00 & 6.00 + j10.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the convolution output in the spatial domain.

$$y(m,n) = \begin{bmatrix} 31 & 35 & 42 & 34 \\ 21 & 18 & 30 & 25 \\ 25 & 28 & 49 & 26 \\ 29 & 24 & 49 & 38 \end{bmatrix}$$

The pointwise product $Y(k,l) = X^*(k,l)H(k,l)$ is

$$Y(k,l) = X(k,l)H(k,l) = \begin{bmatrix} 504.00 + j0.00 & 46.00 - j48.00 & 48.00 + j0.00 & 46.00 + j48.00 \\ 34.00 - j34.00 & -10.00 + j6.00 & 8.00 - j16.00 & -20.00 + j12.00 \\ 36.00 + j0.00 & -24.00 + j6.00 & 0.00 + j0.00 & -24.00 - j6.00 \\ 34.00 + j34.00 & -20.00 - j12.00 & 8.00 + j16.00 & -10.00 - j6.00 \end{bmatrix}$$

The IDFT of $Y(k,l)$ is the correlation output in the spatial domain.

$$r_{hx}(m,n) = \begin{bmatrix} 41 & 40 & 43 & 28 \\ 45 & 34 & 32 & 23 \\ 38 & 32 & 25 & 23 \\ 37 & 32 & 15 & 16 \end{bmatrix}$$

The correlation of $h(m,n)$ and $x(m,n)$ is the reversal of $r_{hx}(m,n)$ in the two directions.

$$r_{xh}(m,n) = \begin{bmatrix} 41 & 28 & 43 & 40 \\ 37 & 16 & 15 & 32 \\ 38 & 23 & 25 & 32 \\ 45 & 23 & 32 & 34 \end{bmatrix}$$

The IDFT of $X(k,l)X^*(k,l)$ is the autocorrelation output of $x(m,n)$ in the spatial domain.

$$r_{xx}(m,n) = \begin{bmatrix} 92 & 48 & 34 & 48 \\ 59 & 42 & 31 & 55 \\ 60 & 42 & 44 & 42 \\ 59 & 55 & 31 & 42 \end{bmatrix}$$

3.7 Compute the DFT of the column vector $x(m) = \{1, 1, -1, -1\}$ and the row vector $x(n) = \{1, 1, -1, -1\}$. Using the separability theorem, verify that the product of the vectors in the time-domain and the 2-D IDFT of the product of their individual DFTs are the same.
Solution

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$X(k) = \{0, 2-j2, 0, 2+j2\}$ and $X(l) = \{0, 2-j2, 0, 2+j2\}$.

$$X(k,l) = X(k)X(l) = \begin{bmatrix} 0 \\ 2-j2 \\ 0 \\ 2+j2 \end{bmatrix} \begin{bmatrix} 0 & 2-j2 & 0 & 2+j2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.00+j0.00 & 0.00+j0.00 & 0.00+j0.00 & 0.00+j0.00 \\ 0.00+j0.00 & 0.00-j8.00 & 0.00+j0.00 & 8.00+j0.00 \\ 0.00+j0.00 & 0.00+j0.00 & 0.00+j0.00 & 0.00+j0.00 \\ 0.00+j0.00 & 8.00+j0.00 & 0.00+j0.00 & 0.00+j8.00 \end{bmatrix}$$

The inverse 2-D DFT is $x(m,n) = x(m)x(n)$.

3.8 Compute the DFT of the column vector $x(m) = \{0.2741, 0.4519, 0.2741\}$ and the row vector $x(n) = \{0.2741, 0.4519, 0.2741\}$. Using the separability theorem, verify that the product of the vectors in the time-domain and the 2-D IDFT of the product of their individual DFTs are the same.
Solution

$$\begin{bmatrix} 0.2741 \\ 0.4519 \\ 0.2741 \end{bmatrix} \begin{bmatrix} 0.2741 & 0.4519 & 0.2741 \end{bmatrix} = \begin{bmatrix} 0.0751 & 0.1238 & 0.0751 \\ 0.1238 & 0.2042 & 0.1238 \\ 0.0751 & 0.1238 & 0.0751 \end{bmatrix}$$

$X(k) = \{1, -j0.4519, 0.0963, j0.4519\}$ and $X(l) = \{1, -j0.4519, 0.0963, j0.4519\}$.

$$X(k,l) = X(k)X(l) = \begin{bmatrix} 1 \\ -j0.4519 \\ 0.0963 \\ j0.4519 \end{bmatrix} \begin{bmatrix} 1 & -j0.4519 & 0.0963 & j0.4519 \end{bmatrix}$$

$$
= \begin{bmatrix} 1.00 + j0.00 & 0.00 - j0.45 & 0.10 + j0.00 & 0.00 + j0.45 \\ 0.00 - j0.45 & -0.20 - j0.00 & 0.00 - j0.04 & 0.20 + j0.00 \\ 0.10 + j0.00 & 0.00 - j0.04 & 0.01 + j0.00 & 0.00 + j0.04 \\ 0.00 + j0.45 & 0.20 + j0.00 & 0.00 + j0.04 & -0.20 + j0.00 \end{bmatrix}
$$

The inverse 2-D DFT is $x(m,n) = x(m)x(n)$.

3.9 Compute the DFT of the column vector $x(m) = \{0, 1, 0, -1\}$ and the row vector $x(n) = \{1, 0, -1, 0\}$. Using the separability theorem, verify that the product of the vectors in the time-domain and the 2-D IDFT of the product of their individual DFTs are the same.

Solution

$$
\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}
$$

$X(k) = \{0, -j2, 0, j2\}$ and $X(l) = \{0, 2, 0, 2\}$.

$$
X(k,l) = X(k)X(l) = \begin{bmatrix} 0 \\ -j2 \\ 0 \\ j2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 & 2 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0.00 + j0.00 & 0.00 + j0.00 & 0.00 + j0.00 & 0.00 + j0.00 \\ 0.00 + j0.00 & 0.00 - j4.00 & 0.00 + j0.00 & 0.00 - j4.00 \\ 0.00 + j0.00 & 0.00 + j0.00 & 0.00 + j0.00 & 0.00 + j0.00 \\ 0.00 + j0.00 & 0.00 + j4.00 & 0.00 + j0.00 & 0.00 + j4.00 \end{bmatrix}
$$

The inverse 2-D DFT is $x(m,n) = x(m)x(n)$.

Chapter 4

enhan t

4.1 Compute the linear convolution of $x(n), n = 0, 1, \ldots,$ and $h(n), n = 0, 1, \ldots,$ using the DFT and IDFT

and verify your answer by directly computing the convolution. Assume zero-padding at the ends.
(i) $x(n) = \{2, 1, 3\}$ and $h(n) = \{1, -2\}$.
(ii) $x(n) = \{-1, 3\}$ and $h(n) = \{1, 3, -2\}$.
(iii) $x(n) = \{4, -1\}$ and $h(n) = \{-3, 1, -2\}$.
(iv) $x(n) = \{-1, 2, 3\}$ and $h(n) = \{-2, 3\}$.
(v) $x(n) = \{2, 4\}$ and $h(n) = \{4, 3, -2\}$.
(i) $Xz(k) = \{6, -1 - j, 4, -1 + j\}$ and $Hz(k) = \{-1, 1 + j2, 3, 1 - j2\}$.
$y(n) = \text{ idft}(Xz(k)Hz(k) = \{-6, 1 - j3, 12, 1 + j3\}) = \{2, -3, 1, -6\}$
$y(n) = x(n) * h(n) = \{2, -3, 1, -6\}$.
(ii) $Xz(k) = \{2, -1 - j3, -4, -1 + j3\}$ and $Hz(k) = \{2, 3 - j3, -4, 3 + j3\}$.
$y(n) = \text{ idft}(Xz(k)Hz(k) = \{4, -12 - j6, 16, -12 + j6\}) = \{-1, 0, 11, -6\}$
$y(n) = x(n) * h(n) = \{-1, 0, 11, -6\}$.
(iii) $Xz(k) = \{3, 4 + j, 5, 4 - j\}$ and $Hz(k) = \{-4, -1 - j1, 6, -1 + j1\}$.
$y(n) = \text{ idft}(Xz(k)Hz(k) = \{-12, -3 - j3, 30, -3 + j3\}) = \{-12, 7, -9, 2\}$
$y(n) = x(n) * h(n) = \{-12, 7, -9, 2\}$.
(iv) $Xz(k) = \{4, -4 - j2, 0, -4 + j2\}$ and $Hz(k) = \{1, -2 - j3, -5, -2 + j3\}$.
$y(n) = \text{ idft}(Xz(k)Hz(k) = \{4, 2 + j16, 0, 2 - j16\}) = \{2, -7, 0, 9\}$
$y(n) = x(n) * h(n) = \{2, -7, 0, 9\}$.
(v) $Xz(k) = \{5, 6 - j3, -1, 6 + j3\}$ and $Hz(k) = \{-4, -1 - j1, 6, -1 + j1\}$.
$y(n) = \text{ idft}(Xz(k)Hz(k) = \{30, -j30, 2, j30\}) = \{8, 22, 8, -8\}$
$y(n) = x(n) * h(n) = \{8, 22, 8, -8\}$.
4.2 Compute the linear convolution of $x(m, n),\ m, n = 0, 1, 2$ and $h(m, n),\ m, n = 0, 1$ using the DFT and

IDFT and verify your answer by directly computing the convolution. Assume zero-padding at the borders.
(i)

$$x(m, n) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad h(m, n) = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

(ii)

$$x(m, n) = \begin{bmatrix} 2 & 2 & 3 \\ 2 & -3 & -1 \\ 1 & -2 & 1 \end{bmatrix} \quad \text{and} \quad h(m, n) = \begin{bmatrix} -2 & -1 \\ 1 & 3 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & -4 & 3 \\ -2 & 3 & 1 \\ 1 & -3 & 1 \end{bmatrix} \quad \text{and} \quad h(m,n) = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

(iv)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 4 \\ 1 & -3 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad \text{and} \quad h(m,n) = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$$

(v)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 2 & 2 & 2 \end{bmatrix} \quad \text{and} \quad h(m,n) = \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix}$$

(i)

$$Xz(k) = \begin{bmatrix} 10.00+j0.00 & -1.00-j1.00 & 8.00+j0.00 & -1.00+j1.00 \\ 2.00+j0.00 & 1.00-j1.00 & 2.00-j6.00 & -5.00-j1.00 \\ 10.00+j0.00 & -3.00-j7.00 & -4.00+j0.00 & -3.00+j7.00 \\ 2.00+j0.00 & -5.00+j1.00 & 2.00+j6.00 & 1.00+j1.00 \end{bmatrix}$$

$$Hz(k) = \begin{bmatrix} 4.00+j0.00 & 3.00-j1.00 & 2.00+j0.00 & 3.00+j1.00 \\ 1.00-j3.00 & 0.00+j0.00 & 3.00+j1.00 & 4.00-j2.00 \\ -2.00+j0.00 & 1.00+j3.00 & 4.00+j0.00 & 1.00-j3.00 \\ 1.00+j3.00 & 4.00+j2.00 & 3.00-j1.00 & 0.00+j0.00 \end{bmatrix}$$

$$y(n) = \text{ idft}(Xz(k)Hz(k) = \begin{bmatrix} 40.00+j0.00 & -4.00-j2.00 & 16.00+j0.00 & -4.00+j2.00 \\ 2.00-j6.00 & 0.00+j0.00 & 12.00-j16.00 & -22.00+j6.00 \\ -20.00+j0.00 & 18.00-j16.00 & -16.00+j0.00 & 18.00+j16.00 \\ 2.00+j6.00 & -22.00-j6.00 & 12.00+j16.00 & 0.00+j0.00 \end{bmatrix})$$

$$= \begin{bmatrix} 2 & 3 & 4 & -3 \\ 5 & -4 & 12 & 5 \\ 4 & 4 & -5 & 1 \\ 1 & 4 & 5 & 2 \end{bmatrix}$$

(ii)

$$Xz(k) = \begin{bmatrix} 5.00+j0.00 & 2.00+j3.00 & 11.00+j0.00 & 2.00-j3.00 \\ 7.00+j2.00 & 2.00-j7.00 & -1.00-j4.00 & -4.00+j1.00 \\ 9.00+j0.00 & -4.00-j3.00 & 3.00+j0.00 & -4.00+j3.00 \\ 7.00-j2.00 & -4.00-j1.00 & -1.00+j4.00 & 2.00+j7.00 \end{bmatrix}$$

$$Hz(k) = \begin{bmatrix} 1.00+j0.00 & -1.00-j2.00 & -3.00+j0.00 & -1.00+j2.00 \\ -3.00-j4.00 & -5.00+j0.00 & -1.00+j2.00 & 1.00-j2.00 \\ -7.00+j0.00 & -3.00+j4.00 & 1.00+j0.00 & -3.00-j4.00 \\ -3.00+j4.00 & 1.00+j2.00 & -1.00-j2.00 & -5.00+j0.00 \end{bmatrix}$$

$$y(n) = \text{ idft}(Xz(k)Hz(k) = \begin{bmatrix} 5.00+j0.00 & 4.00-j7.00 & -33.00+j0.00 & 4.00+j7.00 \\ -13.00-j34.00 & -10.00+j35.00 & 9.00+j2.00 & -2.00+j9.00 \\ -63.00+j0.00 & 24.00-j7.00 & 3.00+j0.00 & 24.00+j7.00 \\ -13.00+j34.00 & -2.00-j9.00 & 9.00-j2.00 & -10.00-j35.00 \end{bmatrix})$$

$$= \begin{bmatrix} -4 & -6 & -8 & -3 \\ -2 & 12 & 14 & 10 \\ 0 & 6 & -10 & -4 \\ 1 & 1 & -5 & 3 \end{bmatrix}$$

(iii)

$$Xz(k) = \begin{bmatrix} 1.00+j0.00 & -5.00+j4.00 & 9.00+j0.00 & -5.00-j4.00 \\ 1.00-j2.00 & -5.00+j4.00 & 3.00+j4.00 & 1.00+j2.00 \\ -3.00+j0.00 & 1.00+j10.00 & 17.00+j0.00 & 1.00-j10.00 \\ 1.00+j2.00 & 1.00-j2.00 & 3.00-j4.00 & -5.00-j4.00 \end{bmatrix}$$

$$Hz(k) = \begin{bmatrix} 6.00+j0.00 & 3.00-j3.00 & 0.00+j0.00 & 3.00+j3.00 \\ 5.00-j1.00 & 2.00+j0.00 & 3.00+j3.00 & 6.00+j2.00 \\ 4.00+j0.00 & 5.00+j1.00 & 6.00+j0.00 & 5.00-j1.00 \\ 5.00+j1.00 & 6.00-j2.00 & 3.00-j3.00 & 2.00+j0.00 \end{bmatrix}$$

$$y(n) = \ \mathrm{idft}(Xz(k)Hz(k) = \begin{bmatrix} 6.00+j0.00 & -3.00+j27.00 & 0.00+j0.00 & -3.00-j27.00 \\ 3.00-j11.00 & -10.00+j8.00 & -3.00+j21.00 & 2.00+j14.00 \\ -12.00+j0.00 & -5.00+j51.00 & 102.00+j0.00 & -5.00-j51.00 \\ 3.00+j11.00 & 2.00-j14.00 & -3.00-j21.00 & -10.00-j8.00 \end{bmatrix})$$

$$= \begin{bmatrix} 4 & -15 & 8 & 3 \\ -9 & 16 & -4 & 7 \\ 6 & -18 & 6 & 3 \\ -1 & 5 & -7 & 2 \end{bmatrix}$$

(iv)

$$Xz(k) = \begin{bmatrix} 13.00+j0.00 & -1.00+j0.00 & 13.00+j0.00 & -1.00+j0.00 \\ 1.00+j0.00 & -1.00+j2.00 & 3.00-j6.00 & -7.00+j0.00 \\ 13.00+j0.00 & 1.00-j6.00 & 1.00+j0.00 & 1.00+j6.00 \\ 1.00+j0.00 & -7.00+j0.00 & 3.00+j6.00 & -1.00-j2.00 \end{bmatrix}$$

$$Hz(k) = \begin{bmatrix} 2.00+j0.00 & 3.00+j1.00 & 4.00+j0.00 & 3.00-j1.00 \\ -2.00-j4.00 & -1.00+j1.00 & 4.00+j0.00 & 3.00-j5.00 \\ -6.00+j0.00 & -1.00+j5.00 & 4.00+j0.00 & -1.00-j5.00 \\ -2.00+j4.00 & 3.00+j5.00 & 4.00+j0.00 & -1.00-j1.00 \end{bmatrix}$$

$$y(n) = \ \mathrm{idft}(Xz(k)Hz(k) = \begin{bmatrix} 26.00+j0.00 & -3.00-j1.00 & 52.00+j0.00 & -3.00+j1.00 \\ -2.00-j4.00 & -1.00-j3.00 & 12.00-j24.00 & -21.00+j35.00 \\ -78.00+j0.00 & 29.00+j11.00 & 4.00+j0.00 & 29.00-j11.00 \\ -2.00+j4.00 & -21.00-j35.00 & 12.00+j24.00 & -1.00+j3.00 \end{bmatrix})$$

$$= \begin{bmatrix} 2 & -5 & 1 & -12 \\ 5 & 0 & 21 & 2 \\ 5 & -11 & -7 & 1 \\ 6 & 10 & 6 & 2 \end{bmatrix}$$

(v)

$$Xz(k) = \begin{bmatrix} 12.00+j0.00 & -2.00+j0.00 & 12.00+j0.00 & -2.00+j0.00 \\ -3.00-j3.00 & 3.00+j3.00 & -1.00-j9.00 & -3.00+j1.00 \\ 6.00+j0.00 & 2.00-j6.00 & -6.00+j0.00 & 2.00+j6.00 \\ -3.00+j3.00 & -3.00-j1.00 & -1.00+j9.00 & 3.00-j3.00 \end{bmatrix}$$

$$Hz(k) = \begin{bmatrix} 2.00+j0.00 & 4.00+j2.00 & 6.00+j0.00 & 4.00-j2.00 \\ 4.00+j2.00 & 6.00-j2.00 & 2.00-j4.00 & 0.00+j0.00 \\ 6.00+j0.00 & 2.00-j4.00 & -2.00+j0.00 & 2.00+j4.00 \\ 4.00-j2.00 & 0.00+j0.00 & 2.00+j4.00 & 6.00+j2.00 \end{bmatrix}$$

$$y(n) = \ \mathrm{idft}(Xz(k)Hz(k) = \begin{bmatrix} 24.00 + j0.00 & -8.00 - j4.00 & 72.00 + j0.00 & -8.00 + j4.00 \\ -6.00 - j18.00 & 24.00 + j12.00 & -38.00 - j14.00 & -0.00 + j0.00 \\ 36.00 + j0.00 & -20.00 - j20.00 & 12.00 + j0.00 & -20.00 + j20.00 \\ -6.00 + j18.00 & -0.00 + j0.00 & -38.00 + j14.00 & 24.00 - j12.00 \end{bmatrix})$$

$$= \begin{bmatrix} 3 & 4 & 4 & 1 \\ 7 & -9 & 7 & 1 \\ 8 & -1 & 21 & -10 \\ 2 & -4 & -4 & -6 \end{bmatrix}$$

4.3
Convolve $x(m,n)$ and a 3×3 Gaussian lowpass filter with $\sigma = 1$. Assume periodicity at the borders.
(i)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 1 & 1 & 4 & 2 \\ 1 & -1 & 2 & -2 \\ 3 & 2 & -2 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 1 & 4 \\ 1 & -1 & 0 & -2 \\ 0 & 2 & -2 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & -1 & -2 & 2 \\ 4 & -1 & 2 & -2 \\ 3 & 2 & -2 & 3 \end{bmatrix}$$

(i)
Zero-padding the row filter, we get

$$hz(m) = \{0.4519, 0.2741, 0, 0.2741\}$$

The 1-D DFT of this filter is

$$H(k) \quad = \quad \{1, 0.4519, -0.0963, 0.4519\}$$

Both the row and column filters have the same coefficients. The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 22.00 + j0.00 & 0.00 + j2.00 & 6.00 + j0.00 & 0.00 - j2.00 \\ 10.00 - j4.00 & 2.00 + j12.00 & -6.00 - j4.00 & -2.00 + j4.00 \\ -2.00 + j0.00 & -4.00 + j2.00 & 6.00 + j0.00 & -4.00 - j2.00 \\ 10.00 + j4.00 & -2.00 - j4.00 & -6.00 + j4.00 & 2.00 - j12.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 22.00 + j0.00 & 0.00 + j2.00 & 6.00 + j0.00 & 0.00 - j2.00 \\ 4.52 - j1.81 & 0.90 + j5.42 & -2.71 - j1.81 & -0.90 + j1.81 \\ 0.19 + j0.00 & 0.39 - j0.19 & -0.58 + j0.00 & 0.39 + j0.19 \\ 4.52 + j1.81 & -0.90 - j1.81 & -2.71 + j1.81 & 0.90 - j5.42 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 22.00+j0.00 & -0.00+j0.90 & -0.58+j0.00 & -0.00-j0.90 \\ 4.52-j1.81 & 0.41+j2.45 & 0.26+j0.17 & -0.41+j0.82 \\ 0.19+j0.00 & 0.17-j0.09 & 0.06+j0.00 & 0.17+j0.09 \\ 4.52+j1.81 & -0.41-j0.82 & 0.26-j0.17 & 0.41-j2.45 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} 1.9736 & 1.6456 & 1.9301 & 2.2581 \\ 1.0975 & 1.4243 & 1.9577 & 1.8762 \\ 0.7787 & 0.9896 & 0.7352 & 0.7854 \\ 1.5058 & 1.1331 & 0.7326 & 1.1766 \end{bmatrix}$$

(ii) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 17.00+j0.00 & 2.00+j3.00 & -5.00+j0.00 & 2.00-j3.00 \\ 12.00-j7.00 & 1.00+j4.00 & -6.00-j3.00 & -7.00-j2.00 \\ -1.00+j0.00 & -4.00-j1.00 & 9.00+j0.00 & -4.00+j1.00 \\ 12.00+j7.00 & -7.00+j2.00 & -6.00+j3.00 & 1.00-j4.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 17.00+j0.00 & 2.00+j3.00 & -5.00+j0.00 & 2.00-j3.00 \\ 5.42-j3.16 & 0.45+j1.81 & -2.71-j1.36 & -3.16-j0.90 \\ 0.10+j0.00 & 0.39+j0.10 & -0.87+j0.00 & 0.39-j0.10 \\ 5.42+j3.16 & -3.16+j0.90 & -2.71+j1.36 & 0.45-j1.81 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 17.00+j0.00 & 0.90+j1.36 & 0.48+j0.00 & 0.90-j1.36 \\ 5.42-j3.16 & 0.20+j0.82 & 0.26+j0.13 & -1.43-j0.41 \\ 0.10+j0.00 & 0.17+j0.04 & 0.08+j0.00 & 0.17-j0.04 \\ 5.42+j3.16 & -1.43+j0.41 & 0.26-j0.13 & 0.20-j0.82 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} 1.7958 & 1.3504 & 1.8327 & 2.0064 \\ 1.5006 & 1.0751 & 1.4203 & 1.8115 \\ 0.6812 & 0.3663 & 0.1055 & 0.4098 \\ 0.8445 & 0.6601 & 0.5600 & 0.5797 \end{bmatrix}$$

(iii) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 17.00+j0.00 & 9.00+j4.00 & 5.00+j0.00 & 9.00-j4.00 \\ 4.00+j5.00 & -2.00+j3.00 & -8.00-j3.00 & -6.00-j1.00 \\ 3.00+j0.00 & -9.00-j4.00 & 15.00+j0.00 & -9.00+j4.00 \\ 4.00-j5.00 & -6.00+j1.00 & -8.00+j3.00 & -2.00-j3.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 17.00+j0.00 & 9.00+j4.00 & 5.00+j0.00 & 9.00-j4.00 \\ 1.81+j2.26 & -0.90+j1.36 & -3.61-j1.36 & -2.71-j0.45 \\ -0.29+j0.00 & 0.87+j0.39 & -1.44+j0.00 & 0.87-j0.39 \\ 1.81-j2.26 & -2.71+j0.45 & -3.61+j1.36 & -0.90-j1.36 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 17.00+j0.00 & 4.07+j1.81 & -0.48+j0.00 & 4.07-j1.81 \\ 1.81+j2.26 & -0.41+j0.61 & 0.35+j0.13 & -1.23-j0.20 \\ -0.29+j0.00 & 0.39+j0.17 & 0.14+j0.00 & 0.39-j0.17 \\ 1.81-j2.26 & -1.23+j0.20 & 0.35-j0.13 & -0.41-j0.61 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} 1.6456 & 0.8985 & 0.9394 & 1.5980 \\ 1.1514 & 0.5470 & 0.3347 & 1.1595 \\ 1.5151 & 0.7378 & -0.0078 & 1.0290 \\ 1.8510 & 1.2833 & 0.8301 & 1.4875 \end{bmatrix}$$

4.4

Convolve $x(m,n)$ and a 3×3 averaging lowpass filter. Assume periodicity at the borders.

(i)

$$x(m,n) = \begin{bmatrix} 1 & -1 & 3 & 4 \\ 1 & 2 & -4 & 2 \\ 1 & -1 & 3 & -2 \\ 1 & -2 & -2 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 3 & -3 & 4 \\ 2 & 0 & 1 & 4 \\ 1 & -1 & 0 & -2 \\ 0 & 2 & -2 & 3 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 2 & 0 & 3 & -2 \\ 2 & -1 & 2 & 2 \\ 4 & -1 & 3 & -2 \\ 1 & 0 & -2 & 3 \end{bmatrix}$$

(i)

Zero-padding the row filter, we get

$$hz(m) = \{1, 1, 0, 1\}$$

The 1-D DFT of this filter is

$$H(k) \quad = \quad \{3, 1, -1, 1\}$$

Both the row and column filters have the same coefficients. The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 7.00+j0.00 & 4.00+j7.00 & 1.00+j0.00 & 4.00-j7.00 \\ 6.00-j3.00 & -3.00+j4.00 & -6.00+j7.00 & 3.00-j8.00 \\ 9.00+j0.00 & -12.00+j1.00 & 15.00+j0.00 & -12.00-j1.00 \\ 6.00+j3.00 & 3.00+j8.00 & -6.00-j7.00 & -3.00-j4.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 21.00+j0.00 & 12.00+j21.00 & 3.00+j0.00 & 12.00-j21.00 \\ 6.00-j3.00 & -3.00+j4.00 & -6.00+j7.00 & 3.00-j8.00 \\ -9.00+j0.00 & 12.00-j1.00 & -15.00+j0.00 & 12.00+j1.00 \\ 6.00+j3.00 & 3.00+j8.00 & -6.00-j7.00 & -3.00-j4.00 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 63.00 + j0.00 & 12.00 + j21.00 & -3.00 + j0.00 & 12.00 - j21.00 \\ 18.00 - j9.00 & -3.00 + j4.00 & 6.00 - j7.00 & 3.00 - j8.00 \\ -27.00 + j0.00 & 12.00 - j1.00 & 15.00 + j0.00 & 12.00 + j1.00 \\ 18.00 + j9.00 & 3.00 + j8.00 & 6.00 + j7.00 & -3.00 - j4.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$9y(m,n) = \begin{bmatrix} 9 & -1 & 3 & 7 \\ 7 & 5 & 6 & 9 \\ 3 & -1 & -3 & 1 \\ 2 & 3 & 3 & 10 \end{bmatrix}$$

(ii)
The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 13.00 + j0.00 & 8.00 + j5.00 & -13.00 + j0.00 & 8.00 - j5.00 \\ 7.00 - j4.00 & 6.00 + j3.00 & -13.00 - j6.00 & 0.00 - j1.00 \\ -7.00 + j0.00 & 2.00 - j5.00 & 3.00 + j0.00 & 2.00 + j5.00 \\ 7.00 + j4.00 & 0.00 + j1.00 & -13.00 + j6.00 & 6.00 - j3.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 39.00 + j0.00 & 24.00 + j15.00 & -39.00 + j0.00 & 24.00 - j15.00 \\ 7.00 - j4.00 & 6.00 + j3.00 & -13.00 - j6.00 & 0.00 - j1.00 \\ 7.00 + j0.00 & -2.00 + j5.00 & -3.00 + j0.00 & -2.00 - j5.00 \\ 7.00 + j4.00 & 0.00 + j1.00 & -13.00 + j6.00 & 6.00 - j3.00 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 117.00 + j0.00 & 24.00 + j15.00 & 39.00 + j0.00 & 24.00 - j15.00 \\ 21.00 - j12.00 & 6.00 + j3.00 & 13.00 + j6.00 & 0.00 - j1.00 \\ 21.00 + j0.00 & -2.00 + j5.00 & 3.00 + j0.00 & -2.00 - j5.00 \\ 21.00 + j12.00 & 0.00 + j1.00 & 13.00 - j6.00 & 6.00 - j3.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$9y(m,n) = \begin{bmatrix} 19 & 4 & 12 & 10 \\ 12 & 4 & 6 & 8 \\ 9 & 3 & 5 & 7 \\ 11 & 1 & 4 & 2 \end{bmatrix}$$

(iii) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 14.00 + j0.00 & 3.00 + j3.00 & 16.00 + j0.00 & 3.00 - j3.00 \\ -1.00 - j3.00 & -2.00 + j2.00 & -3.00 - j7.00 & -2.00 + j4.00 \\ 0.00 + j0.00 & -3.00 - j9.00 & 18.00 + j0.00 & -3.00 + j9.00 \\ -1.00 + j3.00 & -2.00 - j4.00 & -3.00 + j7.00 & -2.00 - j2.00 \end{bmatrix}$$

The partial convolution output $Y(m,l) = X(k,l)H(l)$ in the frequency domain is obtained by pointwise multiplication of each column of $X(k,l)$ by $H(l)$.

$$\begin{bmatrix} 42.00 + j0.00 & 9.00 + j9.00 & 48.00 + j0.00 & 9.00 - j9.00 \\ -1.00 - j3.00 & -2.00 + j2.00 & -3.00 - j7.00 & -2.00 + j4.00 \\ -0.00 + j0.00 & 3.00 + j9.00 - 18.00 + j0.00 & 3.00 - j9.00 & \\ -1.00 + j3.00 & -2.00 - j4.00 & -3.00 + j7.00 & -2.00 - j2.00 \end{bmatrix}$$

The convolution output $Y(k,l) = Y(m,l)H(k)$ in the frequency domain is obtained by pointwise multiplication of each row of $Y(m,l)$ by $H(k)$.

$$\begin{bmatrix} 126.00 + j0.00 & 9.00 + j9.00 & -48.00 + j0.00 & 9.00 - j9.00 \\ -3.00 - j9.00 & -2.00 + j2.00 & 3.00 + j7.00 & -2.00 + j4.00 \\ -0.00 + j0.00 & 3.00 + j9.00 & 18.00 + j0.00 & 3.00 - j9.00 \\ -3.00 + j9.00 & -2.00 - j4.00 & 3.00 - j7.00 & -2.00 - j2.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$9y(m,n) = \begin{bmatrix} 7 & 7 & 5 & 11 \\ 4 & 14 & 4 & 14 \\ 8 & 8 & 4 & 13 \\ 5 & 10 & 2 & 10 \end{bmatrix}$$

4.5

Convolve $x(m,n)$ and a 3×3 Laplacian enhancement filter. Assume periodicity at the borders.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 7 & 7 & 5 & 1 \\ 4 & 4 & 4 & 1 \\ 0 & 8 & 4 & 3 \\ 5 & 0 & 2 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 2 & 5 & 1 \\ 4 & 1 & 0 & 1 \\ 0 & 0 & 3 & 3 \\ 5 & 0 & 2 & 4 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 4 & 0 & -4 & 1 \\ 0 & 1 & 4 & 3 \\ 4 & 0 & -2 & 1 \end{bmatrix}$$

(i) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 56.00 + j0.00 & 1.00 - j13.00 & 6.00 + j0.00 & 1.00 + j13.00 \\ 5.00 - j5.00 & 2.00 + j2.00 & 11.00 + j3.00 & 10.00 + j4.00 \\ 14.00 + j0.00 & -5.00 - j9.00 & -12.00 + j0.00 & -5.00 + j9.00 \\ 5.00 + j5.00 & 10.00 - j4.00 & 11.00 - j3.00 & 2.00 - j2.00 \end{bmatrix}$$

The 2-D DFT of the shifted and zero-padded filter $H(k,l)$ is

$$\begin{bmatrix} 1.00+j0.00 & 3.00+j0.00 & 5.00+j0.00 & 3.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \\ 5.00+j0.00 & 7.00+j0.00 & 9.00+j0.00 & 7.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \end{bmatrix}$$

The convolution output $Y(k,l) = X(k,l)H(k,l)$ in the frequency domain is obtained by pointwise multiplication of each row of $X(k,l)$ by $H(k,l)$.

$$\begin{bmatrix} 56.00+j0.00 & 3.00-j39.00 & 30.00+j0.00 & 3.00+j39.00 \\ 15.00-j15.00 & 10.00+j10.00 & 77.00+j21.00 & 50.00+j20.00 \\ 70.00+j0.00 & -35.00-j63.00 & -108.00+j0.00 & -35.00+j63.00 \\ 15.00+j15.00 & 50.00-j20.00 & 77.00-j21.00 & 10.00-j10.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} 18 & 19 & 11 & -9 \\ 8 & -3 & 6 & -7 \\ -20 & 32 & 3 & 9 \\ 17 & -22 & 0 & -6 \end{bmatrix}$$

(ii) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 32.00+j0.00 & 0.00+j6.00 & 8.00+j0.00 & 0.00-j6.00 \\ 3.00+j5.00 & -5.00-j5.00 & 3.00+j1.00 & 3.00+j3.00 \\ -2.00+j0.00 & -14.00-j2.00 & -2.00+j0.00 & -14.00+j2.00 \\ 3.00-j5.00 & 3.00-j3.00 & 3.00-j1.00 & -5.00+j5.00 \end{bmatrix}$$

The 2-D DFT of the shifted and zero-padded filter $H(k,l)$ is

$$\begin{bmatrix} 1.00+j0.00 & 3.00+j0.00 & 5.00+j0.00 & 3.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \\ 5.00+j0.00 & 7.00+j0.00 & 9.00+j0.00 & 7.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \end{bmatrix}$$

The convolution output $Y(k,l) = X(k,l)H(k,l)$ in the frequency domain is obtained by pointwise multiplication of each row of $X(k,l)$ by $H(k,l)$.

$$\begin{bmatrix} 32.00+j0.00 & 0.00+j18.00 & 40.00+j0.00 & 0.00-j18.00 \\ 9.00+j15.00 & -25.00-j25.00 & 21.00+j7.00 & 15.00+j15.00 \\ -10.00+j0.00 & -98.00-j14.00 & -18.00+j0.00 & -98.00+j14.00 \\ 9.00-j15.00 & 15.00-j15.00 & 21.00-j7.00 & -25.00+j25.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} -7 & 3 & 20 & -6 \\ 17 & -1 & -10 & -3 \\ -12 & -4 & 10 & 7 \\ 20 & -9 & -2 & 9 \end{bmatrix}$$

(iii) The 2-D DFT, $X(k,l)$, of $x(m,n)$ is

$$\begin{bmatrix} 19.00+j0.00 & 11.00+j2.00 & -1.00+j0.00 & 11.00-j2.00 \\ -1.00+j2.00 & 5.00-j6.00 & -1.00+j2.00 & 5.00+j2.00 \\ 11.00+j0.00 & -17.00-j2.00 & -1.00+j0.00 & -17.00+j2.00 \\ -1.00-j2.00 & 5.00-j2.00 & -1.00-j2.00 & 5.00+j6.00 \end{bmatrix}$$

The 2-D DFT of the shifted and zero-padded filter $H(k,l)$ is

$$\begin{bmatrix} 1.00+j0.00 & 3.00+j0.00 & 5.00+j0.00 & 3.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \\ 5.00+j0.00 & 7.00+j0.00 & 9.00+j0.00 & 7.00+j0.00 \\ 3.00+j0.00 & 5.00+j0.00 & 7.00+j0.00 & 5.00+j0.00 \end{bmatrix}$$

The convolution output $Y(k,l) = X(k,l)H(k,l)$ in the frequency domain is obtained by pointwise multiplication of each row of $X(k,l)$ by $H(k,l)$.

$$\begin{bmatrix} 19.00+j0.00 & 33.00+j6.00 & -5.00+j0.00 & 33.00-j6.00 \\ -3.00+j6.00 & 25.00-j30.00 & -7.00+j14.00 & 25.00+j10.00 \\ 55.00+j0.00 & -119.00-j14.00 & -9.00+j0.00 & -119.00+j14.00 \\ -3.00-j6.00 & 25.00-j10.00 & -7.00-j14.00 & 25.00+j30.00 \end{bmatrix}$$

The output is the 2-D IDFT of $Y(k,l)$.

$$y(m,n) = \begin{bmatrix} -2 & 12 & 7 & 0 \\ 17 & -4 & -26 & 1 \\ -12 & 1 & 22 & 9 \\ 17 & -6 & -16 & -1 \end{bmatrix}$$

4.6 Let a 2-D lowpass ideal filter $H(k,l)$ is defined between the limits $-4, -3, -2, -1, 0, 1, 2, 3$ in both the directions. Find the filter coefficients for a given cutoff radius r such that $H(k,l) = 1$ for inside the circle defined by r and zero elsewhere.
(i) $r = 1$
(i) $r = 2$
(i) $r = 3$
(i)

$$H(k,l) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$H(k,l) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$H(k,l) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4.7 Let a 2-D highpass ideal filter $H(k,l)$ is defined between the limits $-4,-3,-2,-1,0,1,2,3$ in both the directions. Find the filter coefficients for a given cutoff radius r such that $H(k,l) = 1$ for outside the circle defined by r and zero elsewhere.

(i) $r = 1$

(i) $r = 2$

(i) $r = 3$

(i)

$$H(k,l) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$H(k,l) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$H(k,l) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Chapter 5

resto

5.1 A signal with with power spectral density

$$|X(k)|^2 = \{64, 0, 0, 0, 0, 0, 0, 0\}$$

has been blurred by a process with finite impulse response

$$\{h_d(0) = 0.5, h_d(1) = 0.5\}$$

and corrupted by an additive Gaussian noise with power spectral density

$$\{0.0002, 0.0071, 0.0273, 0.0121, 0.0220, 0.0121, 0.0273, 0.0071\}$$

The samples of the degraded signal are

$$y(n) = \{0.9898, 0.9759, 1.0319, 1.0313, 0.9135, 0.9970, 0.9835, 1.0628\}$$

Restore the true signal using the Wiener filter.
(i)
The DFT of $h_d(n)$

$$H_d(k) = \{1, 0.8536 - j0.3536, 0.5 - j0.5, 0.1464 - j0.3536, 0, 0.1464 + j0.3536, 0.5 + j0.5, 0.8536 + j0.3536\}$$

Its power spectrum is

$$|H_d(k)|^2 = H_d(k)H_d^*(k) = \{1, 0.8536, 0.5000, 0.1464, 0, 0.1464, 0.5000, 0.8536\}$$

The conjugate of $H_d(k)$, $H_d^*(k)$, is obtained by changing the sign of the imaginary part of the values of $H_d(k)$. With all the required quantities available, the DFT of the Wiener filter $H_r(k)$ is found to be

$$H_r(k) = \{1, 0, 0, 0, 0, 0, 0, 0\}$$

The product $H_r(k)Y(k)$ yields the DFT of the restored signal.

$$H_r(k)Y(k) = \{7.9856, 0, 0, 0, 0, 0, 0, 0\}$$

The IDFT of these values yields the restored signal

$$\hat{x}(n) = \{0.9982, 0.9982, 0.9982, 0.9982, 0.9982, 0.9982, 0.9982, 0.9982\}$$

5.2 A signal with with power spectral density

$$|X(k)|^2 = \{0, 16, 0, 0, 0, 0, 0, 16\}$$

has been blurred by a process with finite impulse response

$$\{h_d(0) = 0.5, h_d(1) = 0.5\}$$

and corrupted by an additive Gaussian noise with power spectral density

$$\{0.0119, 0.0058, 0.0807, 0.0107, 0.1725, 0.0107, 0.0807, 0.0058\}$$

The samples of the degraded signal are

$$y(n) = \{-0.4305, 0.3907, 0.8310, 0.9653, 0.2446, -0.3503, -0.7983, -0.7435\}$$

Restore the true signal using the Wiener filter.
(ii)
The DFT of $h_d(n)$

$$H_d(k) = \{1, 0.8536 - j0.3536, 0.5 - j0.5, 0.1464 - j0.3536, 0, 0.1464 + j0.3536, 0.5 + j0.5, 0.8536 + j0.3536\}$$

Its power spectrum is

$$|H_d(k)|^2 = H_d(k)H_d^*(k) = \{1, 0.8536, 0.5000, 0.1464, 0, 0.1464, 0.5000, 0.8536\}$$

The conjugate of $H_d(k)$, $H_d^*(k)$, is obtained by changing the sign of the imaginary part of the values of $H_d(k)$. With all the required quantities available, the DFT of the Wiener filter $H_r(k)$ is found to be

$$H_r(k) = \{0, 0.9996 + j0.4140, 0, 0, 0, 0, 0, 0.9996 - j0.4140\}$$

The product $H_r(k)Y(k)$ yields the DFT of the restored signal.

$$H_r(k)Y(k) = \{0, 0.0329 - j3.9230, 0, 0, 0, 0, 0, 0.0329 + j3.9230\}$$

The IDFT of these values yields the restored signal

$$\hat{x}(n) = \{0.0082, 0.6993, 0.9808, 0.6877, -0.0082, -0.6993, -0.9808, -0.6877\}$$

5.3 A signal with with power spectral density

$$|X(k)|^2 = \{0, 0, 16, 0, 0, 0, 0, 16, 0\}$$

has been blurred by a process with finite impulse response

$$\{h_d(0) = 0.5, h_d(1) = 0.5\}$$

and corrupted by an additive Gaussian noise with power spectral density

$$\{0.0067, 0.1671, 0.1262, 0.1311, 0.1654, 0.1311, 0.1262, 0.1671\}$$

The samples of the degraded signal are

$$y(n) = \{0.6544, 0.5086, -0.6492, -0.5742, 0.3938, 0.7350, -0.5616, -0.4252\}$$

Restore the true signal using the Wiener filter.
(iii)
The DFT of $h_d(n)$

$$H_d(k) = \{1, 0.8536 - j0.3536, 0.5 - j0.5, 0.1464 - j0.3536, 0, 0.1464 + j0.3536, 0.5 + j0.5, 0.8536 + j0.3536\}$$

Its power spectrum is

$$|H_d(k)|^2 = H_d(k)H_d^*(k) = \{1, 0.8536, 0.5000, 0.1464, 0, 0.1464, 0.5000, 0.8536\}$$

The conjugate of $H_d(k)$, $H_d^*(k)$, is obtained by changing the sign of the imaginary part of the values of $H_d(k)$. With all the required quantities available, the DFT of the Wiener filter $H_r(k)$ is found to be

$$H_r(k) = \{0, 0, 0.9845 + j0.9845, 0, 0, 0, 0.9845 - j0.9845, 0\}$$

The product $H_r(k)Y(k)$ yields the DFT of the restored signal.

$$H_r(k)Y(k) = \{0, 0, 4.4322 + j0.0157, 0, 0, 0, 4.4322 - j0.0157, 0\}$$

The IDFT of these values yields the restored signal

$$\hat{x}(n) = \{1.1080, -0.0039, -1.1080, 0.0039, 1.1080, -0.0039, -1.1080, 0.0039\}$$

5.4 A signal with with power spectral density

$$|X(k)|^2 = \{0, 0, 0, 16, 0, 16, 0, 0\}$$

has been blurred by a process with finite impulse response

$$\{h_d(0) = 0.5, h_d(1) = 0.5\}$$

and corrupted by an additive Gaussian noise with power spectral density

$$\{0.0772, 0.0694, 0.0930, 0.0001, 0.0545, 0.0001, 0.0930, 0.0694\}$$

The samples of the degraded signal are

$$y(n) = \{0.1272, 0.2353, -0.4300, 0.2133, -0.2887, -0.0976, 0.3358, -0.3732\}$$

Restore the true signal using the Wiener filter.
(iv)
The DFT of $h_d(n)$

$$H_d(k) = \{1, 0.8536 - j0.3536, 0.5 - j0.5, 0.1464 - j0.3536, 0, 0.1464 + j0.3536, 0.5 + j0.5, 0.8536 + j0.3536\}$$

Its power spectrum is

$$|H_d(k)|^2 = H_d(k)H_d^*(k) = \{1, 0.8536, 0.5000, 0.1464, 0, 0.1464, 0.5000, 0.8536\}$$

The conjugate of $H_d(k)$, $H_d^*(k)$, is obtained by changing the sign of the imaginary part of the values of $H_d(k)$. With all the required quantities available, the DFT of the Wiener filter $H_r(k)$ is found to be

$$H_r(k) = \{0, 0, 0, 1.0000 + j2.4141, 0, 1.0000 - j2.4141, 0, 0\}$$

The product $H_r(k)Y(k)$ yields the DFT of the restored signal.

$$H_r(k)Y(k) = \{0, 0, 0, 4.0135 + j0.0209, 0, 4.0135 - j0.0209, 0, 0\}$$

The IDFT of these values yields the restored signal

$$\hat{x}(n) = \{1.0034, -0.7132, 0.0052, 0.7058, -1.0034, 0.7132, -0.0052, -0.7058\}$$

5.5 A signal with with power spectral density

$$|X(k)|^2 = \{0, 0, 16, 0, 0, 0, 16, 0\}$$

has been blurred by a process with finite impulse response

$$\{h_d(0) = 0.5, h_d(1) = 0.5\}$$

and corrupted by an additive Gaussian noise with power spectral density

$$\{0.1584, 0.0055, 0.0030, 0.1719, 0.0047, 0.1719, 0.0030, 0.0055\}$$

The samples of the degraded signal are

$$y(n) = \{-0.3581, 0.5292, 0.5198, -0.3412, -0.5804, 0.5697, 0.5835, -0.5244\}$$

Restore the true signal using the Wiener filter.
The DFT of $h_d(n)$

$$H_d(k) = \{1, 0.8536 - j0.3536, 0.5 - j0.5, 0.1464 - j0.3536, 0, 0.1464 + j0.3536, 0.5 + j0.5, 0.8536 + j0.3536\}$$

Its power spectrum is

$$|H_d(k)|^2 = H_d(k)H_d^*(k) = \{1, 0.8536, 0.5000, 0.1464, 0, 0.1464, 0.5000, 0.8536\}$$

The conjugate of $H_d(k)$, $H_d^*(k)$, is obtained by changing the sign of the imaginary part of the values of $H_d(k)$. With all the required quantities available, the DFT of the Wiener filter $H_r(k)$ is found to be

$$H_r(k) = \{0, 0, 0.9996 + j0.9996, 0, 0, 0, 0.9996 - j0.9996, 0\}$$

The product $H_r(k)Y(k)$ yields the DFT of the restored signal.

$$H_r(k)Y(k) = \{0, 0, -0.0774 - j4.0047, 0, 0, 0, -0.0774 + j4.0047, 0\}$$

The IDFT of these values yields the restored signal

$$\hat{x}(n) = \{-0.0193, 1.0012, 0.0193, -1.0012, -0.0193, 1.0012, 0.0193, -1.0012\}$$

Chapter 6

regist

6.1 Using nearest-neighbor interpolation, find the 7×7 interpolated version of the image $x(m, n)$.

(i)

$$\begin{bmatrix} 43 & 50 & 50 & 52 \\ 45 & 49 & 51 & 50 \\ 46 & 46 & 49 & 48 \\ 43 & 44 & 47 & 42 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 68 & 80 & 83 & 78 \\ 39 & 54 & 61 & 66 \\ 41 & 44 & 44 & 67 \\ 55 & 46 & 34 & 66 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 63 & 64 & 64 & 64 \\ 62 & 62 & 62 & 61 \\ 61 & 61 & 61 & 58 \\ 62 & 61 & 60 & 58 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 43 & 50 & 50 & 50 & 50 & 52 & 52 \\ 45 & 49 & 49 & 51 & 51 & 50 & 50 \\ 45 & 49 & 49 & 51 & 51 & 50 & 50 \\ 46 & 46 & 46 & 49 & 49 & 48 & 48 \\ 46 & 46 & 46 & 49 & 49 & 48 & 48 \\ 43 & 44 & 44 & 47 & 47 & 42 & 42 \\ 43 & 44 & 44 & 47 & 47 & 42 & 42 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 68 & 80 & 80 & 83 & 83 & 78 & 78 \\ 39 & 54 & 54 & 61 & 61 & 66 & 66 \\ 39 & 54 & 54 & 61 & 61 & 66 & 66 \\ 41 & 44 & 44 & 44 & 44 & 67 & 67 \\ 41 & 44 & 44 & 44 & 44 & 67 & 67 \\ 55 & 46 & 46 & 34 & 34 & 66 & 66 \\ 55 & 46 & 46 & 34 & 34 & 66 & 66 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 63 & 64 & 64 & 64 & 64 & 64 & 64 \\ 62 & 62 & 62 & 62 & 62 & 61 & 61 \\ 62 & 62 & 62 & 62 & 62 & 61 & 61 \\ 61 & 61 & 61 & 61 & 61 & 58 & 58 \\ 61 & 61 & 61 & 61 & 61 & 58 & 58 \\ 62 & 61 & 61 & 60 & 60 & 58 & 58 \\ 62 & 61 & 61 & 60 & 60 & 58 & 58 \end{bmatrix}$$

6.2 Using bilinear interpolation, find the 7×7 interpolated version of the image $x(m, n)$.
(i)

$$\begin{bmatrix} 34 & 51 & 56 & 53 \\ 38 & 53 & 57 & 54 \\ 40 & 52 & 56 & 52 \\ 39 & 48 & 52 & 49 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 53 & 42 & 39 & 58 \\ 51 & 46 & 44 & 49 \\ 54 & 52 & 58 & 46 \\ 63 & 57 & 63 & 52 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 82 & 84 & 86 & 97 \\ 80 & 80 & 85 & 103 \\ 79 & 77 & 90 & 114 \\ 80 & 84 & 102 & 118 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 34.00 & 42.50 & 51.00 & 53.50 & 56.00 & 54.50 & 53.00 \\ 36.00 & 44.00 & 52.00 & 54.25 & 56.50 & 55.00 & 53.50 \\ 38.00 & 45.50 & 53.00 & 55.00 & 57.00 & 55.50 & 54.00 \\ 39.00 & 45.75 & 52.50 & 54.50 & 56.50 & 54.75 & 53.00 \\ 40.00 & 46.00 & 52.00 & 54.00 & 56.00 & 54.00 & 52.00 \\ 39.50 & 44.75 & 50.00 & 52.00 & 54.00 & 52.25 & 50.50 \\ 39.00 & 43.50 & 48.00 & 50.00 & 52.00 & 50.50 & 49.00 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 53.00 & 47.50 & 42.00 & 40.50 & 39.00 & 48.50 & 58.00 \\ 52.00 & 48.00 & 44.00 & 42.75 & 41.50 & 47.50 & 53.50 \\ 51.00 & 48.50 & 46.00 & 45.00 & 44.00 & 46.50 & 49.00 \\ 52.50 & 50.75 & 49.00 & 50.00 & 51.00 & 49.25 & 47.50 \\ 54.00 & 53.00 & 52.00 & 55.00 & 58.00 & 52.00 & 46.00 \\ 58.50 & 56.50 & 54.50 & 57.50 & 60.50 & 54.75 & 49.00 \\ 63.00 & 60.00 & 57.00 & 60.00 & 63.00 & 57.50 & 52.00 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 82.00 & 83.00 & 84.00 & 85.00 & 86.00 & 91.50 & 97.00 \\ 81.00 & 81.50 & 82.00 & 83.75 & 85.50 & 92.75 & 100.00 \\ 80.00 & 80.00 & 80.00 & 82.50 & 85.00 & 94.00 & 103.00 \\ 79.50 & 79.00 & 78.50 & 83.00 & 87.50 & 98.00 & 108.50 \\ 79.00 & 78.00 & 77.00 & 83.50 & 90.00 & 102.00 & 114.00 \\ 79.50 & 80.00 & 80.50 & 88.25 & 96.00 & 106.00 & 116.00 \\ 80.00 & 82.00 & 84.00 & 93.00 & 102.00 & 110.00 & 118.00 \end{bmatrix}$$

6.3 Using nearest-neighbor interpolation, find the scaled version of the image $x(m,n)$.

(i) $a = 0.5$, $e = 0.75$

$$\begin{bmatrix} 52 & 61 & 57 & 66 \\ 58 & 64 & 69 & 64 \\ 45 & 60 & 74 & 61 \\ 56 & 63 & 74 & 63 \end{bmatrix}$$

(ii) $a = -0.5$, $e = -0.5$

$$\begin{bmatrix} 71 & 56 & 47 & 92 \\ 66 & 51 & 47 & 108 \\ 64 & 55 & 70 & 122 \\ 73 & 57 & 81 & 127 \end{bmatrix}$$

(iii) $a = 0.75$, $e = 0.25$

$$\begin{bmatrix} 48 & 53 & 46 & 62 \\ 54 & 54 & 54 & 80 \\ 64 & 53 & 59 & 78 \\ 57 & 46 & 56 & 55 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 52 & 61 & 66 \\ 45 & 60 & 61 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 127 & 57 \\ 108 & 51 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 48 \\ 54 \\ 57 \end{bmatrix}$$

6.4 Using nearest-neighbor interpolation, find the sheared version of the image $x(m,n)$.

(i) $b = 0$, $d = 1$

$$\begin{bmatrix} 17 & 20 & 26 & 25 \\ 18 & 23 & 30 & 24 \\ 17 & 24 & 32 & 27 \\ 20 & 28 & 30 & 32 \end{bmatrix}$$

(ii) $b = 0$, $d = 0.5$

$$\begin{bmatrix} 63 & 49 & 51 & 54 \\ 66 & 60 & 52 & 56 \\ 57 & 62 & 62 & 57 \\ 57 & 56 & 64 & 61 \end{bmatrix}$$

(iii) $b = 0$, $d = 0.3$

$$\begin{bmatrix} 179 & 178 & 179 & 184 \\ 177 & 178 & 179 & 189 \\ 176 & 177 & 180 & 193 \\ 174 & 175 & 184 & 190 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 17 & 20 & 26 & 25 & 0 & 0 & 0 \\ 0 & 18 & 23 & 30 & 24 & 0 & 0 \\ 0 & 0 & 17 & 24 & 32 & 27 & 0 \\ 0 & 0 & 0 & 20 & 28 & 30 & 32 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 63 & 49 & 51 & 54 & 0 \\ 0 & 60 & 52 & 56 & 0 \\ 0 & 57 & 62 & 62 & 57 \\ 0 & 0 & 56 & 64 & 61 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 179 & 178 & 179 & 184 & 0 \\ 177 & 178 & 179 & 189 & 0 \\ 0 & 176 & 177 & 180 & 193 \\ 0 & 174 & 175 & 184 & 190 \end{bmatrix}$$

6.5 Using nearest-neighbor interpolation, find the sheared version of the image $x(m, n)$.
(i) $b = 1$, $d = 0$

$$\begin{bmatrix} 41 & 36 & 123 & 151 \\ 27 & 10 & 79 & 136 \\ 17 & 17 & 33 & 91 \\ 17 & 30 & 17 & 70 \end{bmatrix}$$

(ii) $b = 0.7$, $d = 0$

$$\begin{bmatrix} 172 & 157 & 115 & 62 \\ 163 & 165 & 118 & 83 \\ 138 & 185 & 128 & 71 \\ 121 & 184 & 126 & 83 \end{bmatrix}$$

(iii) $b = 0.3$, $d = 0$

$$\begin{bmatrix} 95 & 111 & 48 & 32 \\ 96 & 115 & 59 & 26 \\ 89 & 90 & 37 & 24 \\ 86 & 73 & 15 & 21 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 41 & 0 & 0 & 0 \\ 27 & 36 & 0 & 0 \\ 17 & 10 & 123 & 0 \\ 17 & 17 & 79 & 151 \\ 0 & 30 & 33 & 136 \\ 0 & 0 & 17 & 91 \\ 0 & 0 & 0 & 70 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 172 & 0 & 0 & 0 \\ 163 & 157 & 115 & 0 \\ 138 & 165 & 118 & 62 \\ 121 & 185 & 128 & 83 \\ 0 & 184 & 126 & 71 \\ 0 & 0 & 0 & 83 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 95 & 111 & 0 & 0 \\ 96 & 115 & 48 & 32 \\ 89 & 90 & 59 & 26 \\ 86 & 73 & 37 & 24 \\ 0 & 0 & 15 & 21 \end{bmatrix}$$

6.6 Using nearest-neighbor interpolation, find the rotated version of the image $x(m,n)$ in the counterclockwise direction. $\theta = 90°$, $\theta = 180°$ and $\theta = 45°$.

(i)

$$\begin{bmatrix} 95 & 47 & 65 & 55 \\ 74 & 60 & 60 & 47 \\ 105 & 103 & 67 & 46 \\ 103 & 78 & 67 & 58 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 74 & 81 & 67 & 75 \\ 77 & 77 & 77 & 83 \\ 58 & 69 & 69 & 80 \\ 46 & 61 & 69 & 82 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 66 & 77 & 79 & 64 \\ 56 & 68 & 61 & 69 \\ 43 & 51 & 49 & 66 \\ 39 & 45 & 43 & 55 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 55 & 47 & 46 & 58 \\ 65 & 60 & 67 & 67 \\ 47 & 60 & 103 & 78 \\ 95 & 74 & 105 & 103 \end{bmatrix} \quad \begin{bmatrix} 58 & 67 & 78 & 103 \\ 46 & 67 & 103 & 105 \\ 47 & 60 & 60 & 74 \\ 55 & 65 & 47 & 95 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 55 & 0 & 0 \\ 0 & 47 & 60 & 46 & 0 \\ 95 & 60 & 103 & 67 & 58 \\ 0 & 105 & 103 & 78 & 0 \\ 0 & 0 & 103 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 75 & 83 & 80 & 82 \\ 67 & 77 & 69 & 69 \\ 81 & 77 & 69 & 61 \\ 74 & 77 & 58 & 46 \end{bmatrix} \quad \begin{bmatrix} 82 & 69 & 61 & 46 \\ 80 & 69 & 69 & 58 \\ 83 & 77 & 77 & 77 \\ 75 & 67 & 81 & 74 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 75 & 0 & 0 \\ 0 & 81 & 77 & 80 & 0 \\ 74 & 77 & 69 & 69 & 82 \\ 0 & 58 & 69 & 61 & 0 \\ 0 & 0 & 46 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 64 & 69 & 66 & 55 \\ 79 & 61 & 49 & 43 \\ 77 & 68 & 51 & 45 \\ 66 & 56 & 43 & 39 \end{bmatrix} \quad \begin{bmatrix} 55 & 43 & 45 & 39 \\ 66 & 49 & 51 & 43 \\ 69 & 61 & 68 & 56 \\ 64 & 79 & 77 & 66 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 64 & 0 & 0 \\ 0 & 77 & 61 & 66 & 0 \\ 66 & 68 & 51 & 49 & 55 \\ 0 & 43 & 51 & 45 & 0 \\ 0 & 0 & 39 & 0 & 0 \end{bmatrix}$$

6.7 Find the cross-correlation and the correlation coefficients of $x(m,n)$ and $h(m,n)$.

$$h(m,n) = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & 0 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & 0 & 2 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 0 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 1 & 0 \\ 2 & 2 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 2 & 5 & 7 & 9 & 7 & 2 \\ 3 & 10 & 15 & 10 & 12 & 6 \\ 3 & 10 & 23 & 19 & 13 & 4 \\ 4 & 11 & 18 & 15 & 10 & 8 \\ 1 & 4 & 10 & 14 & 5 & 8 \\ 1 & 4 & 4 & 5 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} -0.1000 & 0.2828 & -0.0447 & 0.2236 & 0.1180 & -0.1000 \\ -0.4128 & 0.0286 & 0.0408 & -0.1342 & 0.6261 & 0.8000 \\ -0.5814 & -0.4727 & 0.6200 & 0.3213 & 0.2683 & -0.1512 \\ -0.3124 & -0.4491 & 0.1315 & -0.1315 & 0.4619 & 0.5292 \\ -0.4914 & -0.6252 & 0.0316 & 0.2286 & -0.1890 & 0.5292 \\ -0.1000 & -0.1315 & -0.1315 & -0.1474 & -0.1000 & -0.1000 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 3 & 7 & 6 & 5 & 5 & 2 \\ 0 & 5 & 15 & 10 & 10 & 8 \\ 4 & 7 & 11 & 16 & 13 & 10 \\ 1 & 7 & 12 & 13 & 12 & 10 \\ 1 & 3 & 8 & 10 & 5 & 8 \\ 1 & 3 & 3 & 4 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} -0.1000 & 0.3124 & -0.0164 & 0.0229 & 0.2828 & -0.1000 \\ -0.5500 & -0.3130 & 0.7431 & -0.1732 & 0.4619 & 0.5292 \\ -0.1315 & -0.3953 & -0.4400 & 0.0316 & 0.3464 & 0.4000 \\ -0.4914 & -0.2800 & 0.2121 & -0.5965 & 0.3592 & 0.4000 \\ -0.4914 & -0.6424 & 0.1414 & 0.0359 & -0.1890 & 0.5292 \\ -0.1000 & -0.1414 & -0.1414 & -0.1512 & -0.1000 & -0.1000 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 4 & 8 & 8 & 3 & 0 \\ 0 & 3 & 10 & 13 & 10 & 0 \\ 3 & 9 & 15 & 18 & 11 & 2 \\ 1 & 7 & 14 & 13 & 10 & 8 \\ 2 & 5 & 9 & 8 & 5 & 8 \\ 1 & 2 & 2 & 3 & 2 & 2 \end{bmatrix} \quad \begin{bmatrix} -0.1000 & 0.0707 & 0.0894 & 0.2507 & -0.1000 & 0 \\ -0.5500 & -0.5292 & -0.1342 & 0.4143 & 0.7562 & 0 \\ -0.1414 & -0.3000 & -0.3591 & 0.3124 & 0.3355 & -0.1000 \\ -0.5657 & -0.4490 & 0.3411 & -0.0800 & 0.4619 & 0.5292 \\ -0.3536 & -0.4041 & 0.0800 & -0.2874 & -0.1890 & 0.5292 \\ -0.1000 & -0.1512 & -0.1512 & -0.1414 & -0.1000 & -0.1000 \end{bmatrix}$$

6.8 Find the autocorrelation of $x(m,n)$.

(i)

$$x(m,n) = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 0 & 3 & 1 & 2 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 0 & 2 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 4 & 2 & 8 & 7 & 4 & 5 & 2 \\ 8 & 6 & 11 & 12 & 5 & 4 & 2 \\ 6 & 6 & 9 & 16 & 9 & 12 & 6 \\ 8 & 13 & 17 & 36 & 17 & 13 & 8 \\ 6 & 12 & 9 & 16 & 9 & 6 & 6 \\ 2 & 4 & 5 & 12 & 11 & 6 & 8 \\ 2 & 5 & 4 & 7 & 8 & 2 & 4 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 6 & 2 & 8 & 9 & 3 & 5 & 2 \\ 0 & 9 & 9 & 16 & 10 & 10 & 4 \\ 8 & 9 & 19 & 20 & 16 & 12 & 2 \\ 8 & 16 & 17 & 44 & 17 & 16 & 8 \\ 2 & 12 & 16 & 20 & 19 & 9 & 8 \\ 4 & 10 & 10 & 16 & 9 & 9 & 0 \\ 2 & 5 & 3 & 9 & 8 & 2 & 6 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 6 & 4 & 5 & 11 & 7 & 3 & 0 \\ 6 & 11 & 6 & 9 & 11 & 5 & 0 \\ 4 & 7 & 16 & 16 & 7 & 7 & 6 \\ 10 & 7 & 15 & 42 & 15 & 7 & 10 \\ 6 & 7 & 7 & 16 & 16 & 7 & 4 \\ 0 & 5 & 11 & 9 & 6 & 11 & 6 \\ 0 & 3 & 7 & 11 & 5 & 4 & 6 \end{bmatrix}$$

Chapter 7

Image Reconstruction from Projections

7.1. Find the equation of the straight line, in the normal form, located at a distance s from the origin and the perpendicular to it makes an angle of θ degrees with the x-axis.
(i) $s = 3,\ \theta = 0°$.
(ii) $s = 1,\ \theta = 45°$.
(iii) $s = 2,\ \theta = -60°$.
(iv) $s = 5,\ \theta = 315°$.
(v) $s = 0,\ \theta = 30°$.
(i)

$$x\cos(0°) + y\sin(0°) = 3, \qquad x = 3$$

(ii)

$$x\cos(45°) + y\sin(45°) = 1, \qquad \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1$$

(iii)

$$x\cos(-60°) + y\sin(-60°) = 2, \qquad \frac{x}{2} - \frac{\sqrt{3}y}{2} = 2$$

(iv)

$$x\cos(315°) + y\sin(315°) = 5, \qquad \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = 5$$

(v)

$$x\cos(30°) + y\sin(30°) = 0, \qquad \frac{\sqrt{3}x}{2} + \frac{y}{2} = 0$$

7.2. Find the Radon transform of a circular cylinder with radius 6 and height 3 located at the origin. The cylinder is characterized by

$$f(x,y) = \begin{cases} 3 & \text{for } x^2 + y^2 \le 6^2 \\ 0 & \text{otherwise} \end{cases}$$

$$R(s,\theta) = 6\sqrt{6^2 - s^2}$$

From the transform obtained, and using Radon transform properties, find the Radan transform of
(i)

$$f(x,y) = \begin{cases} 3 & \text{for } x^2 + y^2 \le 3^2 \\ 0 & \text{otherwise} \end{cases}$$

$$R(s,\theta) = 6\sqrt{3^2 - s^2}$$

(ii)

$$f(x,y) = \begin{cases} 3 & \text{for } (x-1)^2 + (y-2)^2 \le 6^2 \\ 0 & \text{otherwise} \end{cases}$$

$$R(s,\theta) = 6\sqrt{6^2 - (s - \cos(\theta) - 2\sin(\theta))^2}$$

(iii)

$$f(x,y) = \begin{cases} 3 & \text{for } (3x)^2 + (3y)^2 \le 6^2 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{3}R(3s,\theta) = 2\sqrt{6^2 - (3s)^2}$$

7.3 Find the Radon transform of the shifted and scaled impulse:
(i) $\delta(x,y)$

$$s = 0$$

(ii) $\delta(x-4, y-4)$

$$s = \sqrt{32}\cos(45 - \theta)$$

(iii) $\delta(x-4, y+4)$

$$s = \sqrt{32}\cos(-45 - \theta)$$

(iv) $\delta(x-1, y)$

$$s = \cos(-\theta)$$

(v) $\delta(x, y-1)$

$$s = \cos(90 - \theta)$$

7.4 Find the Radon transform of the line $f(x,y)$ characterized by the given equation. Using the result, find the transforms of $f(ax,ay)$ and $f(x-p,y-q)$ from the properties of the Radon transform. Find the equation of the lines $f(ax,ay)$ and $f(x-p,y-q)$ and determine the the Radon transform directly. Verify that the results are the same as those obtained using the properties.
(i)

$$f(x,y) = 3x + 2y - 6 = 0, x \text{ is limited from 0 to 2}, a = 2, p = 2, q = 3$$

The square root of the sum of the squares of the coefficients of x and y is

$$\sqrt{(3)^2 + 2^2} = \sqrt{13}$$

Dividing the equation by this constant and rearranging, we get

$$\frac{3x}{\sqrt{13}} + \frac{2y}{\sqrt{13}} = \frac{6}{\sqrt{13}}, \qquad x\cos(33.6901^\circ) + y\sin(33.6901^\circ) = \frac{6}{\sqrt{13}}$$

The terminal coordinates of the line are $(0,3)$ and $(2,0)$ The length of the line is

$$\sqrt{(0-2)^2 + (3-0)^2} = \sqrt{13}$$

$$R(\frac{6}{\sqrt{13}}, 33.6901^\circ) = \sqrt{13}$$

Using the property, the transform of $f(2x, 2y)$ is

$$\frac{1}{2}R(2s, 33.6901°) = \frac{1}{2}\sqrt{13}$$

$$f(2x, 2y) = 3x + 2y - 3 = 0$$

$$\frac{3x}{\sqrt{13}} + \frac{2y}{\sqrt{13}} = \frac{3}{\sqrt{13}}, \qquad x\cos(33.6901°) + y\sin(33.6901°) = \frac{3}{\sqrt{13}}$$

The terminal coordinates of the line are $(0, 1.5)$ and $(1, 0)$ The length of the line is

$$\sqrt{(0-1)^2 + (1.5-0)^2} = \frac{\sqrt{13}}{2}$$

$$R(\frac{3}{\sqrt{13}}, 33.6901°) = \frac{\sqrt{13}}{2}$$

Using the property, the transform of $f(x-2, y-3)$ is

$$R(s - 2\cos(33.6901°) - 3\sin(33.6901°), 33.6901°) = R(s - \frac{12}{\sqrt{13}}, 33.6901°) = \sqrt{13}$$

$$f(x-2, y-3) = 3x + 2y - 18 = 0$$

$$\frac{3x}{\sqrt{13}} + \frac{2y}{\sqrt{13}} = \frac{18}{\sqrt{13}}, \qquad x\cos(33.6901°) + y\sin(33.6901°) = \frac{18}{\sqrt{13}}$$

The length of the line remains the same, $\sqrt{13}$.

$$R(\frac{18}{\sqrt{13}}, 33.6901°) = \sqrt{13}$$

(ii)

$$f(x, y) = 4x + 2y - 8 = 0, x \text{ is limited from 0 to 2}, a = 3, p = 3, q = 2$$

The square root of the sum of the squares of the coefficients of x and y is

$$\sqrt{(4)^2 + 2^2} = \sqrt{20}$$

Dividing the equation by this constant and rearranging, we get

$$\frac{4x}{\sqrt{20}} + \frac{2y}{\sqrt{20}} = \frac{8}{\sqrt{20}}, \qquad x\cos(26.5651°) + y\sin(26.5651°) = \frac{8}{\sqrt{20}}$$

The terminal coordinates of the line are $(0, 3)$ and $(2, 0)$ The length of the line is

$$\sqrt{(0-2)^2 + (4-0)^2} = \sqrt{20}$$

$$R(\frac{4}{\sqrt{5}}, 26.5651°) = \sqrt{20}$$

Using the property, the transform of $f(3x, 3y)$ is

$$\frac{1}{3}R(3s, 26.5651°) = \frac{1}{3}\sqrt{20}$$

$$f(3x, 3y) = 4x + 2y - 8/3 = 0$$

$$\frac{4x}{\sqrt{20}} + \frac{2y}{\sqrt{20}} = \frac{8}{3\sqrt{20}}, \qquad x\cos(26.5651^\circ) + y\sin(26.5651^\circ) = \frac{4}{3\sqrt{5}}$$

The terminal coordinates of the line are $(0, 4)/3$ and $(2, 0)/3$ The length of the line is

$$\sqrt{(0-2)^2 + (4-0)^2} = \frac{\sqrt{20}}{3}$$

$$R(\frac{4}{3\sqrt{5}}, 26.5651^\circ) = \frac{\sqrt{20}}{3}$$

Using the property, the transform of $f(x-3, y-2)$ is

$$R(s - 3\cos(26.5651^\circ) - 2\sin(26.5651^\circ), 26.5651^\circ) = R(s - \frac{8}{\sqrt{5}}, 26.5651^\circ) = \sqrt{20}$$

$$f(x-3, y-2) = 4x + 2y - 24 = 0$$

$$\frac{4x}{\sqrt{20}} + \frac{2y}{\sqrt{20}} = \frac{24}{\sqrt{20}}, \qquad x\cos(26.5651^\circ) + y\sin(26.5651^\circ) = \frac{12}{\sqrt{5}}$$

The length of the line remains the same, $\sqrt{20}$.

$$R(\frac{12}{\sqrt{5}}, 26.5651^\circ) = \sqrt{20}$$

(iii)

$$f(x, y) = x + y - 1 = 0, x \text{ is limited from 0 to } 1, a = -3, p = -3, q = 2$$

The square root of the sum of the squares of the coefficients of x and y is

$$\sqrt{(1)^2 + 1^2} = \sqrt{2}$$

Dividing the equation by this constant and rearranging, we get

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{1}{\sqrt{2}}, \qquad x\cos(45^\circ) + y\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

The terminal coordinates of the line are $(0, 1)$ and $(1, 0)$ The length of the line is

$$\sqrt{(0-1)^2 + (1-0)^2} = \sqrt{2}$$

$$R(\frac{1}{\sqrt{2}}, 45^\circ) = \sqrt{2}$$

Using the property, the transform of $f(-3x, -3y)$ is

$$\frac{1}{3}R(-3s, 45^\circ) = \frac{1}{3}\sqrt{2}$$

$$f(-3x, -3y) = x + y + 1/3 = 0$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = -\frac{1}{3\sqrt{2}}, \qquad x\cos(45^\circ) + y\sin(45^\circ) = -\frac{1}{3\sqrt{2}}$$

The terminal coordinates of the line are $(0, 1)/(-3)$ and $(1, 0)/(-3)$ The length of the line is

$$\sqrt{(0-1)^2 + (1-0)^2} = \frac{\sqrt{2}}{3}$$

$$R(-\frac{1}{3\sqrt{2}}, 45^\circ) = \frac{\sqrt{2}}{3}$$

Using the property, the transform of $f(x+3, y-2)$ is

$$R(s + 3\cos(45^\circ) - 2\sin(45^\circ), 45^\circ) = R(s + \frac{1}{\sqrt{2}}, 45^\circ) = \sqrt{2}$$

$$f(x+3, y-2) = x + y = 0$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 0, \qquad x\cos(45^\circ) + y\sin(45^\circ) = 0$$

The length of the line remains the same, $\sqrt{2}$.

$$R(0, 45^\circ) = \sqrt{2}$$

7.5. Find the Radon transform $R(s, \theta)$ of the image $x(m, n)$ and reconstruct the image from its transform by the backprojection method, using the DFT and the IDFT.
(i)

$$x(m,n) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(iv)

$$x(m,n) = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

(v)

$$x(m,n) = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

The 1-D row DFT of the image is

$$\begin{bmatrix} 3 & -1 \\ 5 & -1 \end{bmatrix}$$

The 1-D column DFT of this partial transform is the 2-DFT, $X(k, l)$, of $x(m, n)$ and it is

$$\begin{bmatrix} 2 & 0 \\ 8 & -2 \end{bmatrix}$$

The 1-D IDFT of the first row coefficinets $\{8, -2\}$ is $\{3, 5\}$. These are the $R(0, 0^\circ)$ and $R(1, 0^\circ)$ coefficients. The 1-D IDFT of the first column coefficinets $\{0, 2\}$ is $\{1, -1\}$. These are the $R(0, 90^\circ)$ and $R(1, 90^\circ)$ coefficients. Remember that the $X(0, 0)$ can be included only in one computation. As we computed the 1-D 2-point IDFT using the 2×2 2-D DFT coefficients, we have to divide these coefficients by 2 to get the true

Radon transform coefficients.
Reconstruction

$$\hat{x}(m,n) = \sum_{\theta} R(m\cos(\theta) + n\sin(\theta), \theta) \tag{7.1}$$

Let us reconstruct the image using Equation (8.6). With $m = 0$, $n = 0$ and $\theta = 0°$, we get $x(0,0) = R(0) = 2$. Proceeding similarly, we get the reconstructed image corresponding to $\theta = 0°$ as

$$x_0(m,n) = 0.5 \begin{bmatrix} 3 & 5 \\ 3 & 5 \end{bmatrix}$$

The reconstructed image corresponding to $\theta = 90°$ is

$$x_{90}(m,n) = 0.5 \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

The sum of the partially reconstructed images is the final image, which is the same as $x(m,n)$.

(ii)

$$x(m,n) = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}$$

The 1-D row DFT of the image is

$$\begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

The 1-D column DFT of this partial transform is the 2-DFT, $X(k,l)$, of $x(m,n)$ and it is

$$\begin{bmatrix} -2 & 0 \\ 6 & 4 \end{bmatrix}$$

The 1-D IDFT of the first row coefficinets $\{6,4\}$ is $\{5,1\}$. These are the $R(0,0°)$ and $R(1,0°)$ coefficients. The 1-D IDFT of the first column coefficinets $\{0,-2\}$ is $\{-1,1\}$. These are the $R(0,90°)$ and $R(1,90°)$ coefficients. Remember that the $X(0,0)$ can be included only in one computation. As we computed the 1-D 2-point IDFT using the 2×2 2-D DFT coefficients, we have to divide these coefficients by 2 to get the true Radon transform coefficients.
Reconstruction

$$\hat{x}(m,n) = \sum_{\theta} R(m\cos(\theta) + n\sin(\theta), \theta) \tag{7.2}$$

Let us reconstruct the image using Equation (8.6). With $m = 0$, $n = 0$ and $\theta = 0°$, we get $x(0,0) = R(0) = 2$. Proceeding similarly, we get the reconstructed image corresponding to $\theta = 0°$ as

$$x_0(m,n) = 0.5 \begin{bmatrix} 5 & 1 \\ 5 & 1 \end{bmatrix}$$

The reconstructed image corresponding to $\theta = 90°$ is

$$x_{90}(m,n) = 0.5 \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

The sum of the partially reconstructed images is the final image, which is the same as $x(m,n)$.
(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The 1-D row DFT of the image is

$$\begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix}$$

The 1-D column DFT of this partial transform is the 2-DFT, $X(k,l)$, of $x(m,n)$ and it is

$$\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

The 1-D IDFT of the first row coefficinets $\{4,0\}$ is $\{2,2\}$. These are the $R(0,0°)$ and $R(1,0°)$ coefficients. The 1-D IDFT of the first column coefficinets $\{0,0\}$ is $\{0,0\}$. These are the $R(0,90°)$ and $R(1,90°)$ coefficients. Remember that the $X(0,0)$ can be included only in one computation. As we computed the 1-D 2-point IDFT using the 2×2 2-D DFT coefficients, we have to divide these coefficients by 2 to get the true Radon transform coefficients.
Reconstruction

$$\hat{x}(m,n) = \sum_{\theta} R(m\cos(\theta) + n\sin(\theta), \theta) \tag{7.3}$$

Let us reconstruct the image using Equation (8.6). With $m = 0$, $n = 0$ and $\theta = 0°$, we get $x(0,0) = R(0) = 2$. Proceeding similarly, we get the reconstructed image corresponding to $\theta = 0°$ as

$$x_0(m,n) = 0.5 \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

The reconstructed image corresponding to $\theta = 90°$ is

$$x_{90}(m,n) = 0.5 \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The sum of the partially reconstructed images is the final image, which is the same as $x(m,n)$.

(iv)

$$x(m,n) = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

The 1-D row DFT of the image is

$$\begin{bmatrix} 7 & -1 \\ 3 & -1 \end{bmatrix}$$

The 1-D column DFT of this partial transform is the 2-DFT, $X(k,l)$, of $x(m,n)$ and it is

$$\begin{bmatrix} -4 & 0 \\ 10 & -2 \end{bmatrix}$$

The 1-D IDFT of the first row coefficinets $\{10,-2\}$ is $\{4,6\}$. These are the $R(0,0°)$ and $R(1,0°)$ coefficients. The 1-D IDFT of the first column coefficinets $\{0,-4\}$ is $\{-2,2\}$. These are the $R(0,90°)$ and $R(1,90°)$ coefficients. Remember that the $X(0,0)$ can be included only in one computation. As we computed

the 1-D 2-point IDFT using the 2×2 2-D DFT coefficients, we have to divide these coefficients by 2 to get the true Radon transform coefficients.
Reconstruction

$$\hat{x}(m,n) = \sum_{\theta} R(m\cos(\theta) + n\sin(\theta), \theta) \tag{7.4}$$

Let us reconstruct the image using Equation (8.6). With $m = 0$, $n = 0$ and $\theta = 0°$, we get $x(0,0) = R(0) = 2$. Proceeding similarly, we get the reconstructed image corresponding to $\theta = 0°$ as

$$x_0(m,n) = 0.5 \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix}$$

The reconstructed image corresponding to $\theta = 90°$ is

$$x_{90}(m,n) = 0.5 \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$$

The sum of the partially reconstructed images is the final image, which is the same as $x(m,n)$.

(v)

$$x(m,n) = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

The 1-D row DFT of the image is

$$\begin{bmatrix} 7 & 1 \\ 5 & 1 \end{bmatrix}$$

The 1-D column DFT of this partial transform is the 2-DFT, $X(k,l)$, of $x(m,n)$ and it is

$$\begin{bmatrix} -2 & 0 \\ 12 & 2 \end{bmatrix}$$

The 1-D IDFT of the first row coefficinets $\{12, 2\}$ is $\{7, 5\}$. These are the $R(0, 0°)$ and $R(1, 0°)$ coefficients. The 1-D IDFT of the first column coefficinets $\{0, -2\}$ is $\{-1, 1\}$. These are the $R(0, 90°)$ and $R(1, 90°)$ coefficients. Remember that the $X(0,0)$ can be included only in one computation. As we computed the 1-D 2-point IDFT using the 2×2 2-D DFT coefficients, we have to divide these coefficients by 2 to get the true Radon transform coefficients.
Reconstruction

$$\hat{x}(m,n) = \sum_{\theta} R(m\cos(\theta) + n\sin(\theta), \theta) \tag{7.5}$$

Let us reconstruct the image using Equation (8.6). With $m = 0$, $n = 0$ and $\theta = 0°$, we get $x(0,0) = R(0) = 2$. Proceeding similarly, we get the reconstructed image corresponding to $\theta = 0°$ as

$$x_0(m,n) = 0.5 \begin{bmatrix} 7 & 5 \\ 7 & 5 \end{bmatrix}$$

The reconstructed image corresponding to $\theta = 90°$ is

$$x_{90}(m,n) = 0.5 \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

The sum of the partially reconstructed images is the final image, which is the same as $x(m,n)$.

7.6. Detect the lines in the 4×4 binary image $x(m, n)$. Choose a suitable threshold.
(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$acc(m,n) = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 2 & 0 & 2 \\ 1 & 1 & 4 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The entry $acc(2, 2) = 4$ in the acc martrix indicates a line at distance 2 from the top left corner (origin) of the image at angle $90 + 90 = 180$ degrees from the m-axis.
(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$acc(m,n) = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 4 & 2 & 2 & 2 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The entry $acc(1, 0) = 4$ in the acc martrix indicates a line at distance 1 from the top left corner (origin) of the image at angle $0 + 90 = 90$ degrees from the m-axis.
The entry $acc(3, 1) = 3$ in the acc martrix indicates a line at distance 3 from the top left corner (origin) of the image at angle $45 + 90 = 135$ degrees from the m-axis.
(iii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$acc(m,n) = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 4 & 2 & 1 & 3 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

The entry $acc(1, 0) = 4$ in the acc martrix indicates a line at distance 1 from the top left corner (origin) of the image at angle $0 + 90 = 90$ degrees from the m-axis.
The entry $acc(3, 2) = 3$ in the acc martrix indicates a line at distance 3 from the top left corner (origin) of the image at angle $90 + 90 = 180$ degrees from the m-axis.

(iv)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$acc(m,n) = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 3 & 1 & 1 & 3 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

The entry $acc(0,3) = 3$ in the acc martrix indicates a line at distance 0 from the top left corner (origin) of the image at angle $135 + 90 = 225$ degrees from the m-axis.
The entry $acc(1,0) = 3$ in the acc martrix indicates a line at distance 1 from the top left corner (origin) of the image at angle $0 + 90 = 90$ degrees from the m-axis.
The entry $acc(1,3) = 3$ in the acc martrix indicates a line at distance 1 from the top left corner (origin) of the image at angle $135 + 90 = 225$ degrees from the m-axis.
(v)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$acc(m,n) = \begin{bmatrix} 0 & 0 & 2 & 1 \\ 4 & 2 & 2 & 2 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The entry $acc(1,0) = 4$ in the acc martrix indicates a line at distance 1 from the top left corner (origin) of the image at angle $0 + 90 = 90$ degrees from the m-axis.
The entry $acc(2,1,0) = 3$ in the acc martrix indicates a line at distance 2 from the top left corner (origin) of the image at angle $45 + 90 = 130$ degrees from the m-axis.

Chapter 8

Morpho

8.1 Find the dilation of $x(m,n)$ and $h(m,n)$.

$$h(m,n) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \mathbf{1} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

8.2 Find the erosion of $x(m,n)$ and $h(m,n)$.

$$h(m,n) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \mathbf{0} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

8.3 Find the opening of $x(m,n)$ and $h(m,n)$.

$$h(m,n) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \mathbf{1} & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

8.4 Find the closing of $x(m,n)$ and $h(m,n)$.

$$h(m,n) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \mathbf{1} & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

8.5 Find the hit-and-miss transformation of $x(m,n)$ and $h_h(m,n)$ and $h_{ms}(m,n)$.

$$h_h(m,n) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \mathbf{0} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad h_{ms}(m,n) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{0} & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

8.6 Find the thinned version of $x(m,n)$.
(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

8.7 Find the skeleton of $x(m,n)$.

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

8.8 Extract the boundary of $x(m,n)$.

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(i)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Chapter 9

Edge

9.1 Find the 6×6 filtered magnitude output and the edge map of $x(m,n)$ obtained using the Sobel filters. The threshold is 1.2 times the average of the magnitude of the values of the filtered output.
(i)

$$x(m,n) = \begin{bmatrix} 227 & 179 & 45 & 10 & 14 & 15 & 14 & 12 \\ 227 & 227 & 177 & 18 & 15 & 17 & 15 & 14 \\ 227 & 227 & 225 & 56 & 8 & 15 & 18 & 15 \\ 227 & 227 & 227 & 98 & 9 & 17 & 18 & 16 \\ 227 & 227 & 227 & 141 & 7 & 17 & 18 & 17 \\ 227 & 227 & 227 & 194 & 26 & 9 & 14 & 14 \\ 227 & 227 & 227 & 214 & 92 & 0 & 11 & 11 \\ 227 & 227 & 227 & 212 & 184 & 64 & 5 & 14 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 168 & 153 & 111 & 58 & 0 & 0 & 0 & 42 \\ 159 & 161 & 114 & 79 & 9 & 2 & 5 & 42 \\ 134 & 181 & 124 & 67 & 86 & 20 & 15 & 26 \\ 117 & 180 & 122 & 79 & 74 & 47 & 46 & 47 \\ 152 & 181 & 132 & 70 & 36 & 38 & 40 & 49 \\ 156 & 171 & 117 & 59 & 73 & 77 & 61 & 67 \\ 174 & 166 & 124 & 84 & 69 & 57 & 55 & 55 \\ 172 & 159 & 129 & 93 & 84 & 31 & 22 & 23 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 13 & 10 & 13 & 9 & 7 & 5 & 4 & 2 \\ 6 & 10 & 12 & 10 & 6 & 6 & 2 & 0 \\ 19 & 19 & 12 & 11 & 6 & 3 & 2 & 2 \\ 19 & 19 & 18 & 18 & 10 & 5 & 2 & 3 \\ 19 & 18 & 16 & 17 & 14 & 10 & 7 & 6 \\ 17 & 14 & 15 & 15 & 10 & 9 & 8 & 6 \\ 13 & 10 & 11 & 13 & 12 & 9 & 8 & 3 \\ 14 & 9 & 9 & 9 & 10 & 10 & 8 & 5 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 49.5025 & 110.4451 & 78.8531 & 6.3738 & 1.2748 & 1.7766 \\ 9.1992 & 87.9275 & 104.8967 & 22.1923 & 3.6443 & 1.1180 \\ 0.3536 & 65.3292 & 111.1987 & 42.2334 & 4.8894 & 0.5590 \\ 0 & 43.4403 & 110.5075 & 66.0350 & 2.4044 & 2.3049 \\ 0 & 22.5534 & 98.9326 & 92.8995 & 12.9735 & 5.3180 \\ 0 & 9.5197 & 68.6740 & 106.8324 & 54.7280 & 5.4458 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 20.1067 & 47.2854 & 47.1981 & 40.9806 & 20.2029 & 16.5548 \\ 7.5166 & 51.5609 & 30.0442 & 33.5023 & 32.7614 & 17.7341 \\ 4.1003 & 53.4278 & 29.1000 & 20.4213 & 15.4364 & 11.5718 \\ 9.4637 & 54.5931 & 35.9627 & 9.8011 & 10.0778 & 10.1119 \\ 18.6078 & 52.1683 & 30.6008 & 12.7046 & 11.5596 & 7.0112 \\ 22.7555 & 43.1350 & 27.3524 & 12.3136 & 19.6866 & 21.1793 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 2.9422 & 1.5910 & 3.0104 & 2.5125 & 2.0691 & 2.1360 \\ 4.7599 & 4.2019 & 4.5962 & 4.5311 & 2.5125 & 1.2748 \\ 1.5207 & 2.1287 & 4.2426 & 6.2774 & 4.7730 & 2.8559 \\ 2.1866 & 1.7678 & 2.4044 & 4.1269 & 3.4731 & 2.8777 \\ 3.5576 & 2.7951 & 2.3251 & 3.0873 & 1.9121 & 2.0156 \\ 2.7443 & 3.0052 & 2.2638 & 1.7410 & 1.5207 & 2.5000 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

9.2 Find the filtered output and the edge map of $x(m,n)$ obtained using a 5×5 LoG filter with $\sigma = 1$. The threshold is 0.75 times the average of the magnitude of the values of the filtered output. Assume replication at the borders.

(i)

$$x(m,n) = \begin{bmatrix} 25 & 17 & 22 & 8 & 118 & 186 & 136 & 133 \\ 15 & 38 & 18 & 13 & 152 & 147 & 131 & 150 \\ 33 & 32 & 14 & 9 & 115 & 165 & 143 & 173 \\ 34 & 14 & 14 & 12 & 21 & 64 & 41 & 179 \\ 15 & 14 & 7 & 18 & 106 & 137 & 92 & 195 \\ 15 & 24 & 39 & 156 & 188 & 194 & 191 & 197 \\ 12 & 24 & 129 & 204 & 204 & 206 & 208 & 195 \\ 0 & 7 & 145 & 206 & 206 & 209 & 211 & 200 \end{bmatrix}$$

(ii)

$$x(m,n) = \begin{bmatrix} 147 & 163 & 179 & 186 & 191 & 194 & 197 & 157 \\ 160 & 175 & 182 & 184 & 184 & 186 & 162 & 50 \\ 141 & 163 & 170 & 175 & 174 & 133 & 38 & 3 \\ 91 & 127 & 135 & 124 & 85 & 16 & 0 & 7 \\ 113 & 126 & 121 & 117 & 18 & 0 & 1 & 10 \\ 136 & 135 & 125 & 151 & 99 & 54 & 8 & 9 \\ 148 & 150 & 159 & 161 & 149 & 106 & 89 & 20 \\ 142 & 164 & 178 & 181 & 168 & 113 & 120 & 91 \end{bmatrix}$$

(iii)

$$x(m,n) = \begin{bmatrix} 72 & 77 & 66 & 62 & 50 & 43 & 66 & 66 \\ 75 & 65 & 61 & 65 & 50 & 36 & 64 & 64 \\ 67 & 57 & 62 & 67 & 48 & 31 & 63 & 63 \\ 55 & 62 & 64 & 62 & 16 & 34 & 61 & 61 \\ 41 & 46 & 47 & 46 & 11 & 33 & 59 & 62 \\ 39 & 28 & 58 & 54 & 25 & 7 & 50 & 61 \\ 41 & 0 & 49 & 48 & 28 & 9 & 6 & 29 \\ 63 & 20 & 6 & 23 & 16 & 10 & 7 & 15 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1.1393 & -1.3247 & 20.6334 & 35.2144 & -20.1352 & -40.2937 & 1.3512 & 12.1893 \\ 0.9546 & -6.4392 & 17.2310 & 23.6173 & -36.0319 & -44.2407 & -5.5452 & -1.8316 \\ -7.4267 & -7.0300 & 17.2427 & 27.4759 & -16.5927 & -22.9585 & 2.6669 & -12.3373 \\ -5.7324 & 4.8810 & 29.0622 & 48.1298 & 31.8933 & 36.2891 & 48.0806 & -0.1095 \\ 5.7233 & 19.8491 & 37.2761 & 33.1484 & 7.8847 & 15.8506 & 32.3539 & -4.7361 \\ 8.5548 & 20.7852 & 10.7965 & -24.3086 & -42.0398 & -28.9790 & -11.6563 & -13.5621 \\ 14.3345 & 20.2479 & -20.4283 & -52.8141 & -38.0404 & -20.7737 & -13.0613 & -4.7126 \\ 26.5096 & 32.6481 & -21.6905 & -45.3147 & -18.1970 & -6.1235 & -5.6080 & 0.2445 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 5.0105 & -0.2727 & -5.3430 & -5.7242 & -10.2161 & -28.1357 & -33.8425 & -15.9979 \\ -8.9723 & -13.5013 & -16.3388 & -18.7095 & -28.2793 & -38.4562 & -18.6954 & 20.0468 \\ 0.5121 & -9.8865 & -17.8006 & -24.2760 & -27.9287 & -12.6175 & 21.2665 & 43.7632 \\ 24.1600 & 9.4774 & -1.3458 & -0.2453 & 18.7616 & 38.9715 & 39.5796 & 24.7139 \\ 14.9449 & 10.6197 & 4.6543 & 8.7492 & 40.4500 & 53.8614 & 35.7797 & 13.2239 \\ -2.4414 & 2.3624 & -2.5966 & -13.3771 & 3.0772 & 20.0810 & 23.4170 & 23.2587 \\ -3.6860 & -3.4955 & -11.0425 & -25.8184 & -23.2475 & -10.5215 & -2.0928 & 14.6576 \\ 2.5690 & -5.4928 & -15.0516 & -24.8636 & -19.8852 & -8.3275 & -14.2503 & -9.5180 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} -3.1158 & -4.0225 & -2.3253 & -2.8234 & 5.1919 & 8.2161 & -4.2257 & -5.3983 \\ -4.1800 & 0.0835 & -1.2931 & -6.4914 & 3.6881 & 9.5863 & -3.8063 & -5.2979 \\ -3.1444 & 0.1023 & -5.9931 & -10.4337 & 5.7108 & 10.6493 & -4.6142 & -5.1380 \\ -0.2920 & -1.9751 & -9.1293 & -5.9373 & 15.2957 & 11.0129 & -7.7909 & -6.0295 \\ 3.8998 & 2.4854 & -6.8059 & -3.0805 & 17.6009 & 10.1186 & -12.1281 & -13.3516 \\ 7.2203 & 9.2935 & -10.5167 & -14.9594 & 7.2599 & 12.4922 & -5.6128 & -14.3111 \\ 3.5515 & 15.5884 & -3.1670 & -15.6817 & 0.2656 & 13.8107 & 9.4087 & 0.7937 \\ -9.8399 & 13.0285 & 12.7044 & -1.7843 & 1.3823 & 9.4791 & 10.1562 & 5.7571 \end{bmatrix}$$

$$e(m,n) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Chapter 10

Segmentation

10.1 Using the averaging method, find the threshold of the 4×4 8-bit image. Let the initial value of the threshold be the average of the gray levels of the image. The iteration stops when the difference between two consecutive threshold values becomes less than 0.5.

$$\begin{bmatrix} 140 & 10 & 5 & 6 \\ 74 & 2 & 7 & 6 \\ 21 & 5 & 6 & 5 \\ 2 & 6 & 5 & 5 \end{bmatrix}$$

10.2 Using the averaging method, find the threshold of the 4×4 8-bit image. Let the initial value of the threshold be the average of the gray levels of the image. The iteration stops when the difference between two consecutive threshold values becomes less than 0.5.

$$\begin{bmatrix} 191 & 102 & 1 & 7 \\ 182 & 45 & 2 & 6 \\ 140 & 10 & 5 & 6 \\ 74 & 2 & 7 & 6 \end{bmatrix}$$

10.3 Using the averaging method, find the threshold of the 4×4 8-bit image. Let the initial value of the threshold be the average of the gray levels of the image. The iteration stops when the difference between two consecutive threshold values becomes less than 0.5.

$$\begin{bmatrix} 184 & 188 & 72 & 2 \\ 188 & 163 & 22 & 5 \\ 191 & 102 & 1 & 7 \\ 182 & 45 & 2 & 6 \end{bmatrix}$$

(i) $19.0625, 41.8590, 56.7500$.
(ii) $49.1250, 73.3091$.
(iii) $85, 94.5714$.

10.4 Using Otsu's method, find the threshold of the 4×4 3-bit image. Find the separability index.

$$\begin{bmatrix} 5 & 2 & 6 & 5 \\ 2 & 5 & 6 & 6 \\ 2 & 7 & 6 & 6 \\ 5 & 6 & 5 & 5 \end{bmatrix}$$

10.5 Using Otsu's method, find the threshold of the 4×4 3-bit image. Find the separability index.

$$\begin{bmatrix} 4 & 0 & 2 & 6 \\ 3 & 6 & 5 & 6 \\ 6 & 1 & 7 & 6 \\ 5 & 2 & 6 & 5 \end{bmatrix}$$

10.6 Using Otsu's method, find the threshold of the 4×4 3-bit image. Find the separability index.

$$\begin{bmatrix} 5 & 6 & 5 & 5 \\ 6 & 5 & 5 & 6 \\ 7 & 6 & 4 & 5 \\ 5 & 5 & 5 & 5 \end{bmatrix}$$

(i)

Gray level, k	0	1	2	3	4	5	6	7
$hn(k)$	0	0	0.1875	0	0	0.3750	0.3750	0.0625
$hc(k)$	0	0	0.1875	0.1875	0.1875	0.5625	0.9375	1
$ha(k)$	0	0	0.3750	0.3750	0.3750	2.2500	4.5000	4.9375
$\sigma_b^2(k)$	0	0	1.9913	1.9913	1.9913	1.1300	0.2836	0

The threshold and the separability index are 3 and 0.8626.
(ii)

Gray level, k	0	1	2	3	4	5	6	7
$hn(k)$	0.0625	0.0625	0.1250	0.0625	0.0625	0.1875	0.3750	0.0625
$hc(k)$	0.0625	0.1250	0.2500	0.3125	0.3750	0.5625	0.9375	1
$ha(k)$	0	0.0625	0.3125	0.5000	0.7500	1.6875	3.9375	4.3750
$\sigma_b^2(k)$	1.2760	2.1451	3.2552	3.5003	3.3844	2.4308	0.4594	0

The threshold and the separability index are 3 and 0.8266.
(iii)

Gray level, k	0	1	2	3	4	5	6	7
$hn(k)$	0	0	0	0	0.0625	0.6250	0.2500	0.0625
$hc1(k)$	0	0	0	0	0.0625	0.6875	0.9375	1
$ha(k)$	0	0	0	0	0.25	3.3750	4.8750	5.3125
$\sigma_b^2(k)$	0	0	0	0	0.1148	0.3580	0.1898	0

The threshold and the separability index are 5 and 0.7702.

10.7 Consider the 8×8 image. The seed pixel is shown in boldface. Use the 4-connectivity to segment the image so that the region is to be made of pixels with gray levels less than or equal to 176.

$$\begin{bmatrix} 255 & 207 & 73 & 38 & 42 & 43 & 42 & 40 \\ 255 & 255 & 205 & 46 & 43 & 45 & 43 & 42 \\ 255 & 255 & 253 & 84 & 36 & 43 & 46 & 43 \\ 255 & 255 & 255 & 126 & 37 & 45 & 46 & 44 \\ 255 & 255 & 255 & 169 & 35 & 45 & 46 & 45 \\ 255 & 255 & 255 & 222 & 54 & 37 & 42 & 42 \\ 255 & 255 & 255 & 242 & 120 & 28 & 39 & 39 \\ 255 & 255 & 255 & 240 & 212 & 92 & 33 & \mathbf{42} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

10.8 Consider the 8×8 image. The seed pixel is shown in boldface. Use the 4-connectivity to segment the image so that the region is to be made of pixels with gray levels less than or equal to 76.

$$\begin{bmatrix} 172 & 157 & 115 & 62 & 4 & 4 & 4 & \mathbf{46} \\ 163 & 165 & 118 & 83 & 13 & 6 & 9 & 46 \\ 138 & 185 & 128 & 71 & 90 & 24 & 19 & 30 \\ 121 & 184 & 126 & 83 & 78 & 51 & 50 & 51 \\ 156 & 185 & 136 & 74 & 40 & 42 & 44 & 53 \\ 160 & 175 & 121 & 63 & 77 & 81 & 65 & 71 \\ 178 & 170 & 128 & 88 & 73 & 61 & 59 & 59 \\ 176 & 163 & 133 & 97 & 88 & 35 & 26 & 27 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

10.9 Consider the 8×8 image. The seed pixel is shown in boldface. Use the 4-connectivity to segment the image so that the region is to be made of pixels with gray levels less than or equal to 56.

$$\begin{bmatrix} 65 & 62 & 65 & 61 & 59 & 57 & 56 & \mathbf{54} \\ 58 & 62 & 64 & 62 & 58 & 58 & 54 & 52 \\ 71 & 71 & 64 & 63 & 58 & 55 & 54 & 54 \\ 71 & 71 & 70 & 70 & 62 & 57 & 54 & 55 \\ 71 & 70 & 68 & 69 & 66 & 62 & 59 & 58 \\ 69 & 66 & 67 & 67 & 62 & 61 & 60 & 58 \\ 65 & 62 & 63 & 65 & 64 & 61 & 60 & 55 \\ 66 & 61 & 61 & 61 & 62 & 62 & 60 & 57 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

10.10 Consider the 8×8 image. Use the 8-connectivity to segment, using the split and merge algorithm, the image so that the region is to be made of pixels with gray levels less than or equal to 45.

$$\begin{bmatrix} 59 & 50 & 40 & 29 & 22 & 20 & 20 & 21 \\ 55 & 48 & 39 & 28 & 22 & 20 & 20 & 20 \\ 53 & 47 & 38 & 28 & 22 & 20 & 20 & 22 \\ 54 & 47 & 38 & 28 & 22 & 20 & 20 & 23 \\ 57 & 49 & 40 & 29 & 23 & 21 & 21 & 22 \\ 62 & 54 & 44 & 32 & 25 & 22 & 22 & 20 \\ 69 & 60 & 49 & 36 & 28 & 23 & 22 & 20 \\ 78 & 68 & 55 & 41 & 31 & 24 & 22 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

10.11 Consider the 8×8 image. Use the 8-connectivity to segment, using the split and merge algorithm, the image so that the region is to be made of pixels with gray levels less than or equal to 240.

$$\begin{bmatrix} 248 & 247 & 242 & 238 & 236 & 235 & 235 & 234 \\ 248 & 247 & 243 & 241 & 238 & 235 & 235 & 234 \\ 249 & 248 & 244 & 242 & 239 & 235 & 235 & 235 \\ 249 & 248 & 246 & 244 & 240 & 235 & 235 & 235 \\ 249 & 248 & 247 & 245 & 241 & 236 & 236 & 235 \\ 247 & 247 & 246 & 244 & 241 & 236 & 236 & 236 \\ 244 & 245 & 244 & 243 & 241 & 236 & 236 & 236 \\ 241 & 243 & 242 & 241 & 239 & 236 & 236 & 236 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

10.12 Consider the 8×8 image. Use the 8-connectivity to segment, using the split and merge algorithm, the image so that the region is to be made of pixels with gray levels less than or equal to 240.

$$
\begin{bmatrix}
239 & 240 & 240 & 240 & 241 & 243 & 238 & 231 \\
239 & 240 & 240 & 240 & 241 & 242 & 239 & 234 \\
239 & 240 & 240 & 240 & 240 & 240 & 240 & 238 \\
239 & 240 & 240 & 240 & 239 & 238 & 240 & 241 \\
239 & 240 & 240 & 240 & 239 & 239 & 242 & 244 \\
237 & 238 & 239 & 240 & 241 & 245 & 245 & 245 \\
236 & 238 & 239 & 240 & 241 & 245 & 246 & 245 \\
236 & 238 & 239 & 240 & 241 & 245 & 246 & 245
\end{bmatrix}
$$

$$
\begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

10.13 Find the seed points of the regions by clustering and finding the centroids. Initial seeds are $\{2, 2\}$ and $\{2, 3\}$.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

10.14 Find the seed points of the regions by clustering and finding the centroids. Initial seeds are $\{2, 2\}$ and $\{2, 3\}$.

$$
\begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1
\end{bmatrix}
$$

10.15 Find the seed points of the regions by clustering and finding the centroids. Initial seeds are $\{2, 2\}$ and $\{2, 3\}$.

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(i)

$$2, 2, 2, 3$$

$$3.0000, 2.0000, 4.0000, 4.3000$$

$$2.8000, 2.4000, 4.3750, 4.6250$$

$$2.8571, 2.7143, 4.8333, 5.0000$$

$$2.8750, 2.8750, 5.2000, 5.2000$$

$$3.0000, 3.0000, 5.5000, 5.5000$$

(ii)

$$2, 2, 2, 3$$

$$2.5000, 2.0000, 4.4286, 5.2857$$

$$2.1818, 3.0000, 5.6429, 6.1429$$

$$2.4000, 3.4000, 6.70006.8000$$

$$2.5000, 3.5000, 7.0000, 7.0000$$

(iii)

$$2, 2, 2, 3$$

$$2, 2, 4, 5$$

$$1.8750, 2.8750, 5.1000, 5.8000$$

$$1.9091, 3.3636, 6.4286, 6.2857$$

$$2.0000, 3.5000, 7.0000, 6.5000$$

10.16 Using the distance transform with masks

$$h_f(m,n) = \begin{bmatrix} \infty & 1 & \infty \\ 1 & 0 & \infty \\ \infty & \infty & \infty \end{bmatrix} \quad \text{and} \quad h_b(m,n) = \begin{bmatrix} \infty & \infty & \infty \\ \infty & 0 & 1 \\ \infty & 1 & \infty \end{bmatrix}$$

find the distance of the pixels of the 8×8 image.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

10.17 Using the distance transform with masks

$$h_f(m,n) = \begin{bmatrix} 4 & 3 & 4 \\ 3 & 0 & \infty \\ \infty & \infty & \infty \end{bmatrix} \quad \text{and} \quad h_b(m,n) = \begin{bmatrix} \infty & \infty & \infty \\ \infty & 0 & 3 \\ 4 & 3 & 4 \end{bmatrix}$$

find the distance of the pixels of the 8×8 image.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

10.18 Using the distance transform with masks

$$h_f(m,n) = \begin{bmatrix} \infty & 11 & \infty & 11 & \infty \\ 11 & 7 & 5 & 7 & 11 \\ \infty & 5 & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \quad \text{and} \quad h_b(m,n) = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 5 & \infty \\ 11 & 7 & 5 & 7 & 11 \\ \infty & 11 & \infty & 11 & \infty \end{bmatrix}$$

find the distance of the pixels of the 8×8 image.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

11.1

$$\begin{bmatrix} 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 9.0 & 10.0 \\ 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 9.0 \\ 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 \\ 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 \\ 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 \\ 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 \\ 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 \\ 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 \end{bmatrix}$$

11.2

$$\begin{bmatrix} 0.0 & 0.0 & 1.0 & 1.3 & 2.3 & 3.3 & 4.3 & 5.3 \\ 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 2.7 & 3.7 & 4.7 \\ 0.0 & 0.0 & 0.0 & 1.0 & 1.3 & 2.3 & 3.3 & 4.3 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & 3.0 & 4.0 \end{bmatrix}$$

11.3

$$\begin{bmatrix} 3.0 & 2.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 3.2 & 2.2 & 1.4 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 3.6 & 2.8 & 2.2 & 1.4 & 1.0 & 0.0 & 0.0 & 0.0 \\ 4.2 & 3.6 & 2.8 & 2.2 & 1.4 & 1.0 & 0.0 & 0.0 \\ 5.0 & 4.2 & 3.6 & 2.8 & 2.0 & 1.0 & 0.0 & 0.0 \\ 5.6 & 5.0 & 4.0 & 3.0 & 2.0 & 1.0 & 0.0 & 0.0 \\ 6.2 & 5.2 & 4.2 & 3.2 & 2.2 & 1.4 & 1.0 & 0.0 \\ 6.4 & 5.4 & 4.4 & 3.6 & 2.8 & 2.0 & 1.0 & 0.0 \end{bmatrix}$$

Chapter 11

Representation

[]

11.1 Find the chain code for the image.

(i)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(i)

$$\{7, 7, 7, 7, 7, 4, 4, 4, 4, 4, 2, 2, 2, 2, 2\}$$

(ii)

$$\{1, 0, 0, 7, 6, 6, 5, 4, 4, 3, 2, 2\}$$

(iii)

$$\{0, 0, 0, 0, 6, 6, 6, 5, 4, 4, 3, 2, 2, 2\}$$

11.2 Find the signature of the image.

(i)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(i) Centroid $\{2, 1\}$

θ	-153	180	90	63	45	0	-45	-90	-135
$d(\theta)$	2.2361	1	1	2.2361	1.4142	1	1.4142	1.0000	1.4142

(ii) Centroid $\{1.5, 1.5\}$

θ	-135	-162	162	135	108	72	45	18	-18	-45	-72	-108
$d(\theta)$	2.1213	1.5811	1.5811	2.1213	1.5811	1.5811	2.1213	1.5811	1.5811	2.1213	1.5811	1.5811

(iii) Centroid $\{1.5, 1.5\}$

θ	-108	-162	162	108	72	18	-18	-72
$d(\theta)$	1.5811	1.5811	1.5811	1.5811	1.5811	1.5811	1.5811	1.5811

11.3 Find the Fourier descriptor of the image. Reconstruct the image from the descriptor and verify that it is the same as the input image.

(i)

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(i) complex data

$$\{0+j1, 0+2, 2, 0+j3, 1+j3, 2+j3, 3+j3, 3+j2, 3+j1, 2+j1, 1+j1\}$$

Fourier descriptor

$$\{15+j20, -1.8743+j1.3618, 0, -0.5949-j1.8310, 0, -1, 0, 0.3031-j0.9329, 0, -11.8339-j8.5978\}$$

(ii) complex data

$$\{0+j0, 0+j1, 0+j2, 0+j3, 1+j3, 2+j3, 3+j3, 2+j2, 1+j1\}$$

Fourier descriptor

$$\{9+j18, -3.2057-j0.8590, -0.9076-j3.3872, 0+j0, -0.3867-j0.1036, -0.3867-j1.4430,$$
$$0+j0, -0.9076-j0.2432, -3.2057-j11.9640\}$$

(iii) complex data

$$\{2+j0, 1+j1, 0+j2, 1+j3, 2+j3, 3+j3, 3+j2, 3+j, 3+j0, \}$$

Fourier descriptor

$$\{18+j15, 1.3803-j0.5371, 2.1665+j0.0717, 0+j0, -0.0271-j0.5967, -1.1577+j0.3974,$$
$$0+j0, 1.2449-j1.1700, -3.6070-j13.1652\}$$

11.4 Find the area, perimeter and compactness of the nonzero region in the 3×3 image.

(i)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(i) area = 9, perimeter = 8, compactness = 1.7671
(ii) area = 5, perimeter = 5.4142, compactness = 2.1434
(iii) area = 6, perimeter = 6.8284, compactness = 1.6170

11.5 Find the Euler number of the 8-connected 4×4 image.

(i)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(i) Euler number = -1
(ii) Euler number = 1
(iii) Euler number = 0

11.6 Find the first two normalized central moments ϕ_1 and ϕ_2 of the 4×4 image.

$$x(k,l) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{m_{00} = 7, \quad m_{10} = 8, \quad m_{01} = 12, \quad \bar{k} = 1.1429, \quad \bar{l} = 1.7143\}$$

$$m_{11} = 1.2857, \quad m_{20} = 4.8571, \quad m_{02} = 3.4286$$

$$e_{11} = 0.0262, e_{20} = 0.0991, e_{02} = 0.0700$$

$$\phi_1 = 0.1691, \quad \phi_2 = 0.0036$$

11.7 Find the first two normalized central moments ϕ_1 and ϕ_2 of the 4×4 image.

$$x(k,l) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\{m_{00} = 5, \quad m_{10} = 12, \quad m_{01} = 9, \quad \bar{k} = 2.4, \quad \bar{l} = 1.8\}$$

$$m_{11} = -0.6, \quad m_{20} = 1.2, \quad m_{02} = 2.8$$

$$e_{11} = -0.024, e_{20} = 0.048, e_{02} = 0.112$$

$$\phi_1 = 0.16, \quad \phi_2 = 0.0064$$

11.8 Find the first two normalized central moments ϕ_1 and ϕ_2 of the 4×4 image.

$$x(k,l) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{m_{00} = 7, \quad m_{10} = 7, \quad m_{01} = 12, \quad \bar{k} = 1, \quad \bar{l} = 1.7143\}$$

$$m_{11} = 0, \quad m_{20} = 4, \quad m_{02} = 3.4286$$

$$e_{11} = 0, e_{20} = 0.0816, e_{02} = 0.07$$

$$\phi_1 = 0.1516, \quad \phi_2 = 0.0001$$

11.9 Find the histogram of the 4×4 8-bit image and derive its histogram-based features.

$$\begin{bmatrix} 108 & 81 & 69 & 92 \\ 114 & 105 & 72 & 101 \\ 117 & 73 & 67 & 92 \\ 105 & 0 & 65 & 101 \end{bmatrix}$$

$$mean = 85.1250, std = 27.7689, Sk = -0.5252, S = 0.0117, U = 0.0859, E = 3.6250$$

11.10 Find the histogram of the 4×4 8-bit image and derive its histogram-based features.

$$\begin{bmatrix} 120 & 103 & 83 & 78 \\ 97 & 99 & 81 & 72 \\ 102 & 96 & 78 & 73 \\ 121 & 107 & 37 & 0 \end{bmatrix}$$

$$mean = 84.1875, std = 29.6104, Sk = -0.5560, S = 0.0133, U = 0.0703, E = 3.8750$$

11.11 Find the histogram of the 4×4 8-bit image and derive its histogram-based features.

$$\begin{bmatrix} 10 & 133 & 175 & 170 \\ 0 & 61 & 171 & 170 \\ 3 & 15 & 131 & 172 \\ 1 & 3 & 70 & 167 \end{bmatrix}$$

$$mean = 90.7500, std = 73.8178, Sk = -0.5647, S = 0.0773, U = 0.0781, E = 3.7500$$

11.12 Find the cooccurrence matrix of the 8×8 3-bit image.

$$\begin{bmatrix} 5 & 2 & 2 & 2 & 2 & 1 & 1 & 2 \\ 5 & 4 & 1 & 2 & 2 & 2 & 1 & 1 \\ 5 & 5 & 2 & 2 & 2 & 2 & 1 & 1 \\ 5 & 5 & 4 & 1 & 2 & 2 & 1 & 1 \\ 6 & 5 & 6 & 2 & 2 & 2 & 2 & 1 \\ 6 & 4 & 5 & 4 & 1 & 2 & 2 & 1 \\ 5 & 5 & 5 & 5 & 2 & 2 & 2 & 1 \\ 5 & 4 & 5 & 5 & 3 & 2 & 2 & 2 \end{bmatrix}$$

Let the spatial relationship of a pair of pixels be $x(m,n)$ and $x(m,n+1)$. That is, a pixel and its immediate right neighbor form the pair. Find the contrast, correlation, energy and homogeneity.

$$\begin{bmatrix} 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 & 4 & 0 & 6 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.0714 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1250 & 0.3036 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0179 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0536 & 0 & 0 & 0 & 0 & 0 & 0.0357 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0536 & 0 & 0.0179 & 0.0714 & 0 & 0.1071 & 0.0179 \\ 0 & 0.0179 & 0 & 0 & 0.0179 & 0 & 0.0179 & 0 \end{bmatrix}$$

$$Contrast: 3.8929, Correlation: 0.7238, Energy: 0.1435, Homogeneity: 0.6710$$

11.13 Find the cooccurrence matrix of the 8×8 3-bit image.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 & 2 & 5 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 & 5 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 1 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 2 & 1 & 0 & 0 & 1 & 2 & 2 & 3 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 & 3 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 & 3 \\ 2 & 1 & 0 & 0 & 1 & 1 & 1 & 3 \end{bmatrix}$$

Let the spatial relationship of a pair of pixels be $x(m,n)$ and $x(m,n+1)$. That is, a pixel and its immediate right neighbor form the pair. Find the contrast, correlation, energy and homogeneity.

$$\begin{bmatrix} 12 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 6 & 0 & 5 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 5 & 1 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.2143 & 0.1429 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.1429 & 0.1071 & 0 & 0.0893 & 0.0536 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0714 & 0 & 0.0893 & 0.0179 & 0 & 0.0357 & 0.0357 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$Contrast: 2.3214, Correlation: 0.6347, Energy: 0.1250, Homogeneity: 0.6455$

11.14 Find the cooccurrence matrix of the 8×8 3-bit image.

$$\begin{bmatrix} 5 & 4 & 4 & 5 & 5 & 2 & 2 & 2 \\ 5 & 2 & 4 & 5 & 5 & 2 & 2 & 2 \\ 3 & 2 & 5 & 5 & 6 & 3 & 2 & 2 \\ 2 & 4 & 4 & 5 & 5 & 4 & 2 & 2 \\ 4 & 5 & 5 & 5 & 6 & 4 & 2 & 2 \\ 5 & 5 & 5 & 5 & 4 & 4 & 2 & 2 \\ 5 & 5 & 5 & 3 & 3 & 4 & 2 & 2 \\ 5 & 5 & 4 & 3 & 4 & 5 & 2 & 2 \end{bmatrix}$$

Let the spatial relationship of a pair of pixels be $x(m,n)$ and $x(m,n+1)$. That is, a pixel and its immediate right neighbor form the pair. Find the contrast, correlation, energy and homogeneity.

$$\begin{bmatrix} 10 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 3 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 4 & 0 & 12 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.1786 & 0 & 0 & 0 & 0.0357 & 0 & 0.0179 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0357 & 0 & 0.0179 & 0 & 0.0357 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0714 & 0 & 0.0179 & 0 & 0.0536 & 0 & 0.0893 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0714 & 0 & 0.0179 & 0 & 0.0714 & 0 & 0.2143 & 0.0357 \\ 0 & 0 & 0.0179 & 0 & 0.0179 & 0 & 0 & 0 \end{bmatrix}$$

$Contrast: 6.8571, Correlation: 0.5032, Energy: 0.1110, Homogeneity: 0.6107$

11.15 Given two 2×2 images, find the corresponding PCA components and their covariance. Then, reconstruct the original images from the PCA components.

$$a(m,n) = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \qquad b(m,n) = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

The mean of the matrices are $am = 2.5$ and $bm = 2.5$. Subtracting the respective means from the matrices, converting them into column vectors and concatenating, we get

$$x(m,n) = \begin{bmatrix} -0.5000 & 0.5000 \\ -1.5000 & -1.5000 \\ 0.5000 & -0.5000 \\ 1.5000 & 1.5000 \end{bmatrix}$$

$$C(m,n) = \begin{bmatrix} 1.6667 & 1.3333 \\ 1.3333 & 1.6667 \end{bmatrix}$$

The two eigenvalues are $\{3, 0.3333\}$. The eigenvectors are

$$\boldsymbol{R} = \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}$$

The principal components are found as

$$\boldsymbol{y} = \boldsymbol{x}\boldsymbol{R} = \begin{bmatrix} -0.5000 & 0.5000 \\ -1.5000 & -1.5000 \\ 0.5000 & -0.5000 \\ 1.5000 & 1.5000 \end{bmatrix} \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} = \begin{bmatrix} -0.0000 & 0.7071 \\ 2.1213 & 0.0000 \\ 0.0000 & -0.7071 \\ -2.1213 & -0.0000 \end{bmatrix}$$

The covariance of the PCA component matrix is the product of its transpose with itself.

$$C(m,n) = \begin{bmatrix} 3.0000 & 0.0000 \\ 0.0000 & 0.3333 \end{bmatrix}$$

The input can be reconstructed by

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{y}\boldsymbol{R}^T + \begin{bmatrix} am & bm \\ am & bm \\ am & bm \\ am & bm \end{bmatrix} = \begin{bmatrix} -0.0000 & 0.7071 \\ 2.1213 & 0.0000 \\ 0.0000 & -0.7071 \\ -2.1213 & -0.0000 \end{bmatrix} \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} + \begin{bmatrix} 2.5 & 2.5 \\ 2.5 & 2.5 \\ 2.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix} \\ &= \begin{bmatrix} -0.5000 & 0.5000 \\ -1.5000 & -1.5000 \\ 0.5000 & -0.5000 \\ 1.5000 & 1.5000 \end{bmatrix} + \begin{bmatrix} 2.5 & 2.5 \\ 2.5 & 2.5 \\ 2.5 & 2.5 \\ 2.5 & 2.5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 3 & 2 \\ 4 & 4 \end{bmatrix} \end{aligned}$$

11.16 Given two 2×2 images, find the corresponding PCA components and their covariance. Then, reconstruct the original images from the PCA components.

$$a(m,n) = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad b(m,n) = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

The mean of the matrices are $am = 1.75$ and $bm = 2.5$. Subtracting the respective means from the matrices, converting them into column vectors and concatenating, we get

$$x(m,n) = \begin{bmatrix} -0.7500 & 0.5000 \\ -0.7500 & -1.5000 \\ 0.2500 & -0.5000 \\ 1.2500 & 1.5000 \end{bmatrix}$$

$$C(m,n) = \begin{bmatrix} 0.9167 & 0.8333 \\ 0.8333 & 1.6667 \end{bmatrix}$$

The two eigenvalues are $\{2.2055, 0.3778\}$. The eigenvectors are

$$\boldsymbol{R} = \begin{bmatrix} -0.5430 & -0.8398 \\ -0.8398 & 0.5430 \end{bmatrix}$$

The principal components are found as

$$\boldsymbol{y} = \boldsymbol{x}\boldsymbol{R} = \begin{bmatrix} -0.7500 & 0.5000 \\ -0.7500 & -1.5000 \\ 0.2500 & -0.5000 \\ 1.2500 & 1.5000 \end{bmatrix} \begin{bmatrix} -0.5430 & -0.8398 \\ -0.8398 & 0.5430 \end{bmatrix} = \begin{bmatrix} -0.0126 & 0.9013 \\ 1.6669 & -0.1846 \\ 0.2841 & -0.4814 \\ -1.9383 & -0.2352 \end{bmatrix}$$

The covariance of the PCA component matrix is the product of its transpose with itself.

$$C(m,n) = \begin{bmatrix} 2.2055 & 0 \\ 0 & 0.3778 \end{bmatrix}$$

The input can be reconstructed by

$$\begin{aligned} \boldsymbol{x} &= \boldsymbol{y}\boldsymbol{R}^T + \begin{bmatrix} am & bm \\ am & bm \\ am & bm \\ am & bm \end{bmatrix} = \begin{bmatrix} -0.0126 & 0.9013 \\ 1.6669 & -0.1846 \\ 0.2841 & -0.4814 \\ -1.9383 & -0.2352 \end{bmatrix} \begin{bmatrix} -0.5430 & -0.8398 \\ -0.8398 & 0.5430 \end{bmatrix} + \begin{bmatrix} 1.75 & 2.5 \\ 1.75 & 2.5 \\ 1.75 & 2.5 \\ 1.75 & 2.5 \end{bmatrix} \\ &= \begin{bmatrix} -0.7500 & 0.5000 \\ -0.7500 & -1.5000 \\ 0.2500 & -0.5000 \\ 1.2500 & 1.5000 \end{bmatrix} + \begin{bmatrix} 1.75 & 2.5 \\ 1.75 & 2.5 \\ 1.75 & 2.5 \\ 1.75 & 2.5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix} \end{aligned}$$

11.17 Given two 2×2 images, find the corresponding PCA components and their covariance. Then, reconstruct the original images from the PCA components.

$$a(m,n) = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \qquad b(m,n) = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

The mean of the matrices are $am = 2.5$ and $bm = 2$. Subtracting the respective means from the matrices, converting them into column vectors and concatenating, we get

$$x(m,n) = \begin{bmatrix} -0.5000 & 0 \\ -1.5000 & 0 \\ 0.5000 & -1.0000 \\ 1.5000 & 1.0000 \end{bmatrix}$$

$$C(m,n) = \begin{bmatrix} 1.6667 & 0.3333 \\ 0.3333 & 0.6667 \end{bmatrix}$$

The two eigenvalues are $\{1.7676, 0.5657\}$. The eigenvectors are

$$\boldsymbol{R} = \begin{bmatrix} -0.9571 & -0.2898 \\ -0.2898 & 0.9571 \end{bmatrix}$$

The principal components are found as

$$\boldsymbol{y} = \boldsymbol{x}\boldsymbol{R} = \begin{bmatrix} -0.5000 & 0 \\ -1.5000 & 0 \\ 0.5000 & -1.0000 \\ 1.5000 & 1.0000 \end{bmatrix} \begin{bmatrix} -0.9571 & -0.2898 \\ -0.2898 & 0.9571 \end{bmatrix} = \begin{bmatrix} 0.4785 & 0.1449 \\ 1.4356 & 0.4347 \\ -0.1888 & -1.1020 \\ -1.7254 & 0.5224 \end{bmatrix}$$

The covariance of the PCA component matrix is the product of its transpose with itself.

$$C(m,n) = \begin{bmatrix} 1.7676 & 0 \\ 0 & 0.5657 \end{bmatrix}$$

The input can be reconstructed by

$$\boldsymbol{x} = \boldsymbol{y}\boldsymbol{R}^T + \begin{bmatrix} am & bm \\ am & bm \\ am & bm \\ am & bm \end{bmatrix} = \begin{bmatrix} 0.4785 & 0.1449 \\ 1.4356 & 0.4347 \\ -0.1888 & -1.1020 \\ -1.7254 & 0.5224 \end{bmatrix} \begin{bmatrix} -0.9571 & -0.2898 \\ -0.2898 & 0.9571 \end{bmatrix} + \begin{bmatrix} 2.5 & 2 \\ 2.5 & 2 \\ 2.5 & 2 \\ 2.5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5000 & 0.0000 \\ -1.5000 & 0.0000 \\ 0.5000 & -1.0000 \\ 1.5000 & 1.0000 \end{bmatrix} + \begin{bmatrix} 2.5 & 2 \\ 2.5 & 2 \\ 2.5 & 2 \\ 2.5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 3 & 1 \\ 4 & 3 \end{bmatrix}$$

Chapter 12

Classification

12.1 The feature vectors of 3 classes are given. Using the 3-nearest neighbors classifier, classify the test vector $\{16, 16\}$.
(i)

Class	Feature 1	Feature 2
Class1	14	16
Class1	14	14
Class1	13	17
Class2	18	14
Class2	17	16
Class2	17	15
Class3	14	17
Class3	17	19
Class3	15	17

(i) The distance between the test vector and the other 9 feature vectors are

$$\{2.0000, 2.8284, 3.1623, 2.8284, 1, 1.4142, 2.2361, 3.1623, 1.4142\}$$

The sorted distances are

$$\{1, 1.4142, 1.4142, 2.0000, 2.2361, 2.8284, 2.8284, 3.1623, 3.1623\}$$

Test vector belongs to class 2.

12.2 The feature vectors of 3 classes are given. Using the 3-nearest neighbors classifier, classify the test vector $\{16, 17\}$.
(i)

Class	Feature 1	Feature 2
Class1	15	16
Class1	16	19
Class1	15	17
Class2	17	14
Class2	18	15
Class2	17	15
Class3	14	17
Class3	17	18
Class3	14	16

(ii) The distance between the test vector and the other 9 feature vectors are

$$\{1.4142, 2.0000, 1.0000, 3.1623, 2.8284, 2.2361, 2.0000, 1.4142, 2.2361\}$$

The sorted distances are

$$\{1.0000, 1.4142, 1.4142, 2.0000, 2.0000, 2.2361, 2.2361, 2.8284, 3.1623\}$$

Test vector belongs to class 1.

12.3 The feature vectors of 3 classes are given. Using the 3-nearest neighbors classifier, classify the test vector $\{17, 16\}$.

(i)

Class	Feature 1	Feature 2
Class1	15	17
Class1	16	20
Class1	14	14
Class2	20	13
Class2	17	14
Class2	18	15
Class3	16	16
Class3	13	18
Class3	16	19

(iii) The distance between the test vector and the other 9 feature vectors are

$$\{2.2361, 4.1231, 3.6056, 4.2426, 2.0000, 1.4142, 1.0000, 4.4721, 3.1623\}$$

The sorted distances are

$$\{1.0000, 1.4142, 2.0000, 2.2361, 3.1623, 3.6056, 4.1231, 4.2426, 4.4721\}$$

Test vector belongs to class 2.

12.4 Let the mean feature vectors $\{10, 10\}$, $\{-10, -10\}$ and $\{10, -10\}$. Let the test vectors be $\{12, -10\}$, $\{8, 9\}$ and $\{-9, -11\}$. Find the three discriminant function of the three classes. Classify the test vectors.

$$\begin{bmatrix} f1 & f2 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} - 0.5 \begin{bmatrix} 10 & 10 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

Simplifying, we get

$$dx(f1, f2) = 10f1 + 10f2 - 100$$

Similarly, for class2, we get

$$dy(f1, f2) = -10f1 - 10f2 - 100$$

For class3, we get

$$dz(f1, f2) = 10f1 - 10f2 - 100$$

For the first feature vector, these three functions yield

$$\{-80, 70, -300\}$$

For the second feature vector, these three functions yield

$$\{-120, -270, 100\}$$

and, for the third feature vector, these three functions yield

$$\{120, -110, -80\}$$

12.5 Let the mean feature vectors $\{22, -20\}$, $\{20, 20\}$ and $\{-18, -19\}$. Let the test vectors be $\{23, -20\}$, $\{21, 18\}$ and $\{-18, -19\}$. Find the three discriminant function of the three classes. Classify the test vectors.

$$\begin{bmatrix} f1 & f2 \end{bmatrix} \begin{bmatrix} 22 \\ -20 \end{bmatrix} - 0.5 \begin{bmatrix} 22 & -20 \end{bmatrix} \begin{bmatrix} 22 \\ -20 \end{bmatrix}$$

Simplifying, we get

$$dx(f1, f2) = 22f1 - 20f2 - 442$$

Similarly, for class2, we get

$$dy(f1, f2) = 20f1 + 20f2 - 400$$

For class3, we get

$$dz(f1, f2) = -18f1 - 19f2 - 342.5$$

For the first feature vector, these three functions yield

$$\{464, -340, -458\}$$

For the second feature vector, these three functions yield

$$\{-340, 380, -1140\}$$

and, for the third feature vector, these three functions yield

$$1000\{-0.3765, -1.0625, 0.3425\}$$

12.6 Let the mean feature vectors $\{15, -17\}$, $\{17, 15\}$ and $\{-16, 16\}$. Let the test vectors be $\{18, -16\}$, $\{18, 14\}$ and $\{-16, 16\}$. Find the three discriminant function of the three classes. Classify the test vectors.

$$\begin{bmatrix} f1 & f2 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix} - 0.5 \begin{bmatrix} 10 & 10 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

Simplifying, we get

$$dx(f1, f2) = 15f1 - 17f2 - 257$$

Similarly, for class2, we get

$$dy(f1, f2) = 17f1 + 15f2 - 257$$

For class3, we get

$$dz(f1, f2) = -16f1 + 16f2 - 256$$

For the first feature vector, these three functions yield

$$\{285, -225, -769\}$$

For the second feature vector, these three functions yield

$$\{-191, 259, -289\}$$

and, for the third feature vector, these three functions yield

$$\{-800, -320, 256\}$$

12.7 Draw the decision tree flowchart for classifying the 5 objects.

Object	F	O	U	R	S
Area	750	964	647	1084	658
Form	132	484	108	233	116

$$\left[\begin{array}{cccc|c} 647 & 658 & 750 & 964 & 1084 \\ 108 & 116 & 132 & 233 & 484 \end{array}\right]$$

f2=(233+484)/2= 358.5 and O classified

$$\left[\begin{array}{ccc|c} 647 & 658 & 750 & 1084 \\ 108 & 116 & 132 & 233 \end{array}\right]$$

f1=(750+1084)/2= 917 and R classified

$$\left[\begin{array}{cc|c} 647 & 658 & 750 \\ 108 & 116 & 132 \end{array}\right]$$

f1=(750+658)/2= 704 and F classified
f1=(647+658)/2= 652.5 and U and S classified

12.8 Draw the decision tree flowchart for classifying the 5 objects.

Object	5	6	7	8	9
Area	1199	1245	822	1478	1229
Form	168	327	205	445	327

$$\left[\begin{array}{c|cccc} 822 & 1199 & 1229 & 1245 & 1478 \\ 168 & 205 & 327 & 327 & 445 \end{array}\right]$$

f1=(822+1199)/2= 1010.5 and 7 classified

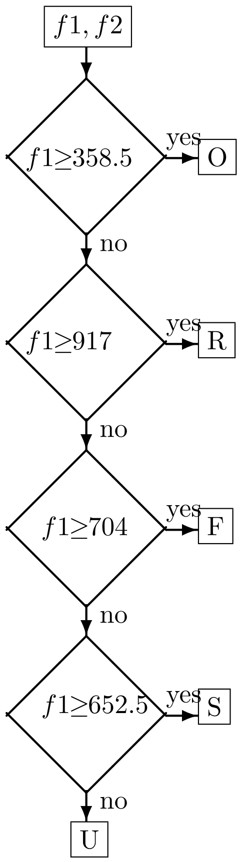
$$\left[\begin{array}{ccc|c} 1199 & 1229 & 1245 & 1478 \\ 168 & 327 & 327 & 445 \end{array}\right]$$

f1=(1245+1478)/2= 1361.5 and 8 classified

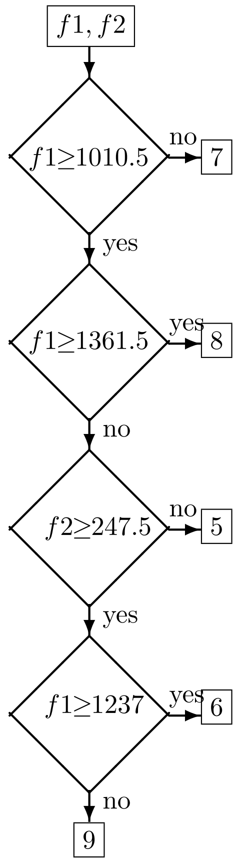
$$\left[\begin{array}{c|cc} 1199 & 1229 & 1245 \\ 168 & 327 & 327 \end{array}\right]$$

f2=(168+327)/2= 247.5 and 5 classified
f1=(1199+1229)/2= 1237 and 6 and 9 classified

12.9 Draw the decision tree flowchart for classifying the 5 objects.



12.7 Flowchart of the decision tree algorithm



12.8 Flowchart of the decision tree algorithm

Object	0	1	2	3	4
Area	1419	880	1090	1023	1118
Form	576	273	183	176	402

$$\left[\begin{array}{rrrr|r} 880 & 1023 & 1090 & 1118 & 1419 \\ 176 & 183 & 273 & 402 & 576 \end{array}\right]$$

f1=(1118+1419)/2= 1268.5 and 0 classified

$$\left[\begin{array}{rrrr|} 880 & 1023 & 1090 & 1118 \\ 273 & 176 & 183 & 402 \end{array}\right]$$

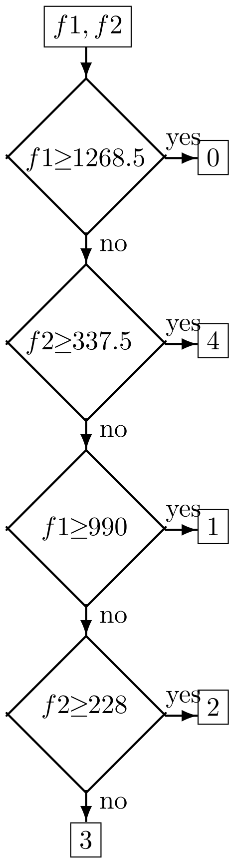
$$\left[\begin{array}{rrr|r} 880 & 1023 & 1090 & 1118 \\ 176 & 183 & 273 & 402 \end{array}\right]$$

f2=(273+402)/2= 337.5 and 4 classified

$$\left[\begin{array}{rrr} 1023 & 1090 & 880 \\ 176 & 183 & 273 \end{array}\right]$$

$$\left[\begin{array}{rrr} 880 & 1090 & 1023 \\ 176 & 183 & 273 \end{array}\right]$$

f1=(880+1090)/2= 990 and 1 classified
f2=(183+273)/2= 228 and 2 and 3 classified



12.9 Flowchart of the decision tree algorithm

12.10 Samples of training data of two classes, $c1(m,n)$ and $c2(m,n)$, are

$$c1(m,n) = \begin{bmatrix} 0.7124 & -1.0637 \\ 1.0219 & -1.3503 \\ 1.0487 & -0.6465 \\ 1.7837 & -0.5444 \\ 2.4486 & -0.3089 \\ 1.4430 & -0.6230 \\ 0.8010 & -0.9970 \\ 1.5931 & -1.0655 \end{bmatrix} \qquad c2(m,n) = \begin{bmatrix} -1.2296 & 0.7281 \\ -1.2066 & 0.6124 \\ -1.7524 & 3.1638 \\ 1.3789 & 0.6478 \\ -2.2385 & -0.5050 \\ -1.6842 & 3.2676 \\ -2.0069 & 2.7461 \\ -2.6606 & 0.6753 \end{bmatrix}$$

Classify the test samples $t(m,n)$ using Bayes classification.

$$t(m,n) = \begin{bmatrix} 0.4804 & -1.2375 \\ 1.0979 & -0.8694 \\ -1.7471 & 0.1364 \\ -0.8781 & 0.2799 \\ -0.9416 & 0.6194 \\ -2.9676 & 1.2279 \end{bmatrix}$$

with
(i) $p(\omega_1) = 0.9$ and $p(\omega_2) = 0.1$.
(ii) $p(\omega_1) = 0.1$ and $p(\omega_2) = 0.9$.

The mean of class1 is $\{1.3566 - 0.8249\}$ and that of class2 is $\{-1.42501.4170\}$. The covariance matrices of class1 and class2, respectively, are

$$\boldsymbol{C}_1 = \frac{cm1(m,n)^T cm1(m,n)}{7} = \begin{bmatrix} 0.3384 & 0.1449 \\ 0.1449 & 0.1197 \end{bmatrix}$$

$$\boldsymbol{C}_2 = \frac{cm2(m,n)^T cm2(m,n)}{7} = \begin{bmatrix} 1.5213 & -0.2589 \\ -0.2589 & 2.0286 \end{bmatrix}$$

The determinants of the matrices are 0.0195 and 3.0192. The inverses are

$$\begin{bmatrix} 6.1322 & -7.4206 \\ -7.4206 & 17.3319 \end{bmatrix} \qquad \begin{bmatrix} 0.6719 & 0.0857 \\ 0.0857 & 0.5039 \end{bmatrix}$$

With $p(\omega_1) = 0.9$ and $p(\omega_2) = 0.1$, the decision function for class1 $d_1(\boldsymbol{x})$ is

$$-3.0661x_1^2 - 8.6659x_2^2 + 7.4206(x_1 + x_2) + 14.4400x_1 - 24.3637x_2 - 17.9806$$

For the test data, the decision function yields

$$\{0.7163, 1.7258, -57.8204, -42.3467, -57.0393, -157.8573\}$$

The decision function for class2 $d_2(\boldsymbol{x})$ is

$$-0.3360x_1^2 - 0.2519x_2^2 - 0.0857(x_1 + x_2) - 0.8360x_1 + 0.5918x_2 - 3.8700$$

For the test data, the decision function yields

$$\{-5.4164, -5.8159, -3.3385, -3.2280, -3.0608, -3.6886\}$$

(ii) With $p(\omega_1) = 0.1$ and $p(\omega_2) = 0.9$, the decision function for class1 $d_1(\boldsymbol{x})$ is

$$-3.0661x_1^2 - 8.6659x_2^2 + 7.4206(x_1 + x_2) + 14.4400x_1 - 24.3637x_2 - 20.1778$$

For the test data, the decision function yields

$$\{-1.4809, -0.4714, -60.0177, -44.5439, -59.2365, -160.0545\}$$

The decision function for class1 $d_2(\boldsymbol{x})$ is

$$-0.3360x_1^2 - 0.2519x_2^2 - 0.0857(x_1 + x_2) - 0.8360x_1 + 0.5918x_2 - 1.6728$$

For the test data, the decision function yields

$$\{-3.2192, -3.6187, -1.1413, -1.0308, -0.8636, -1.4913\}$$

12.11 Samples of training data of two classes, $c1(m,n)$ and $c2(m,n)$, are

$$c1(m,n) = \begin{bmatrix} -1.3587 & -1.7484 \\ 1.5513 & -0.9966 \\ -1.0799 & -1.7673 \\ -1.2003 & -1.9427 \\ 1.0758 & -0.6775 \\ 0.1002 & -1.2125 \\ 1.3904 & -0.5126 \\ 1.6422 & -0.7607 \end{bmatrix} \qquad c2(m,n) = \begin{bmatrix} -1.2082 & -0.4353 \\ 0.4252 & 0.3338 \\ -4.0033 & 1.1938 \\ -1.7136 & 0.5872 \\ -2.7969 & 1.4268 \\ -1.5411 & 1.5656 \\ -0.0826 & -0.3152 \\ 0.1678 & 0.7499 \end{bmatrix}$$

Classify the test samples $t(m,n)$ using Bayes classification.

$$t(m,n) = \begin{bmatrix} -2.5374 & -2.4336 \\ 1.7707 & -0.8282 \\ -1.7326 & 2.1029 \\ -3.5297 & 1.9491 \\ -1.3760 & -0.9096 \\ -2.2018 & 1.1494 \end{bmatrix}$$

with
$p(\omega_1) = 0.5$ and $p(\omega_2) = 0.5$.

The mean of class1 is $\{0.2651, -1.2023\}$ and that of class2 is $\{-1.3441, 0.6383\}$. The covariance matrices of class1 and class2, respectively, are

$$\boldsymbol{C}_1 = \frac{cm1(m,n)^T cm1(m,n)}{7} = \begin{bmatrix} 1.7278 & 0.6868 \\ 0.6868 & 0.3077 \end{bmatrix}$$

$$\boldsymbol{C}_2 = \frac{cm2(m,n)^T cm2(m,n)}{7} = \begin{bmatrix} 2.3405 & -0.6436 \\ -0.6436 & 0.5657 \end{bmatrix}$$

The determinants of the matrices are 0.0600 and 0.9097. The inverses are

$$\begin{bmatrix} 5.1279 & -11.4444 \\ -11.4444 & 28.7907 \end{bmatrix} \qquad \begin{bmatrix} 0.6218 & 0.7075 \\ 0.7075 & 2.5727 \end{bmatrix}$$

With $p(\omega_1) = 0.5$ and $p(\omega_2) = 0.5$, the decision function for class1 $d_1(\boldsymbol{x})$ is

$$-2.5640x_1^2 - 14.3954x_2^2 + 11.4444(x_1 + x_2) + 15.1190x_1 - 37.6489x_2 - 23.9232$$

For the test data, the decision function yields

$$\{-1.7575, -0.6673, -242.3435, -316.0362, -12.9224, -160.8966\}$$

The decision function for class2 $d_2(\boldsymbol{x})$ is

$$-0.3109x_1^2 - 1.2864x_2^2 - 0.7075(x_1 + x_2) - 0.3842x_1 + 0.6913x_2 - 1.1247$$

For the test data, the decision function yields

$$\{-15.8210, -3.1971, -3.0494, -2.3143, -3.7633, -0.9004\}$$

12.12 Samples of training data of two classes, $c1(m,n)$ and $c2(m,n)$, are

$$c1(m,n) = \begin{bmatrix} 2.6140 & -0.5545 \\ 0.5164 & -1.0168 \\ 0.9973 & -1.4846 \\ 1.8727 & -0.5267 \\ 1.1421 & -0.6542 \\ 2.3328 & -0.3331 \\ 1.9811 & -0.2564 \\ 1.2766 & -0.1643 \end{bmatrix} \qquad c2(m,n) = \begin{bmatrix} 0.3066 & 1.8237 \\ -0.6225 & -0.4234 \\ -0.0926 & 1.0912 \\ -0.3983 & 1.8489 \\ -2.8593 & -0.9255 \\ -1.5889 & 1.4916 \\ 0.7320 & 0.7428 \\ -1.0616 & -0.3287 \end{bmatrix}$$

Classify the test samples $t(m,n)$ using Bayes classification.

$$t(m,n) = \begin{bmatrix} 1.5646 & -1.3982 \\ 0.2250 & -1.5204 \\ -1.8283 & -1.3793 \\ -2.0365 & 0.0497 \\ -2.6668 & 0.3246 \\ -0.3692 & 0.8388 \end{bmatrix}$$

with
$p(\omega_1) = 0.4$ and $p(\omega_2) = 0.6$.

The mean of class1 is $\{1.5916, -0.6238\}$ and that of class2 is $\{-0.6981, 0.6651\}$. The covariance matrices of class1 and class2, respectively, are

$$\boldsymbol{C}_1 = \frac{cm1(m,n)^T cm1(m,n)}{7} = \begin{bmatrix} 0.5194 & 0.1800 \\ 0.1800 & 0.1916 \end{bmatrix}$$

$$\boldsymbol{C}_2 = \frac{cm2(m,n)^T cm2(m,n)}{7} = \begin{bmatrix} 1.3019 & 0.6955 \\ 0.6955 & 1.1881 \end{bmatrix}$$

The determinants of the matrices are 0.0671 and 1.0631. The inverses are

$$\begin{bmatrix} 2.8540 & -2.6807 \\ -2.6807 & 7.7371 \end{bmatrix} \qquad \begin{bmatrix} 1.1176 & -0.6542 \\ -0.6542 & 1.2246 \end{bmatrix}$$

With $p(\omega_1) = 0.4$ and $p(\omega_2) = 0.6$, the decision function for class1 $d_1(\boldsymbol{x})$ is

$$-1.4270x_1^2 - 3.8686x_2^2 + 2.6807(x_1 + x_2) + 6.2149x_1 - 9.0933x_2 - 7.3479$$

For the test data, the decision function yields

$$\{-1.8305, -2.0560, -11.5379, -26.6557, -39.7503, -21.0165\}$$

The decision function for class2 $d_2(\boldsymbol{x})$ is

$$-0.5588x_1^2 - 0.6123x_2^2 + 0.6542(x_1 + x_2) - 1.2152x_1 + 1.2711x_2 - 1.3883$$

For the test data, the decision function yields

$$\{-9.0630, -5.2618, -2.3027, -1.2355, -2.3397, -0.5830\}$$

Chapter 13

Compress

13.1 Given a 4×4 image, compute the entropy. Find the Huffman code representation of its unique symbols and the bpp.

(i)

$$\begin{bmatrix} 144 & 113 & 121 & 107 \\ 144 & 110 & 121 & 103 \\ 129 & 109 & 120 & 99 \\ 116 & 108 & 121 & 103 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 209 & 190 & 179 & 179 \\ 143 & 136 & 132 & 129 \\ 131 & 130 & 125 & 117 \\ 113 & 109 & 118 & 143 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 85 & 91 & 91 & 89 \\ 79 & 83 & 88 & 87 \\ 90 & 86 & 86 & 90 \\ 97 & 93 & 88 & 90 \end{bmatrix}$$

(i)

```
[ 99]    '1  0  1  1'
[103]    '1  0  0'
[107]    '1  0  1  0'
[108]    '0  1  0  1'
[109]    '0  1  0  0'
[110]    '0  1  1  1'
[113]    '0  1  1  0'
[116]    '0  0  0  0  1'
[120]    '0  0  0  0  0'
[121]    '1  1'
[129]    '0  0  0  1'
[144]    '0  0  1'
```

$$bpp = 3.5, \qquad entropy = 3.4528$$

(ii)

```
[109]    '0  0  0  1'
[113]    '0  0  0  0'
[117]    '0  0  1  1'
[118]    '0  0  1  0'
[125]    '1  1  0  1'
[129]    '1  1  0  0'
[130]    '1  1  1  1'
[131]    '1  1  1  0'
[132]    '1  0  0  1'
[136]    '1  0  0  0'
[143]    '0  1  1'
[179]    '0  1  0'
[190]    '1  0  1  1'
[209]    '1  0  1  0'
```

$$bpp = 3.75, \qquad entropy = 3.75$$

(iii)

```
[79]    '0  1  0  1'
[83]    '0  1  0  0'
[85]    '0  1  1  1'
[86]    '1  0  1'
[87]    '0  1  1  0'
[88]    '1  0  0'
[89]    '0  0  0  0  1'
[90]    '1  1'
[91]    '0  0  1'
[93]    '0  0  0  0  0'
[97]    '0  0  0  1'
```

$$bpp = 3.375, \qquad entropy = 3.3278$$

13.2 Given a 4×4 image, decompose it into 4 bit planes and represent each of them by run-length coding. Use both the methods.

(i)

$$\begin{bmatrix} 6 & 6 & 15 & 14 \\ 8 & 8 & 4 & 11 \\ 9 & 9 & 10 & 12 \\ 10 & 14 & 13 & 1 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 11 & 9 & 15 & 14 \\ 13 & 7 & 6 & 7 \\ 6 & 5 & 5 & 5 \\ 11 & 4 & 4 & 2 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 13 & 9 & 12 & 10 \\ 13 & 13 & 12 & 9 \\ 13 & 13 & 12 & 9 \\ 11 & 13 & 12 & 10 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 6 & 6 & 15 & 14 \\ 8 & 8 & 4 & 11 \\ 9 & 9 & 10 & 12 \\ 10 & 14 & 13 & 1 \end{bmatrix} = 2^3 \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\{22, 0211, 04, 031\ 04, 211, 31, 121\ 04, 31, 211, 022\ 211, 31, 022, 22\}$$

$$\{32, 1241, 14, 13\ 14, 31, 41, 22,\ 14, 41, 31, 12\ 31, 41, 12, 32\}$$

(ii)

$$\begin{bmatrix} 11 & 9 & 15 & 14 \\ 13 & 7 & 6 & 7 \\ 6 & 5 & 5 & 5 \\ 11 & 4 & 4 & 2 \end{bmatrix} = 2^3 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + 2^2 \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\{04, 013, 4, 013\ 22, 04, 04, 121\ 0112, 13, 013, 0121\ 031, 0211, 13, 013\}$$

$$\{14, 11, 10, 11\ 32, 14, 14, 22\ 1132, 23, 11, 1141\ 13, 1241, 23, 11\}$$

(iii)

$$\begin{bmatrix} 13 & 9 & 12 & 10 \\ 13 & 13 & 12 & 9 \\ 13 & 13 & 12 & 9 \\ 11 & 13 & 12 & 10 \end{bmatrix} = 2^3 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} + 2^2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} + 2 \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\{04, 04, 04, 04\ 01111, 031, 031, 121\ 31, 4, 4, 0121\ 022, 0211, 0211, 022\}$$

$$\{14, 14, 14, 14\ 1131, 13, 13, 22\ 41, 10, 10, 1141\ 12, 1241, 1241, 12\}$$

13.3 Given a 4×4 image, find the linear predictive code. Find the entropies of the input and the code.

(i)

$$\begin{bmatrix} 15 & 16 & 20 & 20 \\ 15 & 15 & 19 & 22 \\ 15 & 16 & 19 & 20 \\ 15 & 17 & 19 & 16 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 158 & 157 & 154 & 149 \\ 168 & 153 & 157 & 149 \\ 170 & 152 & 157 & 149 \\ 166 & 153 & 157 & 142 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 106 & 103 & 98 & 99 \\ 121 & 122 & 108 & 93 \\ 102 & 102 & 100 & 99 \\ 100 & 101 & 102 & 96 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 15 & 1 & 4 & 0 \\ 15 & 0 & 4 & 3 \\ 15 & 1 & 3 & 1 \\ 15 & 2 & 2 & -3 \end{bmatrix}$$

$$\{2.3829, 2.7028\}$$

(ii)

$$\begin{bmatrix} 158 & -1 & -3 & -5 \\ 168 & -15 & 4 & -8 \\ 170 & -18 & 5 & -8 \\ 166 & -13 & 4 & -15 \end{bmatrix}$$

$$\{3.0778, 3.6250\}$$

(iii)

$$\begin{bmatrix} 106 & -3 & -5 & 1 \\ 121 & 1 & -14 & -15 \\ 102 & 0 & -2 & -1 \\ 100 & 1 & 1 & -6 \end{bmatrix}$$

$$\{3.4528, 3.5\}$$

13.4 Given a 1-digit sequence, find its arithmetic code. Reconstruct the sequence from the code. Let there be 3 symbols $\{1, 2, 3\}$ and number of occurrences of the symbols, respectively, be $\{2, 1, 1\}$ in a sequence of length 4.
(i) $\{1\}$
(ii) $\{2\}$
(i)

$$l1 = l + \left\lfloor \frac{(u - l + 1) \times c\_count(1)}{total} \right\rfloor = 0 + \left\lfloor \frac{16 \times 0}{4} \right\rfloor = 0 = (0000)_2$$

$$u1 = 0 + \left\lfloor \frac{(u - l + 1) \times c\_count(2)}{total} \right\rfloor - 1 = 0 + \left\lfloor \frac{16 \times 2}{4} \right\rfloor - 1 = 8 = (0111)_2$$

$code = \{0\}$.

$$l1 = 0 = (0000)_2, \qquad u1 = 15 = (1111)_2$$

The final *code* is $\{00000\}$.

To decode, we read the first 4 bits of the code into $tag = (0000)_2 = 0$.

$$tem = \left\lfloor \frac{(tag - l + 1) \times total - 1}{u - l + 1} \right\rfloor = \left\lfloor \frac{1 \times 4 - 1}{16} \right\rfloor = 0 = (0000)_2$$

The index is 1. The decoded symbol, 1, is with this index in *sym* list. Now, we update the limits.

$$l1 = 0 + \left\lfloor \frac{16 \times 0}{4} \right\rfloor = 0 = (0000)_2$$

$$u1 = 0 + \left\lfloor \frac{16 \times 2}{4} \right\rfloor - 1 = 7 = (0111)_2$$

$$l1 = 0 = (0000)_2, u1 = 15 = (1111)_2, tag = 0 = (0000)_2$$

(ii)

$$l1 = l + \left\lfloor \frac{(u - l + 1) \times c\_count(1)}{total} \right\rfloor = 0 + \left\lfloor \frac{16 \times 2}{4} \right\rfloor = 8 = (1000)_2$$

$$u1 = 0 + \left\lfloor \frac{(u - l + 1) \times c\_count(2)}{total} \right\rfloor - 1 = 0 + \left\lfloor \frac{16 \times 3}{4} \right\rfloor - 1 = 11 = (1011)_2$$

$code = \{1\}$.

$$l1 = 0 = (0000)_2, \qquad u1 = 7 = (0111)_2$$

$code = \{10\}$.

$$l1 = 0 = (0000)_2, \qquad u1 = 15 = (0111)_2$$

The final $code$ is $\{100000\}$.

To decode, we read the first 4 bits of the code into $tag = (1000)_2 = 0$.

$$tem = \left\lfloor \frac{(tag - l + 1) \times total - 1}{u - l + 1} \right\rfloor = \left\lfloor \frac{9 \times 4 - 1}{16} \right\rfloor = 2 = (0010)_2$$

The index is 2. The decoded symbol, 2, is with this index in sym list. Now, we update the limits.

$$l1 = 0 + \left\lfloor \frac{16 \times 2}{4} \right\rfloor = 8 = (1000)_2$$

$$u1 = 0 + \left\lfloor \frac{16 \times 3}{4} \right\rfloor - 1 = 11 = (1011)_2$$

$$l1 = 0 = (0000)_2, u1 = 7 = (0111)_2, tag = 0 = (0000)_2$$

$$l1 = 0 = (0000)_2, u1 = 15 = (1111)_2, tag = 0 = (0000)_2$$

13.5 Given a sequence $x(n)$, find the 1-level DWT coefficients using the 9/7 filter. Assume whole-point symmetry at the borders. Verify that the reconstructed signal is the same as the input.

(i)

$$\{1, -4, 1, 3, 3, 1, 3, 0, 2, 2, 3, 1, -5, 2, 0, 3\}$$

(ii)

$$\{1, 2, 1, 1, 3, 4, 0, 3, 1, 3, 2, -1, 0, 1, 4, -3\}$$

(iii)

$$\{-2, 0, 2, -2, 1, 0, 3, 1, -2, 1, 2, -1, 2, 0, -2, 1\}$$

(i)

$$\{-2.3039, 0.2557, 3.8341, 2.4144, 1.6731, 4.0880, -3.5067, 2.2929,$$
$$3.6913, -1.0733, 1.6485, 1.8253, 0.6833, -1.5910, -3.6983, -1.5573\}$$

(ii)

$$\{2.3203, 1.4371, 4.2909, 2.2174, 2.9378, 2.4055, -0.4746, 2.2891,$$
$$-0.8769, 1.0635, -1.8660, -1.9852, -1.0298, 1.4647, 0.3339, 5.7916\}$$

(iii)

$$\{-1.9768, 1.2504, -0.6304, 3.2213, -1.3664, 1.7194, 1.1290, -1.1528,$$
$$-0.2750, 2.7667, 1.6317, -0.5233, -1.1112, 2.7597, 0, -2.7190\}$$

13.6 Given a 4×4 image, find its compressed version using the 1-level Haar DWT and the Huffman code. What is the bpp and SNR.

(i)

$$\begin{bmatrix} 170 & 168 & 164 & 173 \\ 179 & 167 & 167 & 167 \\ 184 & 179 & 173 & 166 \\ 183 & 179 & 184 & 173 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 172 & 173 & 170 & 171 \\ 171 & 176 & 173 & 172 \\ 174 & 178 & 172 & 170 \\ 176 & 175 & 171 & 170 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 162 & 163 & 163 & 161 \\ 162 & 164 & 161 & 161 \\ 163 & 165 & 164 & 162 \\ 167 & 161 & 162 & 164 \end{bmatrix}$$

(i)

```
[ 1]    '0  1  1  1'
[ 0]    '0  1  0'
[-1]    '0  1  1  0'
[ 3]    '0  0  0  1'
[ 4]    '0  0  0  0'
[ 5]    '0  0  1  1'
[47]    '0  0  1  0'
[51]    '1  1  0  1'
[54]    '1  1  0  0'
[63]    '1  1  1  1'
[-5]    '1  1  1  0'
[-2]    '1  0'
```

$$bpp = 3.375, SNR = 47.9747$$

(ii)

```
[57]    '0  1  0  1'
[59]    '0  1  0  0'
[63]    '0  1  1  1'
[56]    '0  1  1  0'
[ 1]    '0  0  1'
[-1]    '0  0  0'
[ 0]    '1'
```

$$bpp = 2.375, SNR = 46.9179$$

(iii)

```
[61]    '1  0  1'
[59]    '0  1  0  1'
[63]    '0  1  0  0'
[-3]    '0  1  1  1'
[ 2]    '1  0  0'
[ 1]    '1  1'
[-1]    '0  1  1  0'
[ 0]    '0  0'
```

$$bpp = 2.75, SNR = 54.5465$$

Chapter 14

Color Image

14.1 Given the RGB components of a 4×4 color image, find the CMY components of its CMY model.

(i)

$$\begin{bmatrix} 229 & 226 & 238 & 214 \\ 239 & 238 & 225 & 233 \\ 252 & 247 & 222 & 242 \\ 214 & 213 & 240 & 224 \end{bmatrix} \begin{bmatrix} 215 & 212 & 231 & 213 \\ 222 & 225 & 214 & 224 \\ 242 & 235 & 209 & 229 \\ 201 & 196 & 225 & 211 \end{bmatrix} \begin{bmatrix} 71 & 41 & 73 & 72 \\ 90 & 70 & 74 & 93 \\ 91 & 76 & 53 & 89 \\ 50 & 33 & 69 & 79 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 171 & 228 & 231 & 229 \\ 180 & 225 & 231 & 221 \\ 192 & 225 & 229 & 213 \\ 204 & 221 & 229 & 216 \end{bmatrix} \begin{bmatrix} 133 & 54 & 64 & 44 \\ 130 & 51 & 63 & 42 \\ 99 & 48 & 63 & 40 \\ 71 & 50 & 65 & 44 \end{bmatrix} \begin{bmatrix} 125 & 120 & 130 & 113 \\ 125 & 115 & 129 & 105 \\ 119 & 114 & 127 & 102 \\ 116 & 114 & 128 & 109 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 64 & 68 & 67 & 66 \\ 61 & 59 & 65 & 65 \\ 68 & 64 & 66 & 69 \\ 74 & 71 & 68 & 69 \end{bmatrix} \begin{bmatrix} 102 & 107 & 108 & 108 \\ 93 & 100 & 106 & 104 \\ 107 & 102 & 102 & 106 \\ 115 & 110 & 103 & 107 \end{bmatrix} \begin{bmatrix} 55 & 64 & 63 & 54 \\ 55 & 56 & 63 & 59 \\ 65 & 59 & 58 & 62 \\ 69 & 66 & 60 & 56 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 26 & 29 & 17 & 41 \\ 16 & 17 & 30 & 22 \\ 3 & 8 & 33 & 13 \\ 41 & 42 & 15 & 31 \end{bmatrix} \begin{bmatrix} 40 & 43 & 24 & 42 \\ 33 & 30 & 41 & 31 \\ 13 & 20 & 46 & 26 \\ 54 & 59 & 30 & 44 \end{bmatrix} \begin{bmatrix} 184 & 214 & 182 & 183 \\ 165 & 185 & 181 & 162 \\ 164 & 179 & 202 & 166 \\ 205 & 222 & 186 & 176 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 84 & 27 & 24 & 26 \\ 75 & 30 & 24 & 34 \\ 63 & 30 & 26 & 42 \\ 51 & 34 & 26 & 39 \end{bmatrix} \begin{bmatrix} 122 & 201 & 191 & 211 \\ 125 & 204 & 192 & 213 \\ 156 & 207 & 192 & 215 \\ 184 & 205 & 190 & 211 \end{bmatrix} \begin{bmatrix} 130 & 135 & 125 & 142 \\ 130 & 140 & 126 & 150 \\ 136 & 141 & 128 & 153 \\ 139 & 141 & 127 & 146 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 191 & 187 & 188 & 189 \\ 194 & 196 & 190 & 190 \\ 187 & 191 & 189 & 186 \\ 181 & 184 & 187 & 186 \end{bmatrix} \begin{bmatrix} 153 & 148 & 147 & 147 \\ 162 & 155 & 149 & 151 \\ 148 & 153 & 153 & 149 \\ 140 & 145 & 152 & 148 \end{bmatrix} \begin{bmatrix} 200 & 191 & 192 & 201 \\ 200 & 199 & 192 & 196 \\ 190 & 196 & 197 & 193 \\ 186 & 189 & 195 & 199 \end{bmatrix}$$

14.2 Given the RGB components of a 4×4 color image, find the HSI components of its HSI model.

(i)

$$\begin{bmatrix} 232 & 234 & 235 & 235 \\ 235 & 235 & 237 & 235 \\ 236 & 236 & 234 & 234 \\ 236 & 237 & 234 & 236 \end{bmatrix} \begin{bmatrix} 49 & 48 & 57 & 71 \\ 52 & 55 & 61 & 78 \\ 53 & 61 & 61 & 84 \\ 57 & 64 & 69 & 92 \end{bmatrix} \begin{bmatrix} 162 & 169 & 174 & 188 \\ 166 & 175 & 181 & 194 \\ 169 & 180 & 185 & 200 \\ 172 & 184 & 191 & 206 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 210 & 148 & 149 & 155 \\ 201 & 152 & 146 & 150 \\ 190 & 152 & 140 & 144 \\ 180 & 157 & 134 & 137 \end{bmatrix} \begin{bmatrix} 103 & 37 & 29 & 34 \\ 64 & 37 & 28 & 34 \\ 48 & 35 & 27 & 34 \\ 37 & 34 & 28 & 31 \end{bmatrix} \begin{bmatrix} 137 & 111 & 120 & 125 \\ 113 & 115 & 119 & 124 \\ 125 & 120 & 115 & 122 \\ 135 & 131 & 110 & 115 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 98 & 94 & 86 & 82 \\ 94 & 90 & 83 & 80 \\ 88 & 85 & 80 & 78 \\ 80 & 79 & 76 & 74 \end{bmatrix} \begin{bmatrix} 131 & 127 & 120 & 116 \\ 124 & 122 & 116 & 112 \\ 116 & 115 & 111 & 108 \\ 109 & 106 & 103 & 103 \end{bmatrix} \begin{bmatrix} 82 & 80 & 77 & 74 \\ 80 & 80 & 78 & 74 \\ 75 & 75 & 74 & 71 \\ 69 & 70 & 69 & 67 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 0.8952 & 0.8893 & 0.8881 & 0.8782 \\ 0.8942 & 0.8864 & 0.8837 & 0.8738 \\ 0.8923 & 0.8841 & 0.8776 & 0.8680 \\ 0.8907 & 0.8817 & 0.8737 & 0.8650 \end{bmatrix} \begin{bmatrix} 0.6682 & 0.6807 & 0.6330 & 0.5688 \\ 0.6556 & 0.6452 & 0.6180 & 0.5385 \\ 0.6528 & 0.6164 & 0.6187 & 0.5135 \\ 0.6323 & 0.6041 & 0.5810 & 0.4831 \end{bmatrix} \begin{bmatrix} 0.5791 & 0.5895 & 0.6092 & 0.6458 \\ 0.5922 & 0.6078 & 0.6261 & 0.6627 \\ 0.5987 & 0.6235 & 0.6275 & 0.6771 \\ 0.6078 & 0.6340 & 0.6458 & 0.6980 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0.9497 & 0.8864 & 0.8705 & 0.8716 \\ 0.9426 & 0.8844 & 0.8684 & 0.8676 \\ 0.9089 & 0.8760 & 0.8671 & 0.8636 \\ 0.8831 & 0.8655 & 0.8680 & 0.8648 \end{bmatrix} \begin{bmatrix} 0.3133 & 0.6250 & 0.7081 & 0.6752 \\ 0.4921 & 0.6349 & 0.7133 & 0.6688 \\ 0.6033 & 0.6580 & 0.7128 & 0.6600 \\ 0.6847 & 0.6832 & 0.6912 & 0.6714 \end{bmatrix} \begin{bmatrix} 0.5882 & 0.3869 & 0.3895 & 0.4105 \\ 0.4941 & 0.3974 & 0.3830 & 0.4026 \\ 0.4745 & 0.4013 & 0.3686 & 0.3922 \\ 0.4601 & 0.4209 & 0.3556 & 0.3699 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 0.2815 & 0.2865 & 0.3015 & 0.3046 \\ 0.2829 & 0.2967 & 0.3140 & 0.3099 \\ 0.2831 & 0.2947 & 0.3092 & 0.3048 \\ 0.2905 & 0.2947 & 0.3021 & 0.3040 \end{bmatrix} \begin{bmatrix} 0.2090 & 0.2027 & 0.1837 & 0.1838 \\ 0.1946 & 0.1781 & 0.1552 & 0.1654 \\ 0.1935 & 0.1818 & 0.1623 & 0.1712 \\ 0.1977 & 0.1765 & 0.1653 & 0.1762 \end{bmatrix} \begin{bmatrix} 0.4065 & 0.3935 & 0.3699 & 0.3556 \\ 0.3895 & 0.3817 & 0.3621 & 0.3477 \\ 0.3647 & 0.3595 & 0.3464 & 0.3359 \\ 0.3373 & 0.3333 & 0.3242 & 0.3190 \end{bmatrix}$$

14.3 Given the RGB components of a 4×4 color image, find the YIQ components of its NTSC model.

(i)

$$\begin{bmatrix} 236 & 239 & 242 & 247 \\ 236 & 239 & 244 & 249 \\ 235 & 240 & 245 & 250 \\ 236 & 241 & 245 & 249 \end{bmatrix} \begin{bmatrix} 193 & 190 & 187 & 186 \\ 190 & 187 & 185 & 184 \\ 189 & 185 & 184 & 182 \\ 188 & 184 & 182 & 180 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 3 & 6 & 6 \\ 2 & 6 & 9 & 9 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 209 & 210 & 232 & 250 \\ 206 & 212 & 235 & 248 \\ 208 & 213 & 233 & 238 \\ 212 & 212 & 220 & 219 \end{bmatrix} \begin{bmatrix} 32 & 28 & 37 & 43 \\ 29 & 29 & 40 & 42 \\ 31 & 33 & 43 & 39 \\ 35 & 38 & 39 & 32 \end{bmatrix} \begin{bmatrix} 51 & 54 & 66 & 76 \\ 46 & 51 & 65 & 69 \\ 47 & 49 & 61 & 61 \\ 50 & 49 & 53 & 48 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 222 & 223 & 227 & 231 \\ 226 & 227 & 232 & 235 \\ 233 & 234 & 235 & 237 \\ 238 & 239 & 238 & 238 \end{bmatrix} \begin{bmatrix} 185 & 185 & 186 & 189 \\ 183 & 183 & 183 & 186 \\ 179 & 179 & 179 & 180 \\ 175 & 174 & 174 & 174 \end{bmatrix} \begin{bmatrix} 3 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 0.7214 & 0.7176 & 0.7142 & 0.7178 \\ 0.7141 & 0.7111 & 0.7128 & 0.7164 \\ 0.7106 & 0.7086 & 0.7135 & 0.7147 \\ 0.7104 & 0.7088 & 0.7102 & 0.7103 \end{bmatrix} \begin{bmatrix} 0.3426 & 0.3541 & 0.3643 & 0.3771 \\ 0.3471 & 0.3561 & 0.3686 & 0.3814 \\ 0.3458 & 0.3580 & 0.3670 & 0.3809 \\ 0.3467 & 0.3577 & 0.3654 & 0.3769 \end{bmatrix} \begin{bmatrix} -0.1988 & -0.1914 & -0.1828 & -0.1766 \\ -0.1939 & -0.1840 & -0.1746 & -0.1684 \\ -0.1927 & -0.1766 & -0.1668 & -0.1585 \\ -0.1873 & -0.1701 & -0.1590 & -0.1516 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 0.3415 & 0.3348 & 0.3867 & 0.4260 \\ 0.3288 & 0.3381 & 0.3966 & 0.4183 \\ 0.3362 & 0.3476 & 0.3994 & 0.3961 \\ 0.3515 & 0.3579 & 0.3714 & 0.3519 \end{bmatrix} \begin{bmatrix} 0.3897 & 0.3926 & 0.4192 & 0.4422 \\ 0.3922 & 0.3999 & 0.4242 & 0.4474 \\ 0.3935 & 0.4005 & 0.4213 & 0.4373 \\ 0.3947 & 0.3928 & 0.4054 & 0.4169 \end{bmatrix} \begin{bmatrix} 0.1700 & 0.1827 & 0.1971 & 0.2120 \\ 0.1676 & 0.1786 & 0.1923 & 0.2038 \\ 0.1663 & 0.1688 & 0.1796 & 0.1919 \\ 0.1651 & 0.1577 & 0.1672 & 0.1746 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 0.6875 & 0.6878 & 0.6952 & 0.7081 \\ 0.6862 & 0.6874 & 0.6933 & 0.7037 \\ 0.6852 & 0.6864 & 0.6876 & 0.6922 \\ 0.6819 & 0.6807 & 0.6796 & 0.6796 \end{bmatrix} \begin{bmatrix} 0.3160 & 0.3208 & 0.3278 & 0.3302 \\ 0.3313 & 0.3336 & 0.3453 & 0.3491 \\ 0.3519 & 0.3543 & 0.3566 & 0.3602 \\ 0.3679 & 0.3713 & 0.3690 & 0.3690 \end{bmatrix} \begin{bmatrix} -0.1916 & -0.1932 & -0.1907 & -0.1899 \\ -0.1878 & -0.1870 & -0.1828 & -0.1865 \\ -0.1738 & -0.1730 & -0.1722 & -0.1725 \\ -0.1615 & -0.1586 & -0.1594 & -0.1594 \end{bmatrix}$$

14.4 Given the RGB components of a 4×4 color image, find the YCbCr components of its YCbCr model.

(i)

$$\begin{bmatrix} 142 & 142 & 150 & 149 \\ 144 & 144 & 143 & 163 \\ 145 & 144 & 152 & 157 \\ 147 & 153 & 155 & 126 \end{bmatrix} \begin{bmatrix} 15 & 15 & 24 & 24 \\ 17 & 17 & 12 & 35 \\ 18 & 18 & 16 & 24 \\ 18 & 21 & 20 & 18 \end{bmatrix} \begin{bmatrix} 44 & 44 & 49 & 48 \\ 46 & 47 & 37 & 59 \\ 47 & 46 & 43 & 51 \\ 48 & 49 & 42 & 28 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 227 & 230 & 230 & 229 \\ 224 & 215 & 214 & 215 \\ 208 & 207 & 208 & 209 \\ 191 & 197 & 198 & 195 \end{bmatrix} \begin{bmatrix} 0 & 6 & 5 & 3 \\ 4 & 0 & 0 & 0 \\ 2 & 2 & 2 & 1 \\ 0 & 6 & 6 & 3 \end{bmatrix} \begin{bmatrix} 27 & 32 & 32 & 32 \\ 30 & 20 & 19 & 22 \\ 19 & 15 & 14 & 16 \\ 13 & 15 & 14 & 13 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 226 & 226 & 226 & 225 \\ 226 & 226 & 225 & 223 \\ 226 & 226 & 225 & 222 \\ 226 & 226 & 225 & 223 \end{bmatrix} \begin{bmatrix} 239 & 239 & 238 & 235 \\ 239 & 239 & 238 & 235 \\ 239 & 239 & 238 & 235 \\ 239 & 239 & 238 & 236 \end{bmatrix} \begin{bmatrix} 219 & 221 & 221 & 220 \\ 219 & 221 & 220 & 218 \\ 219 & 221 & 220 & 218 \\ 219 & 221 & 221 & 219 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 64 & 64 & 71 & 71 \\ 66 & 66 & 62 & 81 \\ 67 & 67 & 67 & 73 \\ 68 & 71 & 70 & 60 \end{bmatrix} \begin{bmatrix} 122 & 122 & 120 & 120 \\ 122 & 122 & 120 & 120 \\ 122 & 122 & 120 & 120 \\ 122 & 121 & 118 & 116 \end{bmatrix} \begin{bmatrix} 182 & 182 & 182 & 181 \\ 182 & 182 & 184 & 183 \\ 182 & 181 & 186 & 184 \\ 183 & 184 & 186 & 175 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 77 & 81 & 81 & 79 \\ 78 & 73 & 73 & 73 \\ 72 & 72 & 72 & 72 \\ 66 & 71 & 71 & 69 \end{bmatrix} \begin{bmatrix} 106 & 106 & 107 & 107 \\ 107 & 105 & 105 & 106 \\ 105 & 103 & 103 & 104 \\ 105 & 104 & 103 & 104 \end{bmatrix} \begin{bmatrix} 226 & 225 & 225 & 225 \\ 223 & 221 & 221 & 221 \\ 217 & 217 & 218 & 218 \\ 211 & 211 & 212 & 212 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 216 & 216 & 216 & 214 \\ 216 & 216 & 215 & 213 \\ 216 & 216 & 215 & 213 \\ 216 & 216 & 215 & 214 \end{bmatrix} \begin{bmatrix} 121 & 122 & 122 & 123 \\ 121 & 122 & 122 & 122 \\ 121 & 122 & 122 & 122 \\ 121 & 122 & 122 & 122 \end{bmatrix} \begin{bmatrix} 124 & 124 & 124 & 125 \\ 124 & 124 & 124 & 124 \\ 124 & 124 & 124 & 124 \\ 124 & 124 & 124 & 124 \end{bmatrix}$$

14.5 An 8-bit gray level image is to be converted to a color image by pseudocoloring. What is the color map for the given color assigment.

(i)

Gray level range	0	1-64	65-128	129-192	255
color_map	Cyan	Green	Blue	magenta	Red

(ii)

Gray level range	0	1-64	65-128	129-192	255
color_map	Yellow	Red	magenta	Cyan	Green

(iii)

Gray level range	0	1-64	65-128	129-192	255
color_map	magenta	Yellow	Red	Cyan	Blue

(i)

$$color\_map1 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(ii)

$$color\_map1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(iii)

$$color\_map1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

14.6 Given the RGB components of a 4×4 color image, find its complement.

(i)

$$\begin{bmatrix} 76 & 69 & 93 & 147 \\ 129 & 88 & 77 & 163 \\ 67 & 114 & 142 & 163 \\ 100 & 92 & 160 & 165 \end{bmatrix} \begin{bmatrix} 96 & 90 & 121 & 179 \\ 153 & 113 & 102 & 180 \\ 99 & 142 & 169 & 191 \\ 132 & 122 & 188 & 186 \end{bmatrix} \begin{bmatrix} 50 & 47 & 58 & 82 \\ 78 & 59 & 49 & 100 \\ 34 & 69 & 96 & 111 \\ 57 & 54 & 109 & 104 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 69 & 114 & 59 & 41 \\ 78 & 82 & 141 & 56 \\ 138 & 47 & 81 & 106 \\ 128 & 131 & 52 & 79 \end{bmatrix} \begin{bmatrix} 81 & 131 & 82 & 65 \\ 90 & 99 & 167 & 82 \\ 157 & 61 & 104 & 136 \\ 153 & 150 & 70 & 106 \end{bmatrix} \begin{bmatrix} 42 & 73 & 40 & 27 \\ 49 & 48 & 93 & 37 \\ 93 & 30 & 53 & 68 \\ 83 & 88 & 35 & 51 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 128 & 164 & 139 & 142 \\ 174 & 199 & 178 & 163 \\ 179 & 204 & 204 & 169 \\ 101 & 174 & 202 & 149 \end{bmatrix} \begin{bmatrix} 160 & 190 & 169 & 171 \\ 197 & 215 & 199 & 189 \\ 200 & 217 & 218 & 193 \\ 124 & 189 & 216 & 175 \end{bmatrix} \begin{bmatrix} 74 & 93 & 71 & 88 \\ 102 & 105 & 99 & 105 \\ 90 & 100 & 109 & 100 \\ 42 & 70 & 104 & 82 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 179 & 186 & 162 & 108 \\ 126 & 167 & 178 & 92 \\ 188 & 141 & 113 & 92 \\ 155 & 163 & 95 & 90 \end{bmatrix} \begin{bmatrix} 159 & 165 & 134 & 76 \\ 102 & 142 & 153 & 75 \\ 156 & 113 & 86 & 64 \\ 123 & 133 & 67 & 69 \end{bmatrix} \begin{bmatrix} 205 & 208 & 197 & 173 \\ 177 & 196 & 206 & 155 \\ 221 & 186 & 159 & 144 \\ 198 & 201 & 146 & 151 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 186 & 141 & 196 & 214 \\ 177 & 173 & 114 & 199 \\ 117 & 208 & 174 & 149 \\ 127 & 124 & 203 & 176 \end{bmatrix} \begin{bmatrix} 174 & 124 & 173 & 190 \\ 165 & 156 & 88 & 173 \\ 98 & 194 & 151 & 119 \\ 102 & 105 & 185 & 149 \end{bmatrix} \begin{bmatrix} 213 & 182 & 215 & 228 \\ 206 & 207 & 162 & 218 \\ 162 & 225 & 202 & 187 \\ 172 & 167 & 220 & 204 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 127 & 91 & 116 & 113 \\ 81 & 56 & 77 & 92 \\ 76 & 51 & 51 & 86 \\ 154 & 81 & 53 & 106 \end{bmatrix} \begin{bmatrix} 95 & 65 & 86 & 84 \\ 58 & 40 & 56 & 66 \\ 55 & 38 & 37 & 62 \\ 131 & 66 & 39 & 80 \end{bmatrix} \begin{bmatrix} 181 & 162 & 184 & 167 \\ 153 & 150 & 156 & 150 \\ 165 & 155 & 146 & 155 \\ 213 & 185 & 151 & 173 \end{bmatrix}$$

14.7 Given the RGB components of a 4×4 color image with intensity values varying from 0 to 255, find the histogram-equalized version of its intensity component of its HSI model.

(i)

$$\begin{bmatrix} 81 & 80 & 81 & 90 \\ 78 & 73 & 84 & 100 \\ 75 & 79 & 93 & 93 \\ 74 & 84 & 102 & 91 \end{bmatrix} \begin{bmatrix} 108 & 107 & 107 & 116 \\ 105 & 99 & 110 & 129 \\ 101 & 106 & 120 & 122 \\ 100 & 110 & 130 & 120 \end{bmatrix} \begin{bmatrix} 38 & 36 & 37 & 45 \\ 37 & 31 & 41 & 55 \\ 35 & 38 & 50 & 49 \\ 34 & 43 & 59 & 49 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} 58 & 110 & 89 & 25 \\ 61 & 106 & 89 & 81 \\ 64 & 83 & 130 & 157 \\ 62 & 88 & 173 & 162 \end{bmatrix} \begin{bmatrix} 79 & 129 & 112 & 39 \\ 82 & 124 & 111 & 83 \\ 86 & 100 & 130 & 139 \\ 85 & 92 & 149 & 134 \end{bmatrix} \begin{bmatrix} 30 & 73 & 45 & 7 \\ 29 & 66 & 49 & 45 \\ 27 & 43 & 70 & 90 \\ 27 & 39 & 97 & 94 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} 114 & 127 & 144 & 114 \\ 129 & 136 & 131 & 84 \\ 144 & 137 & 124 & 76 \\ 143 & 139 & 145 & 120 \end{bmatrix} \begin{bmatrix} 84 & 99 & 139 & 128 \\ 106 & 120 & 135 & 95 \\ 127 & 135 & 133 & 86 \\ 143 & 148 & 155 & 130 \end{bmatrix} \begin{bmatrix} 69 & 68 & 71 & 55 \\ 98 & 71 & 57 & 39 \\ 111 & 80 & 64 & 34 \\ 89 & 90 & 96 & 76 \end{bmatrix}$$

(i) The intensity component and its equalized version are

$$\begin{bmatrix} 76 & 74 & 75 & 84 \\ 73 & 68 & 78 & 95 \\ 70 & 74 & 88 & 88 \\ 69 & 79 & 97 & 87 \end{bmatrix} \begin{bmatrix} 128 & 96 & 112 & 175 \\ 64 & 16 & 143 & 239 \\ 48 & 96 & 223 & 223 \\ 32 & 159 & 255 & 191 \end{bmatrix}$$

(ii) The intensity component and its equalized version are

$$\begin{bmatrix} 56 & 104 & 82 & 24 \\ 57 & 99 & 83 & 70 \\ 59 & 75 & 110 & 129 \\ 58 & 73 & 140 & 130 \end{bmatrix} \begin{bmatrix} 32 & 191 & 143 & 16 \\ 48 & 175 & 159 & 96 \\ 80 & 128 & 207 & 223 \\ 64 & 112 & 255 & 239 \end{bmatrix}$$

(iii) The intensity component and its equalized version are

$$\begin{bmatrix} 89 & 98 & 118 & 99 \\ 111 & 109 & 108 & 73 \\ 127 & 117 & 107 & 65 \\ 125 & 126 & 132 & 109 \end{bmatrix} \begin{bmatrix} 48 & 64 & 191 & 80 \\ 159 & 143 & 112 & 32 \\ 239 & 175 & 96 & 16 \\ 207 & 223 & 255 & 143 \end{bmatrix}$$